# BEAM-WAVE INTERACTION IN HELIX-TRAVELLING-WAVE TUBES THROUGH

$$\vec{E} = -\nabla V; \nabla.\vec{E} = \frac{\rho}{\varepsilon}; \bar{J} = \rho\vec{v}; \nabla.\vec{J} + \frac{\partial\rho}{\partial t} = 0$$


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“Ali said to Kaamil (al-insān al-kāmil):

**Knowledge is better than wealth. Knowledge protects you while you have to protect your wealth. Knowledge is a judge, while wealth has to be judged on. Wealth decreases when it is expended, while knowledge purifies when it is given**.”

“Ali said in a poem:

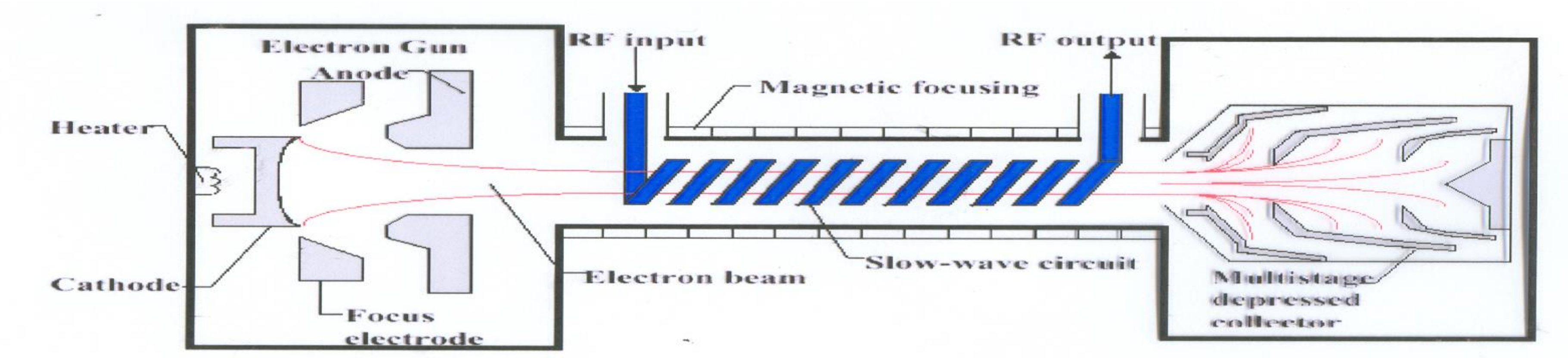
**Succeed with knowledge and live energetically forever; men are all dead, only the possessors of knowledge are truly alive**."

**“What man ‘learns’ is really what he discovers by taking the cover off his own soul, which is a mine of infinite knowledge.”**

## Dedication

**Professor N. B. Chakrabarty**, who at the Indian Institute of Technology, Kharagpur, India, mentored my doctoral research in the area of nonlinear Eulerian hydrodynamic analysis of double-stream amplifier (Haeff tube) and beam-plasma amplifier.

**Professor Alexander Scott Gilmour, Jr.**, who responded to my request and authored a paper entitled "An overview of my efforts to bridge the gap in the microwave tube area between what universities provide and what the industry needs" in the Special Issue on "Microwave Tubes and Applications" in the Journal of Electromagnetic Waves and Applications (Taylor and Francis) (issue 17, vol. 31, 2017), which I guest-edited. Dr. Gilmour is the recipient of J. R. Pierce award in 2018. I was one of his references for this award.




Travelling-Wave Tube

- Two basic constituents:
  - Electron beam
  - Electromagnetic interaction structure

- Principal parts:
  - Electron gun: beam formation
  - Focusing structure: beam confinement
  - Collector: collection of spent beam
  - Slow-wave structure (SWS)
  - RF input and output couplers
  - Attenuator

“We learn from history that we do not learn from history.” — GWF Hegel

From the historical time line we know
who invented what.

“Success has many fathers, but failure is an orphan.”

Who did invent transmission and reception of radio waves?
(a) G. Marconi
(b) A. S. Popov
(c) J. C. Bose
(d) M. N. Saha

Answer: (c) J. C. Bose

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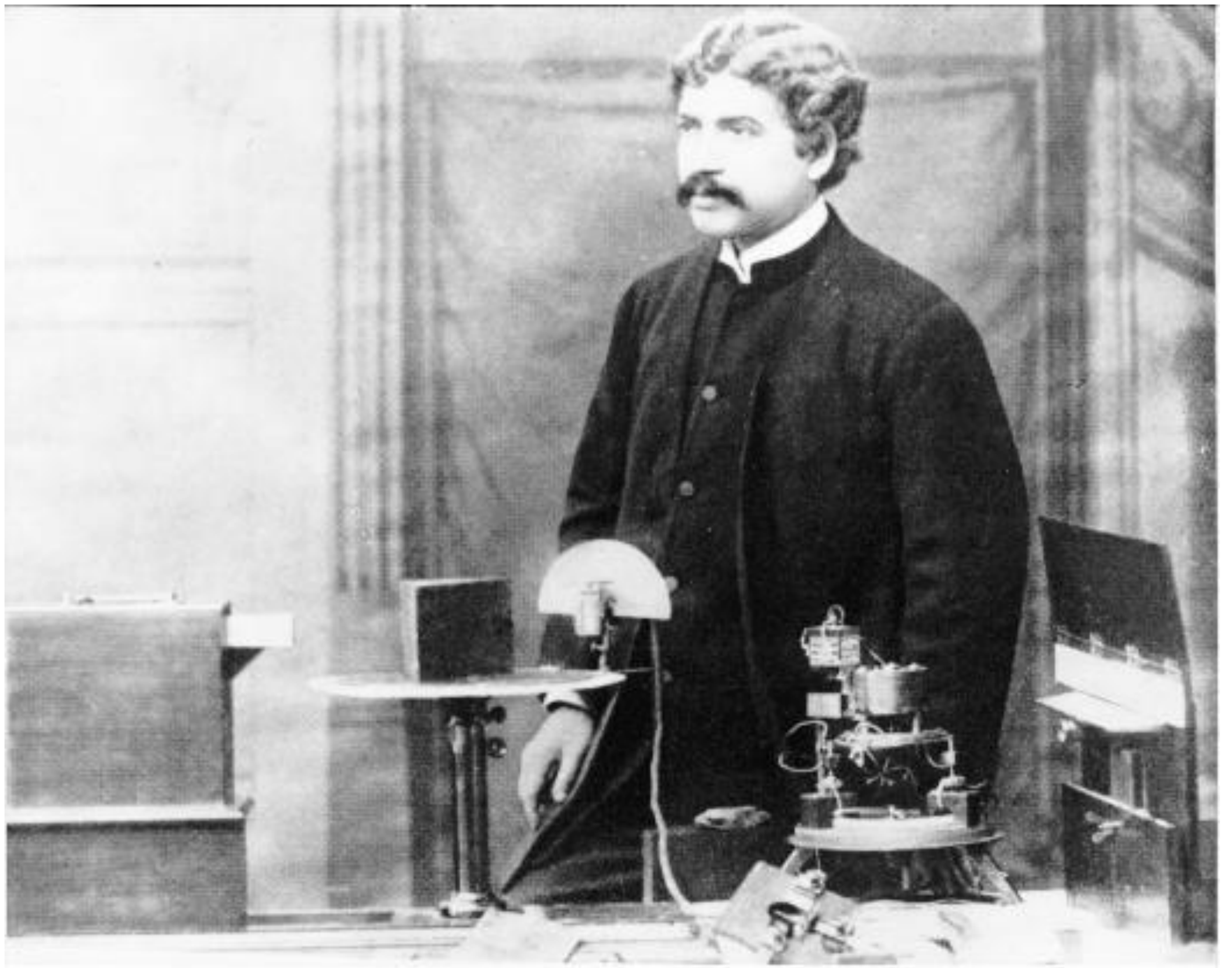
Who did invent travelling-wave tube?
(a) R. Kompfner
(b) N. E. Lindenblad
(c) J. R. Pierce
(d) A. Haeff

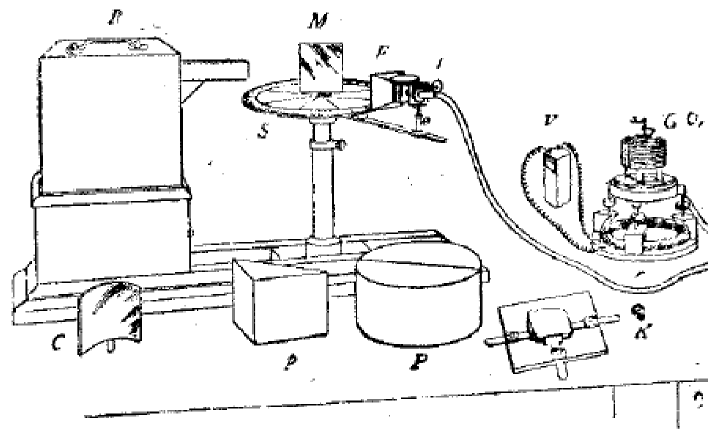
Answer: (d) A. Haeff

## J.C. Bose (1858-1937) at the Royal Institution, London, 1897



# J C Bose


IEEE Milestone Plaque for Sir JC Bose




R, radiator; S, spectrometer-circle; M, plane mirror; C, cylindrical mirror; p, totally reflecting prism; P, semi-cylinders; K, crystal-holder; F, collecting funnel attached to the spiral spring receiver; t, tangent screw, by which the receiver is rotated; V, voltaic cell; r, circular rheostat; G, galvanometer.

By 1895, Sir. J. C. Bose made the first public demonstration of radio waves in the Kolkata town hall. Details of the apparatus used are vague, but at a distance of 75 feet, he remotely rang an electric bell and ignited a small charge of gunpowder. He called it *Adrisya Alok,* or "Invisible Light". The frequency of operation is nearly 60 GHz. He termed horn antenna as collecting funnel.

*Courtesy: C Subhradeep (CEERI)*

## IEEE MILESTONE IN ELECTRICAL ENGINEERING AND COMPUTING

**First Millimeter-Wave Communication Equipment by JC Bose, 1894-1896**

**Sir Jagadish Chadra Bose, in 1895, first demonstrated at Presidency College, Calcutta, India, transmission and reception of electromagnetic waves at 60 GHz over a distance of 23 meters, through two intercepting walls by remotely ringing a bell and detonating gunpowder. For this communication system, Bose developed entire millimetre-wave components such as: a spark transmitter, coherer, dielectric lens, polarizer, horn antenna and cylindrical diffraction grating.**

**September 2012**

**IEEE Monogram**

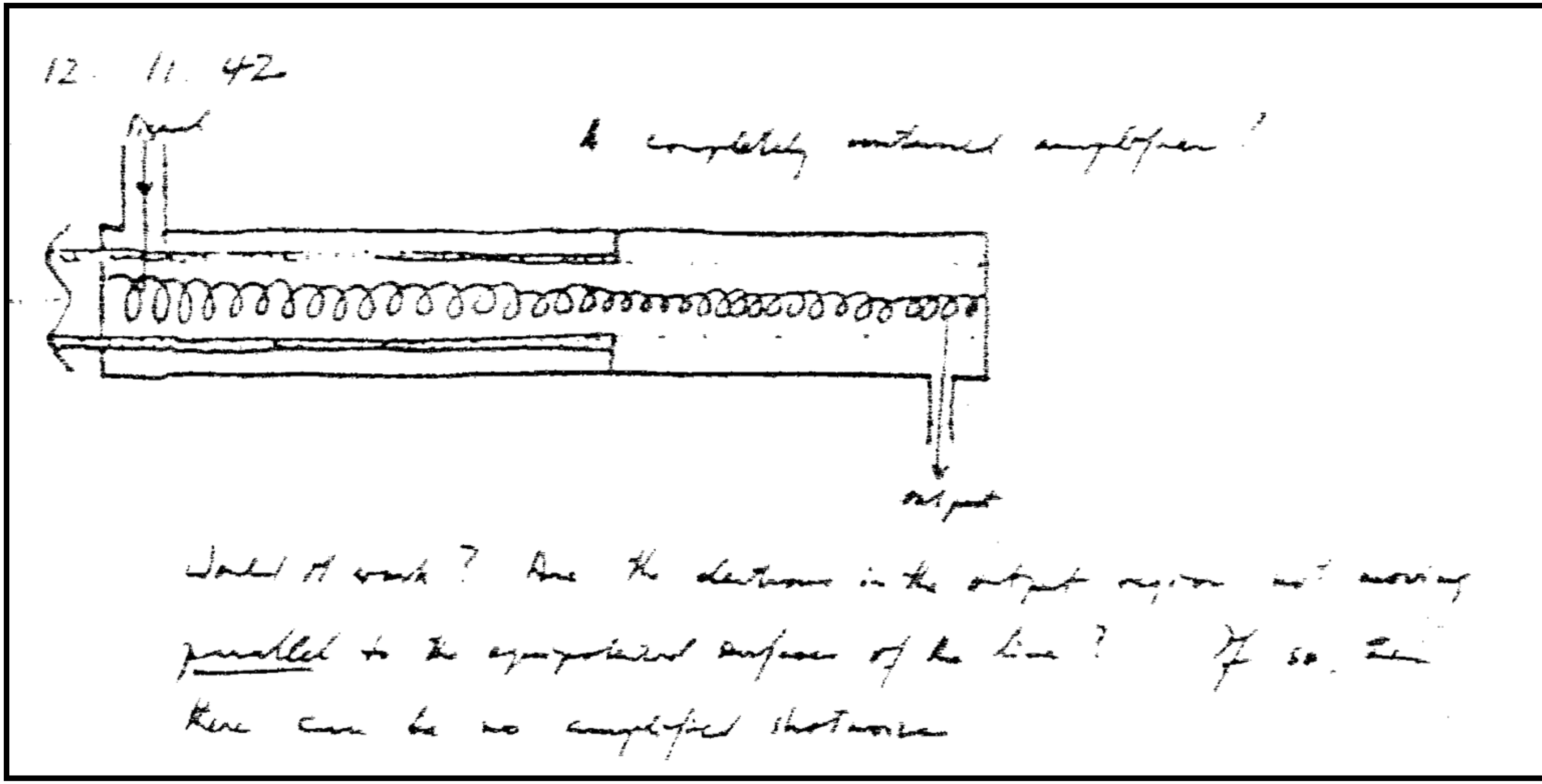
*Courtesy: C Subhradeep (CEERI)*

“In 1895 Bose gave his first public demonstration of electromagnetic waves, using them to ring a bell remotely and to explode some gunpowder. In 1896 the Daily Chronicle of England reported: *"The inventor (J.C. Bose) has transmitted signals to a distance of nearly a mile and herein lies the first and obvious and exceedingly valuable application of this new theoretical marvel."*

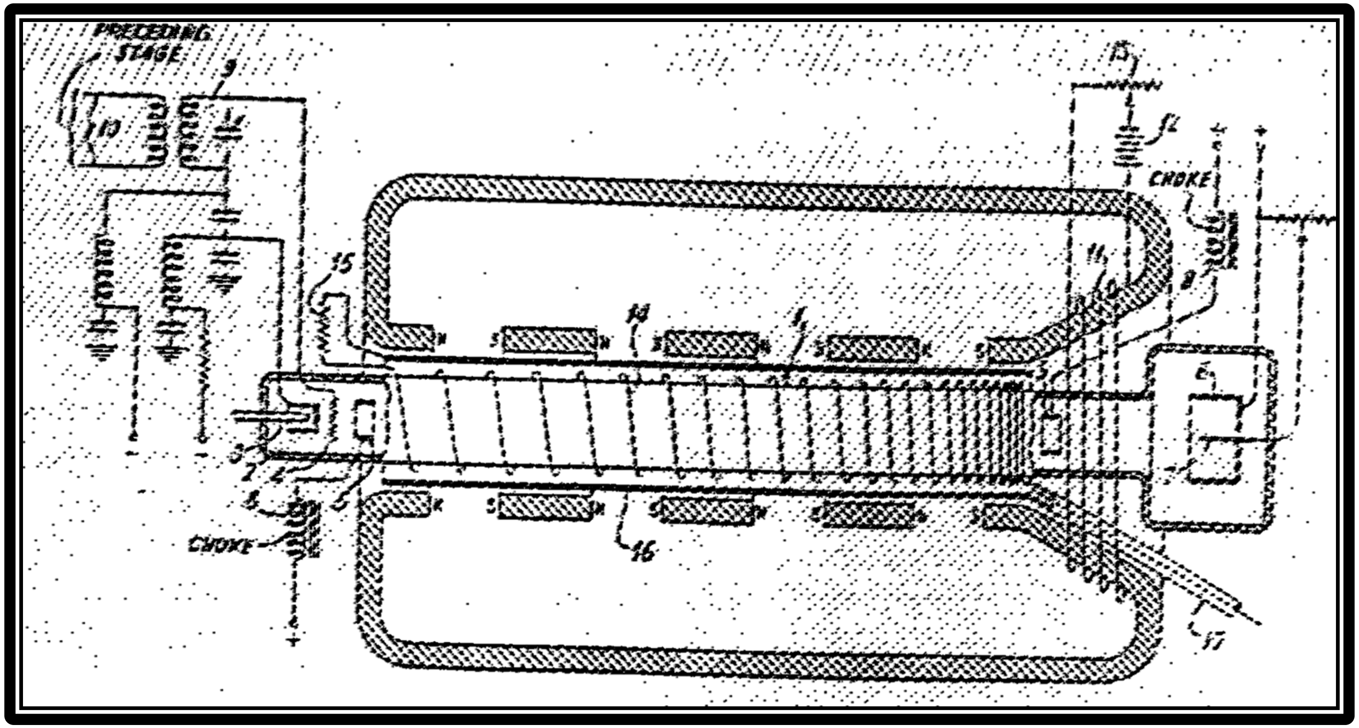
“Popov in Russia was doing similar experiments, but had written in December 1895 that he was still entertaining the hope of remote signaling with radio waves.”

“The first successful wireless signaling experiment by Marconi on Salisbury Plain in England was not until May 1897.”

Source: D. T. Emerson, “The work of Jagadis Chunder Bose: 100 years of mm-wave research,” *IEEE Trans. Microwave Th. Tech.*, December 1997, 45, No. 12 (2267-2273 ).

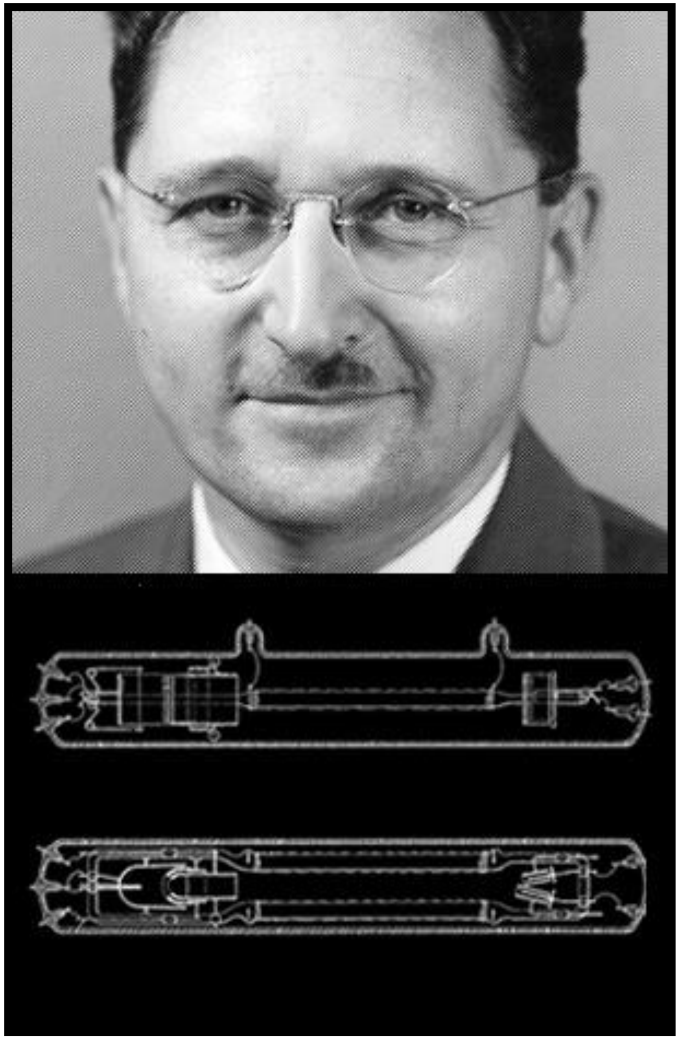



Sketch of the travelling-wave tube from R. Kompfner’s note book (1942). The helix pitch is tapered for velocity re-synchronization. (Fig. 12.2 of the book: A.S. Gilmour, “Klystrons, Traveling Wave Tubes, Magnetrons, Crossed-Field Amplifiers, and Gyrotrons,” (Artech House, Norwood, 2011))




N. E. Lindenblad's travelling-wave tube amplification at 390 MHz over a 30 MHz band (U. S. Patent 2,300,052, filed on May 4, 1940.
(Fig. 12.1 of the book: A.S. Gilmour, "Klystrons, Traveling Wave Tubes, Magnetrons, Crossed-Field Amplifiers, and Gyrotrons," (Artech House, Norwood, 2011))

Helix wound around the outside the glass envelope. Signal applied to the grid of the electron gun (also applied to the helix in other experiments). Series of permanent magnets (non-periodic). The helix pitch is tapered for velocity re-synchronization.



“The patent Andrei Haeff filed in 1933 for a primitive type of traveling-wave tube has been largely ignored.”

*Courtesy SK Datta*

- Haeff invented TWT in 1933.

(Haeff also invented the double-stream amplifier (Haeff tube), in which two electron beams with slightly different DC velocities are intimately mixed such that the slow space-charge wave of the faster beam couples to the fast space-charge wave of the slower beam resulting in growing waves).

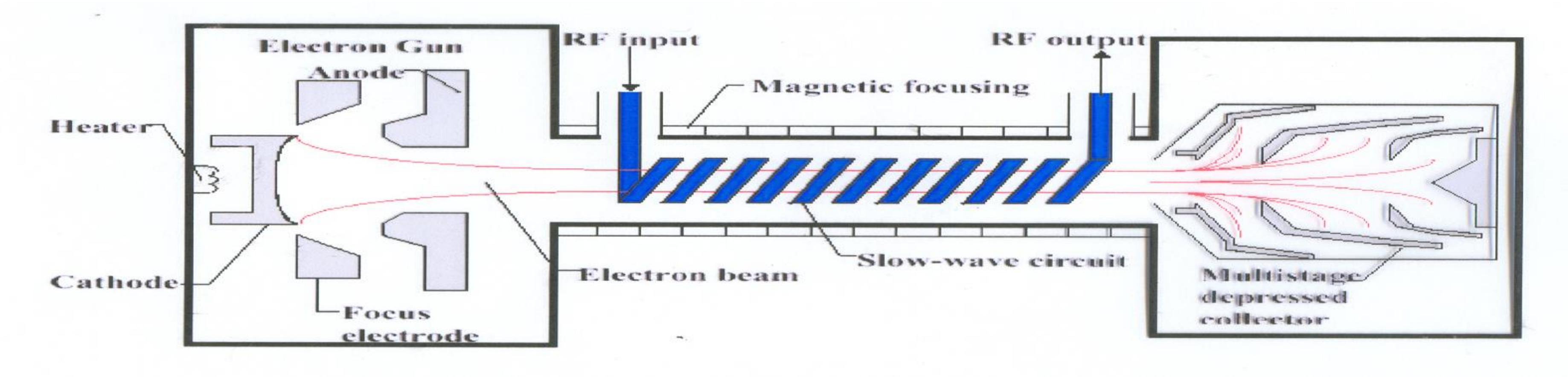
- Lindenblad invented TWT in 1940.
- Kompfner invented TWT, however, not before 1943.
- Pierce and Field significantly contributed to the development of the TWT, however, not before 1947.

## Scope of This Presentation

- Historical timeline of the invention of the travelling-wave tube
- Electron bunching and requirement of near-synchronism
- Space-charge waves and coupling to structure wave
- Gain equation
- Hot attenuation
- Johnson’s start oscillation condition
- Some broadbanding aspects

## Prerequisite

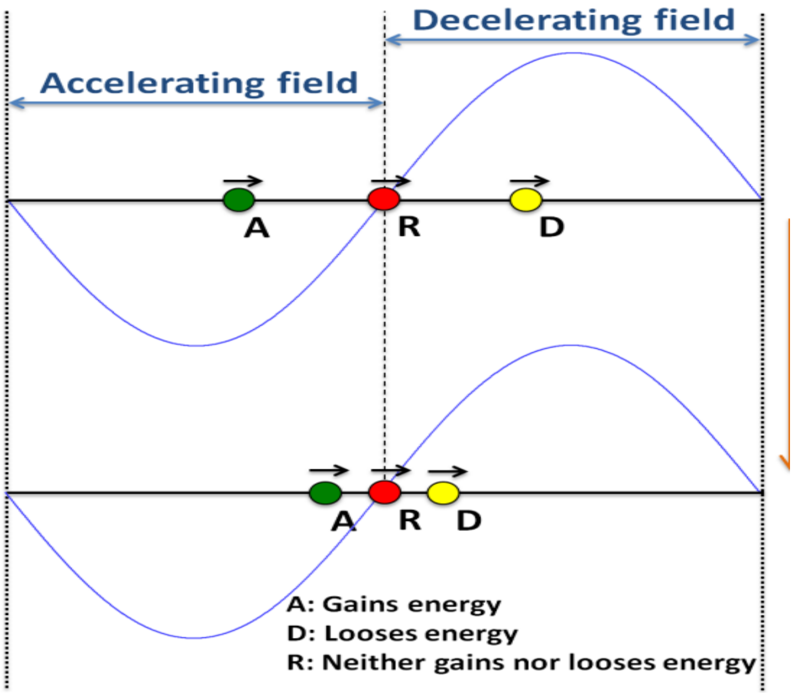
$$\bar{J} = \rho\vec{v};\ \vec{E} = -\nabla V;\ \nabla.\vec{J} + \frac{\partial\rho}{\partial t} = 0;\ \nabla.\vec{E} = \frac{\rho}{\varepsilon}$$




The TWT is a growing-wave device (tube). It is an amplifier.
It is a slow-wave tube in which the interaction structure is a slow-wave structure (such as helix) that supports RF waves of phase velocity less than the speed *c* of light.
The applied DC magnetic field confines the electron beam in the device; it does not take place in beam-wave interaction.

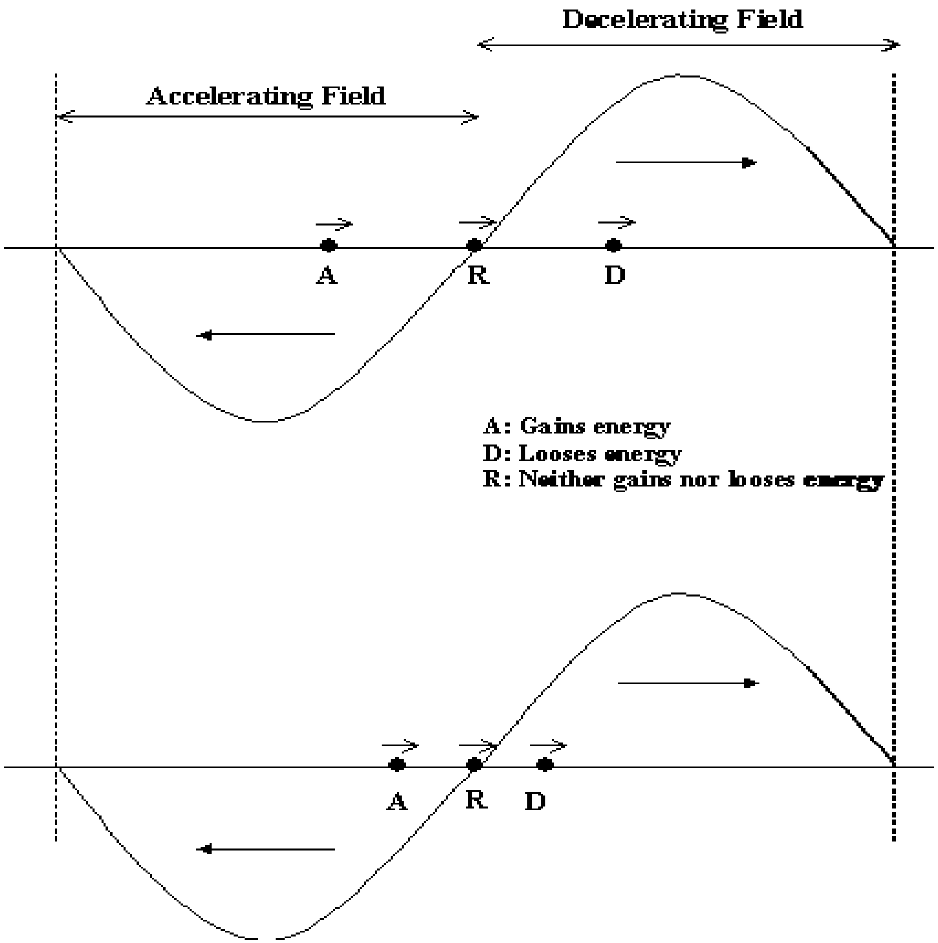
It belongs to the class of linear beam, O-type, Cerenkov radiation type of vacuum electron devices/ microwave tubes. In this tube, the bunched electrons transfer their axial kinetic energy to RF waves.

## Axial Bunching in a Travelling-Wave Tube




Bunching of typically two electrons ‘A’ and ‘D’ subjected to the accelerating and decelerating RF electric fields, respectively, in the interaction region of a TWT around a reference electron ‘R’ that experiences no such fields.

Electrons are *bunched* though there is *no net energy transfer* from the electron beam to RF waves.




$$v_0 = v_{ph}$$

Exact synchronism: no net energy transfer between the beam to RF waves

$$v_0 \underset{\sim}{>} v_{ph}$$

Near-synchronism: net energy transfer from the beam to RF waves

## Millimetre-Wave Consideration in Conventional Microwave Tubes

B. N. Basu, *Electromagnetic Theory and Applications in Beam-wave Electronics* (World Scientific, Singapore,1996)
Reduction of structure size
Reduction of beam radius

Larger magnetic field for beam confinement for

- Smaller beam radius $b$
- Larger beam current $I_0$
- Smaller beam voltage $V_0$
- Larger beam perveance $I_0 / (V_0)^{3/2}$

$$B^2_{\text{Brillouin}} = \frac{\sqrt{2} I_0}{\pi \varepsilon_0 |\eta|^{3/2} V_0^{1/2} b^2}$$

Heavy solenoids or advanced magnetic materials are required
Larger cathode emission densities entailing the risk of cathode life

See Chapter 7 in B. N. Basu, *Electromagnetic Theory and Applications in Beam-wave electronics* (World Scientific, Singapore, 1996).

Higher beam voltage can increase the beam power and also reduce the required magnetic field but is associated with a reduced beam perveance making it difficult to contain thermal electrons, and has limitation arising from backward-wave oscillation in wideband helix TWTs.

Lower beam current can reduce magnetic field but it reduces the beam power and is associated with a reduced beam perveance making it difficult to contain thermal electrons.

Tight tolerances are required for tiny interaction structures.
Thermal management becomes difficult.
Pressure fitting, instead of more effective brazed-helix technology, becomes difficult to implement.
Special thermally conducting materials, such as Type II-A diamond, for dielectric helix-supports may be used.
Plasma spraying of beryllia on the surface of the helix can be done.

A.S. Gilmour, Jr., *Microwave Tubes* (Artech House, Washington, 1986)

# COMMERCIALLY AVAILABLE MILLIMETER-WAVE TWT'S

## Communication

| Type | Frequency Range | Power Output | Duty Cycle | Saturated Gain |
|---|---|---|---|---|
| 814H | 91.0-96.0 GH | 0.10 kW | CW | 25 dB |

## Space

| Type | Frequency Range | Power Output | Duty Cycle | Saturated Gain |
|---|---|---|---|---|
| 944H | 42.0-42.5 GHz | 100 W | CW | 44 dB |

## Pulsed Radar And ECM

| Type | Frequency Range | Power Output | Duty Cycle | Saturated Gain |
|---|---|---|---|---|
| 982H | 93.0-95.0 GHz | 12kW | 0.5 | 50dB |

## CW Radar and ECM

| Type | Frequency Range | Power Output | Duty Cycle | Saturated Gain |
|---|---|---|---|---|
| 920H | 59.7-60.3 GHz | 0.05 kW | CW | 35 dB |

**B. N. Basu, *Electromagnetic Theory and Applications in Beam-wave electronics* (World Scientific, Singapore, New Jersey, London, Hong Kong, 1996).**

The above book will provide a self-contained analysis of the gain equation of a TWT obtained by combining the circuit and electronic equations taking due care to involve the interaction impedance instead of the characteristic impedance in the circuit equation, for the sake of rigour in the analysis.

## Space-Charge Waves

The mechanism of interaction of a slow-wave device such as the TWT is based on the property of an electron beam to support space-charge waves.

The electron beam supports two space-charge waves—Hahn and Ramo fast and slow space-charge waves with phase velocities greater and slower, respectively, than the DC electron beam velocity

$$\vec{J} = \rho\vec{v}$$

(current density equation)

$$\nabla.\vec{J} + \frac{\partial \rho}{\partial t}$$

(continuity equation)

$$\nabla.\vec{E} = \frac{\rho}{\varepsilon}$$

(Poisson's equation)

$$\left.\begin{aligned} J &= \rho v \\ J &= J_0 + J_1 \\ v &= v_0 + v_1 \\ \rho &= \rho_0 + \rho_1 \end{aligned}\right\} \Rightarrow$$

$$J = J_0 + J_1 = (\rho_0 + \rho_1)(v_0 + v_1) = \rho_0 v_0 + \rho_0 v_1 + v_0 \rho_1 + \rho_1 v_1$$

$$J_0 + J_1 = \rho_0 v_0 + \rho_0 v_1 + v_0 \rho_1 + \rho_1 v_1 \Leftarrow J_0 = \rho_0 v_0$$

$$\Downarrow$$

$$J_1 = \rho_0 v_1 + v_0 \rho_1 + \rho_1 v_1 \cong \rho_0 v_1 + v_0 \rho_1$$

$$\Downarrow$$

$$\frac{\partial J_1}{\partial z} = \rho_0 \frac{\partial v_1}{\partial z} + v_0 \frac{\partial \rho_1}{\partial z} \Leftarrow \frac{\partial J_1}{\partial z} + \frac{\partial \rho_1}{\partial t} = 0 \Leftarrow \nabla.\vec{J} + \frac{\partial \rho}{\partial t}$$

$$\Downarrow$$

$$\left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right)\rho_1 = -\rho_0 \frac{\partial v_1}{\partial z} \Leftarrow \boxed{D = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}}$$

$$\Downarrow$$

$$D\rho_1 = -\rho_0 \frac{\partial v_1}{\partial z}$$

$$D\rho_1 = -\rho_0 \frac{\partial v_1}{\partial z}$$ (rewritten)

$$D = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}$$

$$\nabla.\vec{E} = \frac{\rho}{\varepsilon}$$

(Poisson's equation)

$$D^2\rho_1 = -\rho_0 D \frac{\partial v_1}{\partial z} = -\rho_0 \frac{\partial}{\partial z} D v_1$$

$$Dv_1 = \eta E_z$$ (force equation)

$$D^2\rho_1 = -\rho_0 \frac{\partial}{\partial z} D v_1 = -\rho_0 \frac{\partial}{\partial z} \eta E_z = -\eta \rho_0 \frac{\partial E_z}{\partial z}$$

$$\frac{\partial E_z}{\partial z} = -\frac{\rho_1}{\varepsilon_0}$$

$$D^2\rho_1 = -\eta \rho_0 \frac{\partial E_z}{\partial z} = -\eta \rho_0 \frac{\rho_1}{\varepsilon_0} = -\frac{\eta \rho_0}{\varepsilon_0} \rho_1$$

$$D^2\rho_1 = -\frac{\eta \rho_0}{\varepsilon_0} \rho_1 = -\frac{|\eta||\rho_0|}{\varepsilon_0} \rho_1 = -\omega_p^2 \rho_1$$

$$\omega_p^2 = \frac{|\eta||\rho_0|}{\varepsilon_0}$$

$$D^2 = -\omega_p^2$$

$$D = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}$$

$$D = \pm j\omega_p$$

$$\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} = \pm j\omega_p$$

$$\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} = \pm j\omega_p$$

Taking RF dependence as $\exp(j\omega t - \Gamma z)$

$$j\omega - v_0 \Gamma = \pm j\omega_p$$

$\Gamma = j\beta$

$$j\omega - j\beta = j(\omega - v_0\beta) = \pm j\omega_p$$

$$\omega - v_0\beta = \pm\omega_p$$

$$\beta = \frac{\omega \mp \omega_p}{v_o}$$

$$v_{ph} = \frac{\omega}{\beta} = \frac{\omega}{\frac{\omega \mp \omega_p}{v_o}} = \frac{\omega}{\omega \mp \omega_p} v_o$$

The upper sign refers to the fast space-charge wave on the electron beam with phase velocity greater than the DC beam velocity.

The lower sign refers to the slow space-charge wave on the electron beam with phase velocity less than the DC beam velocity.

## Space-Charge Waves

$$\omega - \beta v_0 \mp \omega_p = 0$$

$$\beta = \beta_e \mp \beta_p$$

$$\beta = \frac{\omega}{v_p} = \frac{\omega}{v_0} \mp \frac{\omega_p}{v_0} = \frac{\omega \mp \omega_p}{v_0}$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega \mp \omega_p} v_0$$

$$\beta_e = \frac{\omega}{v_0} \qquad \beta_p = \frac{\omega_p}{v_0} \qquad \omega_p = \left(\frac{|\eta|\rho_0}{\varepsilon_0}\right)^{1/2}$$

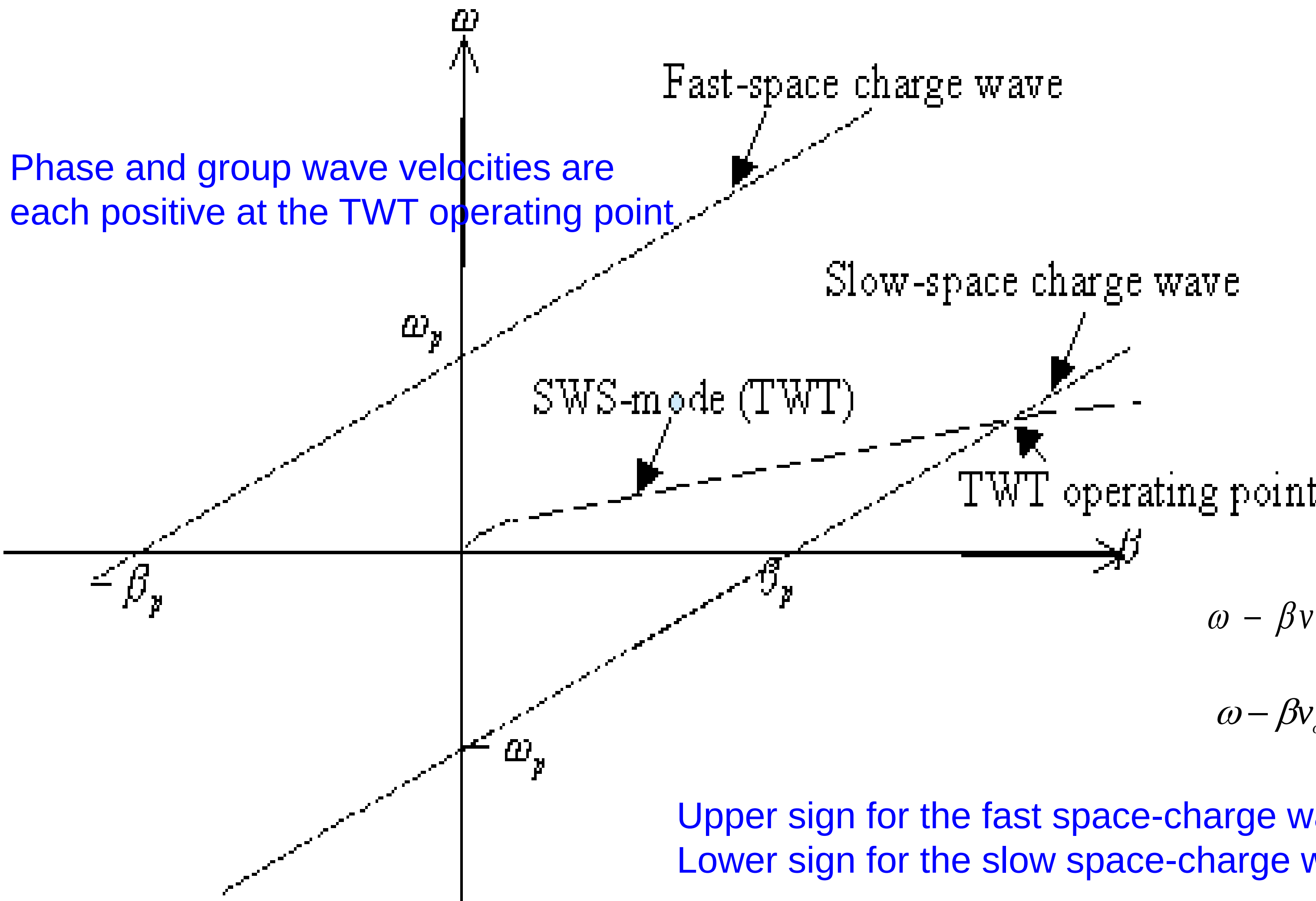
Upper sign for the fast wave and lower sign for the slow wave

**Amplification of space-charge waves takes place on an**

- electron beam of uniform diameter in a resistive-wall waveguide.
- electron beam in a rippled-wall (varying diameter) conducting-wall waveguide.
- electron beam of varying diameter in a conducting smooth-wall waveguide.
- electron beam mixed with another beam of a slightly different DC electron beam velocity (two-stream amplifier/Haeff tube).
- electron beam penetrating through a plasma (beam-plasma amplifier).
- electron beam interacting with RF waves supported by a slow-wave structure (TWT).

## Intersection between slow space-charge and circuit/structure waves at the TWT operating point in the dispersion plot

Phase and group wave velocities are each positive at the TWT operating point




$$\omega - \beta v_0 \mp \omega_p = 0$$

$$\omega - \beta v_o = \pm \omega_p$$

Upper sign for the fast space-charge wave
Lower sign for the slow space-charge wave

## Some TWT features

- Cerenkov radiation type
- Magnetic field for beam confinement
- Larger magnetic field at higher frequencies for beam confinement
- Conversion of axial beam kinetic energy
- Axial non-relativistic electron bunching
- Near-synchronism between DC beam velocity and circuit phase velocity
- Electron beam velocity to be slightly greater than RF phase velocity
- Slow space-charge wave on electron beam to couple to forward circuit wave
- Space-charge-limited operation
- Pierce gun
- Smaller structure sizes at higher frequencies
- BWO absolute instability above the pi-point frequency

## Pierce’s theory for the beam-present dispersion relation of a travelling-wave tube and for its gain

Pierce’s elegant method is to find the ratio $\frac{V}{i}$

of the circuit voltage $V$ to RF beam current $i$

by the circuit theory approach and find the same ratio by the electronic equation approach

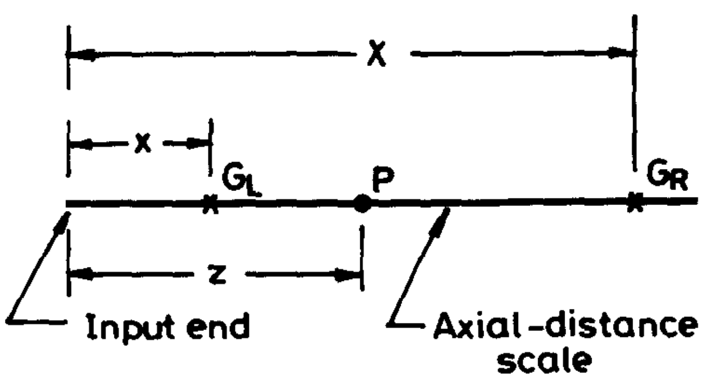
and then equate these ratios found by these two approaches.

## Circuit Equation

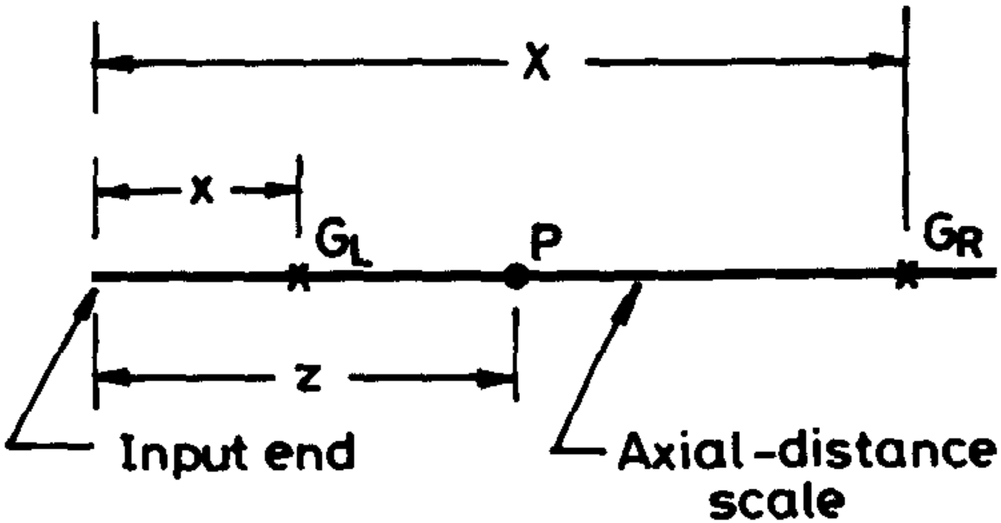
Let us deduce the circuit equation

$$\frac{V}{i} = -\frac{\Gamma_0^3 K}{(\Gamma^2 - \Gamma_0^2)\Gamma}$$ (to be deduced)

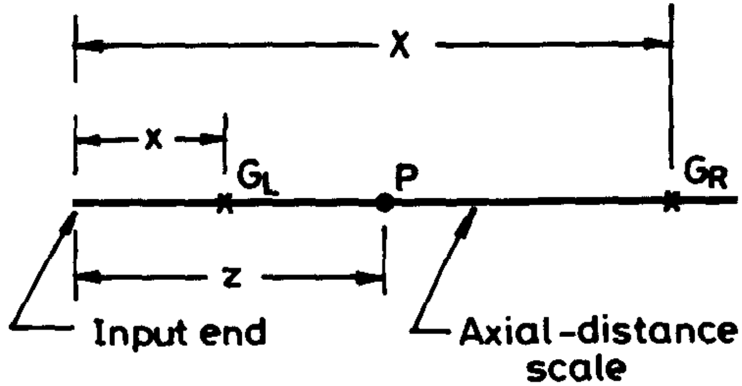
being the ratio of circuit voltage *V* to RF beam current *i* in terms of the interaction impedance *K* of the slow-wave structure, cold (beam-absent) propagation constant $\Gamma_0$ of the isolated structure and hot (beam-present) propagation constant $\Gamma$ of the beam-circuit coupled system. The slow-wave structure is considered as a circuit. **The effect of the element of a modulated beam at a point on the circuit is simulated by an infinitesimal current generator at that point.** Such an infinitesimal generator sends two circuit waves in opposite directions, one to the left and one to the right such that the amplitudes of the circuit electric field intensity associated with these waves are equal. We find the contribution to the circuit field intensity at a circuit point from all such infinitesimal current generators distributed along the circuit both to the left and to the right of the point. These contributions are then added to the circuit field intensity at the point caused by the power injected at the input end of the circuit to find the total circuit field intensity at the point in the presence of the modulated beam.



The scale to measure the axial distances of the point (z) where to find the electric field intensity, shown as a dot (P), as well as of the points of locations of two arbitrarily chosen infinitesimal current generators (x), shown as crosses, one ($G_L$) to the left and the other ($G_R$) to the right of P.




The effect of the element of a modulated beam at a point on the circuit is simulated by an infinitesimal current generator at that point that 'sees' half the characteristic impedance of the transmission line, being equivalent to two such characteristic impedances in parallel, corresponding to two halves of the supposedly matched line.



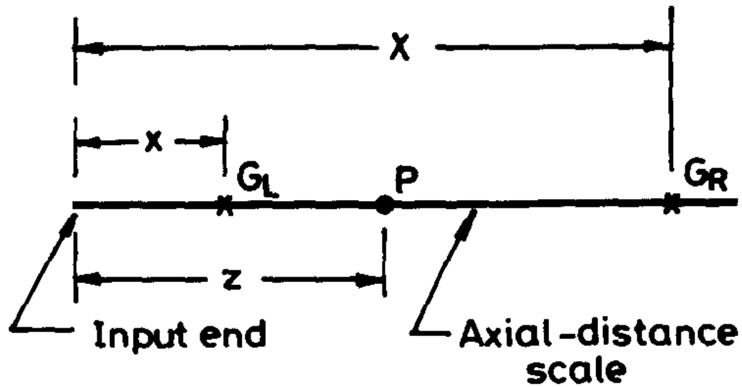

Let $dE_R$ and $dE_L$ be the amplitudes of the electric field intensity at a point on the circuit associated with two waves, one travelling to the right and another to the left of the point, respectively, both launched by an infinitesimal current generator. The electric field intensity at a point on the circuit caused by an infinitesimal current generator to the left of the point is $dE_R \exp -j\beta_0(z-x)$, which is associated with a wave travelling to the right from the generator, where $\beta_0$ is the axial phase propagation constant; $z$ is the distance of the point and $x$ is the distance of the infinitesimal current generator, both measured from the input end of the circuit. Similarly, the electric field intensity at the point caused by a current generator to the right of the point is $dE_L \exp -j\beta_0(x-z)$, which is associated with a wave travelling to the left from the generator.

$$E(z) = E_i \exp(-j\beta_0 z) + \int_0^z dE_R(x) \exp - j\beta_0(z-x) dx + \int_z^\ell dE_L(x) \exp - j\beta_0(x-z) dx$$

$$E(z) = E_i \exp(-j\beta_0 z) + \int_0^z \xi_R(x) \exp - j\beta_0(z-x) dx + \int_z^\ell \xi_L(x) \exp - j\beta_0(x-z) dx$$ (to be recalled)

where $E_i$ is the amplitude of the circuit electric field intensity injected at the input end ($z = 0$) of the circuit. Here, we have defined

$$\left.\begin{aligned} dE_R &= \varsigma_R(x) dx \\ dE_L &= \varsigma_L(x) dx \end{aligned}\right\}$$




The current generators see identical halves of the matched line both to its left and to its right.

$$\left.\begin{aligned} dE_R &= dE_L = dE \\ \varsigma_R(x) &= \varsigma_L(x) = \varsigma(x) \end{aligned}\right\}$$

⇓

$$E(z) = E_i \exp(-j\beta_0 z) + \int_0^z \xi_R(x) \exp - j\beta_0(z-x)dx + \int_z^\ell \xi_L(x) \exp - j\beta_0(x-z)dx$$ (recalled)

⇓

$$E(z) = E_i \exp(-j\beta_0 z) + \int_0^z \xi(x) \exp - j\beta_0(z-x)dx + \int_z^\ell \xi(x) \exp - j\beta_0(x-z)dx$$

⇓

$$E(z) = E_i \exp(-j\beta_0 z) + I_1 + I_2$$

(to be recalled)

$$\left.\begin{aligned} I_1 &= \int_0^z \xi(x) \exp - j\beta_0(z-x)dx \\ I_2 &= \int_z^\ell \xi(x) \exp - j\beta_0(x-z)dx \end{aligned}\right\}$$

$E(z) = E_i \exp(-j\beta_0 z) + I_1 + I_2$ (recalled)

Upon differentiation

$$\frac{dE}{dz} = -j\beta_0 E_i \exp(-j\beta_0 z) + \frac{dI_1}{dz} + \frac{dI_2}{dz}$$

$$\left.\begin{aligned} I_1 &= \int_0^z \xi(x) \exp - j\beta_0 (z-x) dx \\ I_2 &= \int_z^\ell \xi(x) \exp - j\beta_0 (x-z) dx \end{aligned}\right\}$$

Differentiating definite integrals using Leibnitz formula

$$\left.\begin{aligned} \frac{dI_1}{dz} &= -j\beta_0 I_1 + \zeta(z) \\ \frac{dI_2}{dz} &= j\beta_0 I_2 - \zeta(z) \end{aligned}\right\}$$

(to be recalled)

$$\frac{dE}{dz} = -j\beta_0 E_i \exp(-j\beta_0 z) - j\beta_0 (I_1 + I_2)$$ (to be recalled)

$$\frac{dE}{dz} = -j\beta_0 E_i \exp(-j\beta_0 z) - j\beta_0 (I_1 + I_2)$$ (recalled)

Upon differentiation

$$\frac{dI_1}{dz} = -j\beta_0 I_1 + \zeta(z)$$
$$\frac{dI_2}{dz} = j\beta_0 I_2 - \zeta(z)$$
(recalled)

$$\frac{d^2E}{dz^2} = -\beta_0^2 E_i \exp(-j\beta_0 z) - j\beta_0 \left(\frac{dI_1}{dz} - \frac{dI_2}{dz}\right)$$

$$\frac{d^2E}{dz^2} = -\beta_0^2 E_i \exp(-j\beta_0 z) - j\beta_0 [(I_1 + I_2) + 2\varsigma(z)]$$

$$\frac{d^2E}{dz^2} = -\beta_0^2 [E_i \exp(-j\beta_0 z) + I_1 + I_2] - 2j\beta_0 \varsigma(z))$$

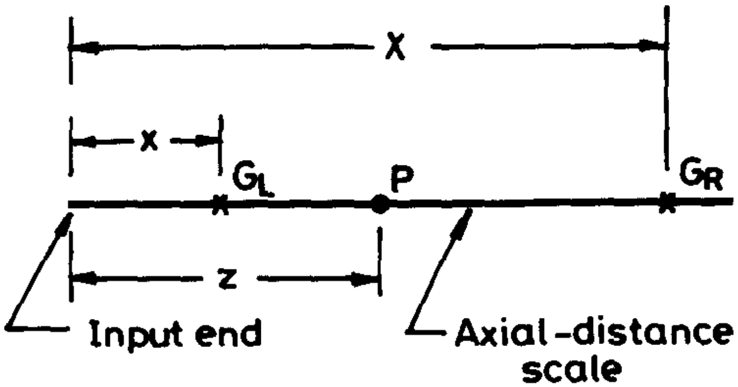
$$E(z) = E_i \exp(-j\beta_0 z) + I_1 + I_2$$ (recalled)

$$dE_R = \varsigma_R(x)dx; dE_L = \varsigma_L(x)dx$$
$$dE_R = dE_L = dE = \varsigma_R(x)dx = \varsigma_L(x)dx = \varsigma(x)dx = \varsigma(z)dz$$
$$dE = \varsigma(z)dz$$
$$\varsigma(z) = \frac{dE}{dz}$$

$$\boxed{\frac{d^2E}{dz^2} = \beta_0^2 E - 2j\beta_0 \frac{dE}{dz}}$$

(to be recalled)

Let us now proceed to express the above expression in a form that explicitly involves a beam parameter.




For this purpose, let us find the increment d$P$ of circuit power at a point due to an infinitesimal current generator simulating the effect of a modulated electron beam-element of length d$z$ coupled to the circuit at the point.

$$dP = dP_R + dP_L$$

where $dP_R$ and $dP_L$ are the increments of circuit power $dP$ at the point associated with two waves sent in the right and left directions by the infinitesimal current generator situated to the left right of the point, respectively.

$$dP = dP_R + dP_L$$ (rewritten and to be recalled later)

Interaction impedance of the slow-wave structure (circuit) is defined as

$$K = \frac{|V_z|^2}{2P}$$

where $P$ is the power propagating down the structure. $V_z$ is the longitudinal voltage which may be found by taking the negative integral of the axial electric field intensity $E_z = E_z \sin\beta z$ between the limits $z=0$, the reference point where the electric field intensity is zero, and $z=\lambda g/4$ where the intensity is maximum, where $E_z(0)$ is the peak value of intensity, and $\lambda g$ $(=2\pi/\beta)$ is the guide wavelength of the wave supported by the waveguiding slow-wave structure. Thus, we get

$$V_z = -\int_{z=\lambda_g/4}^{z=0} E_z dz = -\int_{z=\lambda_g/4}^{z=0} E_z(0)\sin\beta z dz = \frac{E_z(0)}{\beta}$$

⇓

$$K = \frac{|V_z|^2}{2P}$$

⇓ ⇐ Interpreting $\beta$ as the cold structure propagation constant $\beta_0$

$$K = \frac{E_z^2(0)}{2\beta_0^2 P}$$

$$\left.\begin{aligned} 2\beta_0^2 K dP_R &= d(E_R^2) = 2E_R dE_R \\ 2\beta_0^2 K dP_L &= d(E_L^2) = 2E_L dE_L \end{aligned}\right\} \Longleftarrow \left.\begin{aligned} dP &= dP_R + dP_L \\ K &= \frac{E_z^2(0)}{2\beta_0^2 P} \\ dE_R &= dE_L = dE \end{aligned}\right\} \text{(recalled)}$$

$$\Downarrow$$

$$dP = dP_R + dP_L = \frac{E_R + E_L}{\beta_0^2 K} dE \quad \text{(to be recalled)}$$

(increment of circuit power)

Since the increment of circuit power $dP$ at the point owes to the modulation of the beam element of length $dz$, $dP$ may also be equated to the power lost by the beam element of length $dz$ subjected to the circuit field at the point. In order to find the latter, the beam element is divided into two halves, each of length $dz/2$, which experience the electric field intensities $E_L$ and $E_R$ associated with the power propagating to the left and to the right of the circuit point coupled to the beam element, respectively.

The power lost, that is the energy lost per second, by an electron belonging to that half of the modulated beam element which is subjected to the electric field intensity $E_L$ may be found as $-eE_L v_1$, being equal to the product of $-eE_L$, the force on the electron, and $v_1$, the distance moved by it per second under rf modulation. The power lost per electron thus found becomes positive, since *e* carrier a negative sign, $v_1$ and $E_L$ being interpreted to have the same direction. Multiplying this quantity by $n\alpha dz/2$, the number of electrons in the element half considered, one may find the power lost by this half as $(-eE_L v_1)(n\ \alpha\ dz/2)$, where $n$ is the rf number density of electrons of the perturbed beam element and $\alpha$ is the beam cross-sectional area. Similarly, the power lost by the remaining half which is subjected to the circuit electric field intensity $E_R$ may be found as $(-eE_R\ v_1)(n\ \alpha\ dz/2)$. Adding these quantities one may then find the power lost by the complete beam element of length $dz$ which may be equated to $dP$. Thus one gets

$$dP = -eE_L v_1 n\alpha dz/2 - eE_R v_1 n\alpha dz/2$$

⟸

$$\left.\begin{aligned} i &= J_1\alpha \\ J_1 &= nev_1 \end{aligned}\right\}$$

(RF beam current $i$ and RF beam current density $J_1$)

(power lost by the beam elemental length *dz*)

⇓

$$dP = -i(E_R + E_L)\frac{dz}{2}$$

(power lost by the beam elemental length $dz$)

$$dP = -i(E_R + E_L)\frac{dz}{2}$$ (rewritten)

(power lost by the beam elemental length *dz*)

(negative sign indicating that the power lost is positive if *i* and $(E_R + E_L)$ are directed oppositely)

$$dP = dP_R + dP_L = \frac{E_R + E_L}{\beta_0^2 K} dE$$ (recalled)

(increment of circuit power)

On equating power lost by the beam element with increment of circuit power

$$\frac{dE}{dz} = -\frac{\beta_0^2 K i}{2}$$ (to be recalled)

$$\frac{d^2E}{dz^2} = \beta_0^2 E - 2j\beta_0 \frac{dE}{dz}$$ (recalled) ⟸ $$\frac{dE}{dz} = -\frac{\beta_0^2 Ki}{2}$$ (recalled)

⇓

$$\frac{d^2E}{dz^2} = -\beta_0^2 E + j\beta_0^3 Ki$$

⇓ ⟸ Assuming RF quantities to vary with *z* as exp $(-\Gamma z)$

$$\Gamma^2 E = -\beta_0^2 E + j\beta_0^3 Ki$$ ⟸ $$E = -\Gamma V$$ ⟸ $$E = -\frac{\partial V}{\partial z}$$ ⟸ $$\vec{E} = -\nabla V$$

⇓

$$\frac{V}{i} = -\frac{j\beta_0^3 K}{(\Gamma^2 + \beta_0^2)\Gamma}$$ ⟸ $$\Gamma_0 = j\beta_0$$

⇓

$$\frac{V}{i} = -\frac{\Gamma_0^3 K}{(\Gamma^2 - \Gamma_0^2)\Gamma}$$ ⟸ Putting $\Gamma \approx \Gamma_0$ since we do not expect $\Gamma$ to deviate much from $\Gamma_0$

⇓

$$\boxed{\frac{V}{i} = -\frac{\Gamma\Gamma_0 K}{\Gamma^2 - \Gamma_0^2}}$$ (to be recalled)

(circuit equation)

# Electronic Equation

Let us deduce the electronic equation

$$\frac{V}{i}=\left(\frac{2V_0}{I_0}\right)\left(\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}\right)$$ (to be deduced)

being the ratio of circuit voltage $V$ to RF beam current $i$.

Let us now proceed to find this ratio by studying the motion of the electrons subjected to the circuit electric field intensity $E$ plus the space-charge electric field intensity $E_s$.

$Dv_1=\eta E_z$ (force equation) (recalled) ⟸ Replacing $E_z$ by $E+E_s$

$$D=\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial z}$$

⇓

$Dv_1=\eta(E+E_s)$ ⟸ $D=\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial z}=j\omega-v_0\Gamma$ ⟸ Considering RF dependence as $\exp(j\omega t-\Gamma z)$

⇓

$$(j\omega-v_0\Gamma)v_1=\eta(E+E_s)$$

⇓

$$v_1=\frac{\eta(E+E_s)}{j\omega-v_0\Gamma}$$ (to be recalled)

$$\nabla . \vec{E} = \frac{\rho}{\varepsilon}$$ (Poisson's equation)

$$\frac{\partial E_s}{\partial z} = \frac{\rho_1}{\varepsilon_0}$$

Considering RF dependence as $\exp(j\omega t - \Gamma z)$

$$-\Gamma E_s = \frac{\rho_1}{\varepsilon_0}$$

$$E_s = -\frac{\rho_1}{\Gamma \varepsilon_0}$$

$$\nabla . \vec{J} + \frac{\partial \rho}{\partial t}$$

(continuity equation)

$$-\Gamma J_1 = -j\omega \rho_1$$

$$\rho_1 = \frac{\Gamma J_1}{j\omega}$$ (to be recalled)

$$E_s = -\frac{\Gamma J_1}{j\omega}\frac{1}{\Gamma \varepsilon_0} = -\frac{J_1}{j\omega \varepsilon_0}$$

$$v_1 = \frac{\eta(E + E_s)}{j\omega - v_0 \Gamma}$$ (recalled)

$$v_1 = \frac{\eta\left(E - \frac{J_1}{j\omega \varepsilon_0}\right)}{j\omega - v_0 \Gamma}$$ (to be recalled)

$$\left.\begin{array}{l} v_1 = \dfrac{\eta\left(E - \dfrac{J_1}{j\omega\varepsilon_0}\right)}{j\omega - v_0\Gamma} \\ \rho_1 = \dfrac{\Gamma J_1}{j\omega} \end{array}\right\}$$ (recalled)

⇓

$$J_1 = \rho_0 v_1 + v_0 \rho_1$$ (recalled)

⇓

$$J_1((j\omega - v_0\Gamma)^2 + \omega_p^2) = j\omega\eta\rho_0 E$$ (to be recalled)

$$\boxed{\omega_p^2 = \frac{|\eta||\rho_0|}{\varepsilon_0}}$$

⇓ ⇐ $E = \Gamma V$ ⇐ $E = -\dfrac{\partial V}{\partial z}$ ⇐ $\vec{E} = -\nabla V$ ⇐ $\left.\begin{array}{l} i = J_1\alpha \\ i_0 = J_0\alpha \end{array}\right\}$

$\alpha =$ beam cross-sectional area

$$\frac{V}{i} = \frac{(j\omega - v_0\Gamma)^2 + \omega_p^2}{j\beta_e \eta i_0 \Gamma}$$

⇓ ⇐ $\left.\begin{array}{l} \beta_e = \dfrac{\omega}{v_0} \\ \beta_p = \dfrac{\omega_p}{v_0} \end{array}\right\}$

⇓

$$\frac{V}{i} = \frac{((j\beta_e - \Gamma)^2 + \beta_p^2)v_0^2}{j\beta_e \eta i_0 \Gamma}$$

$$\frac{V}{i} = \frac{((j\beta_e - \Gamma)^2 + \beta_p^2)v_0^2}{j\beta_e \eta i_0 \Gamma}$$ (rewritten)

$$\left.\begin{aligned} \eta &= -|\eta|; i_0 = -I_0 \\ v_0^2 &= 2|\eta| V_0 \end{aligned}\right\}$$

$V_0 =$ beam voltage

$I_0 =$ beam current

$$\boxed{\frac{V}{i} = \left(\frac{2V_0}{I_0}\right)\left(\frac{(j\beta_e - \Gamma)^2 + \beta_p^2}{j\beta_e \Gamma}\right)}$$

(electronic equation)

$$\frac{V}{i} = -\frac{\Gamma\Gamma_0 K}{\Gamma^2 - \Gamma_0^2}$$ (circuit equation) (recalled)

$$\frac{V}{i} = \left(\frac{2V_0}{I_0}\right)\left(\frac{(j\beta_e - \Gamma)^2 + \beta_p^2}{j\beta_e\Gamma}\right)$$ (electronic equation) (recalled)

Equating the right hand sides of the above two equations we deduce the dispersion of a TWT as follows:

$$-\frac{\Gamma\Gamma_0 K}{\Gamma^2 - \Gamma_0^2} = \left(\frac{2V_0}{I_0}\right)\left(\frac{(j\beta_e - \Gamma)^2 + \beta_p^2}{j\beta_e\Gamma}\right)$$

**(Dispersion relation of a travelling-wave tube)**

## Weak beam-structure coupling:

$$-\frac{\Gamma\Gamma_0 K}{\Gamma^2-\Gamma_0^2}=\left(\frac{2V_0}{I_0}\right)\left(\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}\right)$$ (dispersion relation of a travelling-wave tube) (deduced)

⇓

$$(\Gamma^2-\Gamma_0^2)((j\beta_e-\Gamma)^2+\beta_p^2)=-\frac{j\Gamma^2\Gamma_0 K\beta_e}{2V_0}I_0$$

⇓ ⟸ For small beam current ($I_0$~0)

$$(\Gamma^2-\Gamma_0^2)((j\beta_e-\Gamma)^2+\beta_p^2)=0$$

⇓

$$\left.\begin{aligned}&\Gamma^2-\Gamma_0^2=0\\&(j\beta_e-\Gamma)^2+\beta_p^2=0\end{aligned}\right\}$$

⇓

$$\left.\begin{aligned}&\Gamma=\pm\Gamma_0\ \text{(circuit waves)}\\&\Gamma=j(\beta_e\mp\beta_p)\,\text{(space-charge waves)}\end{aligned}\right\}$$ ⟸ For small beam current ($I_0$~0), the circuit/structure waves and the space-charge waves get separated corresponding to weak beam-structure coupling.

$$-\frac{\Gamma\Gamma_0 K}{\Gamma^2-\Gamma_0^2}=\left(\frac{2V_0}{I_0}\right)\left(\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}\right)$$ (dispersion relation of a travelling-wave tube) (deduced)

It is a fourth-degree equation.

(1) An electron beam supports slow and fast space-charge waves, both with forward phase velocities.

(2) A slow-wave structure, treated as a transmission line circuit equivalent, supports two circuit waves, one with forward phase velocity and another with backward phase velocity.

(3) A beam-circuit coupled system such as a travelling-wave tube is therefore expected to support forward waves — three forward waves and one backward wave.

(4) The dispersion relation of a travelling-wave tube is a fourth-degree equation giving four solutions corresponding to these three forward waves and one backward wave.

(5) We look forward to the solutions of the dispersion relation for three forward waves considering the structure to be matched such that no backward wave is excited.

$$-\frac{\Gamma\Gamma_0 K}{\Gamma^2-\Gamma_0^2}=\left(\frac{2V_0}{I_0}\right)\left(\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}\right)$$

(dispersion relation of a travelling-wave tube) (recalled)

Following Pierce's approach, we look for the solutions for $\Gamma$ for the three forward waves, considering the structure to be matched such that no backward wave is excited. Hence, expecting that $\Gamma$ will not differ appreciably from the beam propagation constant $\beta_e$, we can express $\Gamma$ in terms of the dimensionless quantities $C$ and $\delta$, as follows:

$$-\Gamma=-j\beta_e+\beta_e C\delta$$

$$\left.\begin{aligned}&\beta_e=\frac{\omega}{v_0}\\&C=\left(\frac{KI_0}{4V_0}\right)^{1/3}\\&C\delta<<1\end{aligned}\right\}$$

$C$ is tacitly defined so by Pierce from the feedback of the end result of his analysis leading to his famous gain equation $G=A+BCN$.

## Nearly non-synchronous beam ( $v_p \neq v_0$ ):

Let us put the cold circuit axial-propagation constant as:

$$\beta_0 = \beta_e (1 + bC)$$

$$b = \frac{\beta_0 - \beta_e}{\beta_e C} = \frac{v_0 - v_p}{v_p C}$$ (Pierce's velocity synchronization parameter)

For a non-synchronous beam and in the presence of the circuit loss defined by Pierce's loss parameter *d*:

$$\Gamma_0 = \beta_e C d + j\beta_e (1 + bC) \quad \Longleftarrow \quad \left.\begin{aligned} \alpha_0 &= \beta_e C d \\ \Gamma_0 &= j\beta_0 \end{aligned}\right\}$$

$$\left.\begin{aligned} \Gamma + \Gamma_0 &= j\beta_e + j\beta_e (1 + bC) + \beta_e (Cd - C\delta) \\ \Gamma - \Gamma_0 &= -\beta_e C (\delta + d + jb) \end{aligned}\right\} \quad \Longleftarrow \quad j\beta_e - \Gamma = \beta_e C\delta \quad \Longleftarrow \quad -\Gamma = -j\beta_e + \beta_e C\delta$$

$$\Longleftarrow \quad \left.\begin{aligned} C\delta &<< 1 \\ Cd &<< 1 \\ bC &<< 1 \\ \Gamma_0 &= j\beta_e \end{aligned}\right\}$$

$$\left.\begin{aligned} \Gamma + \Gamma_0 &= 2j\beta_e \\ \Gamma - \Gamma_0 &= -\beta_e C (\delta + d + jb) \end{aligned}\right\}$$

$$-\frac{\Gamma\Gamma_0 K}{\Gamma^2-\Gamma_0^2}=\left(\frac{2V_0}{I_0}\right)\left(\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}\right)$$ (dispersion relation of a travelling-wave tube) (recalled)

⇓

$$-\frac{\Gamma\Gamma_0 K}{(\Gamma+\Gamma_0)(\Gamma-\Gamma_0)}=\left(\frac{2V_0}{I_0}\right)\left(\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}\right)$$

⇓ ⇐

$$\left.\begin{aligned}\Gamma+\Gamma_0&=2j\beta_e\\ \Gamma-\Gamma_0&=-\beta_e C(\delta+d+jb)\end{aligned}\right\}$$ (recalled)

$$\Gamma\approx\Gamma_0\approx j\beta_e$$

$$j\beta_e-\Gamma=\beta_e C\delta$$ (recalled)

$$(\delta^2+4QC)(j\delta+jd-b)=1$$ (cubic equation) (to be recalled)

For the matched structure with no backward wave excited, let us express the circuit voltage in terms of three forward-wave components as follows at a distance $z$ from the input end:

$$V(z) = V_1(z) + V_2(z) + V_3(z)$$

$$V(z) = V_1(0)\exp(-\Gamma_1 z) + V_2(0)\exp(-\Gamma_2 z) + V_3(0)\exp(-\Gamma_3 z)$$

$$-\Gamma = -j\beta_e + \beta_e C\delta$$

$$\delta = x + jy$$

$$\left.\begin{aligned} \delta_1 &= x_1 + jy_1 \\ \delta_2 &= x_2 + jy_2 \\ \delta_3 &= x_3 + jy_3 \end{aligned}\right\}$$

$$\left.\begin{aligned} -\Gamma_1 &= -j\beta_e + \beta_e C\delta_1 \\ -\Gamma_2 &= -j\beta_e + \beta_e C\delta_2 \\ -\Gamma_1 &= -j\beta_e + \beta_e C\delta_3 \end{aligned}\right\}$$

$$\left.\begin{aligned} -\Gamma_1 &= -j\beta_e + \beta_e C(x_1 + jy_1) \\ -\Gamma_2 &= -j\beta_e + \beta_e C(x_2 + jy_2) \\ -\Gamma_1 &= -j\beta_e + \beta_e C(x_3 + jy_3) \end{aligned}\right\}$$

$$(\delta^2 + 4QC)(j\delta + jd - b) = 1$$

(recalled)

$$\begin{aligned} V(z) = {} & V_1(0)\exp(-j\beta_e(1 - Cy_1)z)\exp(\beta_e C x_1 z) \\ & + V_2(0)\exp(-j\beta_e(1 - Cy_2)z)\exp(\beta_e C x_2 z) \\ & + V_3(0)\exp(-j\beta_e(1 - Cy_3)z)\exp(\beta_e C x_3 z) \end{aligned}$$

(to be recalled)

$$V(z) = V_1(0)\exp(-j\beta_e(1-Cy_1)z)\exp(\beta_e Cx_1 z) + V_2(0)\exp(-j\beta_e(1-Cy_2)z)\exp(\beta_e Cx_2 z) + V_3(0)\exp(-j\beta_e(1-Cy_3)z)\exp(\beta_e Cx_3 z)$$ (recalled)

$\Leftarrow$ $z = 0$ $\Leftarrow$

$$V_{\text{in}} = V_1(0) + V_2(0) + V_3(0)$$

(to be recalled)

$$V_{\text{out}} = V_1(0)\exp(-j\beta_e(1-Cy_1)l)\exp(\beta_e Cx_1 l)$$

(output at the end of the interaction length $l$ of the TWT contributed by the dominating growing-wave component) (to be recalled)

We invoke the condition that at the entry of the interaction structure the electron beam is not modulated enabling us to write:

$$\left.\begin{aligned} v_{1,\text{in}} &= 0 \\ J_{1,\text{in}} &= 0 \end{aligned}\right\}$$

$\Downarrow$

$$\left.\begin{aligned} v_{1,\text{in}} &= v_1(0) + v_2(0) + v_3(0) = 0 \\ J_{1,\text{in}} &= J_1(0) + J_2(0) + J_3(0) = 0 \end{aligned}\right\}$$

(to be recalled)

$$J_1((j\omega - v_0\Gamma)^2 + \omega_p^2) = j\omega\eta\rho_0 E$$ (recalled)

$\Downarrow \Leftarrow E = \Gamma V$

$$J_1 = \frac{j\beta_e \eta J_0 \Gamma}{(j\omega - v_0\Gamma)^2 + \omega_p^2} V \Leftarrow \boxed{V' = \frac{V}{1 + \frac{4QC}{\delta^2}}} \Leftarrow \boxed{QC = \frac{\omega_p{}^2}{4\omega_e{}^2 C^2} = \frac{\beta_p{}^2}{4\beta_e{}^2 C^2}}$$ (Pierce's space-charge parameter)

(defined) (defined)

$\Downarrow \Leftarrow$ $$\left.\begin{aligned} -\Gamma &= -j\beta_e + \beta_e C\delta \\ \Gamma &\approx j\beta_e \end{aligned}\right\}$$

$$J_1 = -\frac{\eta J_0 V}{v_0^2 C^2 \delta^2 (1 + \frac{4QC}{\delta^2})} \Rightarrow J_1 = -\frac{\eta J_0}{v_0{}^2 C^2 \delta^2} V' \Rightarrow J_1 \propto \frac{V'}{\delta^2}$$

(to be recalled in the analysis of 'hot' attenuation)

$$
\left.\begin{aligned}
v_1 &= \frac{\eta\left(E - \dfrac{J_1}{j\omega\varepsilon_0}\right)}{j\omega - v_0\Gamma} \\
J_1\left((j\omega - v_0\Gamma)^2 + \omega_p^2\right) &= j\omega\eta\rho_0 E
\end{aligned}\right\}
$$

(recalled) ⟸

$$
\left.\begin{aligned}
QC &= \frac{\omega_p^{\,2}}{4\omega_e^{\,2}C^2} = \frac{\beta_p^{\,2}}{4\beta_e^{\,2}C^2} \\
E &= \Gamma V \\
-\Gamma &= -j\beta_e + \beta_e C\delta \\
\Gamma &\approx j\beta_e
\end{aligned}\right\}
$$

⇓

$$
v_1 = \frac{j\eta V}{v_0 C\delta\left(1 + \dfrac{4QC}{\delta^2}\right)}
$$

⟸

$$
V' = \frac{V}{1 + \dfrac{4QC}{\delta^2}}
$$

(recalled)

⇓

$$
v_1 = \frac{j\eta V'}{v_0 C\delta}
$$

⟹

$$
v_1 \propto \frac{V'}{\delta}
$$

(to be recalled in the analysis of ‘hot’ attenuation)

$$V(z) = V_1(0)\exp(-\Gamma_1 z) + V_2(0)\exp(-\Gamma_2 z) + V_3(0)\exp(-\Gamma_3 z)$$

$$\left.\begin{aligned} -\Gamma_1 &= -j\beta_e + \beta_e C(x_1 + jy_1) \\ -\Gamma_2 &= -j\beta_e + \beta_e C(x_2 + jy_2) \\ -\Gamma_1 &= -j\beta_e + \beta_e C(x_3 + jy_3) \end{aligned}\right\}$$

(recalled)

$$\begin{aligned} V(z) = {} & V_1(0)\exp(\beta_e C x_1 z)\exp - j\beta_e(1 - Cy_1)z \\ & + V_2(0)\exp(\beta_e C x_2 z)\exp - j\beta_e(1 - Cy_2)z \\ & + V_3(0)\exp(\beta_e C x_3 z)\exp - j\beta_e(1 - Cy_3)z \end{aligned}$$

$$V_{\text{in}} = V_1(0) + V_2(0) + V_3(0)$$

$$V' = \frac{V}{1 + \dfrac{4QC}{\delta^2}}$$

(recalled)

$$V_{\text{in}} = V_1'(0) + V_2'(0) + V_3'(0)) + 4QC\left(\frac{V_1'(0)}{\delta_1^2} + \frac{V_2'(0)}{\delta_2^2} + \frac{V_3'(0)}{\delta_3^2}\right)$$

$$V_{\text{out}} = V(l) = V_1(0)\exp(-\Gamma_1 l)$$

$$-\Gamma_1 = -j\beta_e + \beta_e C\delta_1 = -j\beta_e + \beta_e C(x_1 + jy_1)$$

Solving three simultaneous equations representing the device input conditions

$$V_{\mathrm{in}} = V_1(0) + V_2(0) + V_3(0)$$

and

$$\left.\begin{aligned} v_{1,\mathrm{in}} &= v_1(0) + v_2(0) + v_3(0) = 0 \\ J_{1,\mathrm{in}} &= J_1(0) + J_2(0) + J_3(0) = 0 \end{aligned}\right\} \quad \text{(recalled)}$$

the latter two interpreted with the help of

$$v_1 = \frac{j\eta V}{v_0 C\delta\left(1 + \dfrac{4QC}{\delta^2}\right)} \quad \text{(recalled)} \qquad \text{and} \qquad J_1 = -\frac{\eta J_0 V}{v_0^2 C^2 \delta^2 \left(1 + \dfrac{4QC}{\delta^2}\right)} \quad \text{(recalled)}$$

we obtain

$$V_1(0) = \left(1 + 4QC/\delta_1^2\right)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right) V_{in} \quad \text{(to be recalled)}$$

Also, we obtain

$$V_{\mathrm{out}} = V(l) = V_1(0)\exp(-\Gamma_1 l)$$

$$-\Gamma_1 = -j\beta_e + \beta_e C\delta_1 = -j\beta_e + \beta_e C(x_1 + jy_1)$$

$$V_{\mathrm{out}} = V_{\mathrm{in}}(1 + 4QC/\delta_1^2)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right) \times \exp - j\beta_e(1 - Cy_1)l \times \exp(\beta_e C x_1 l) \quad \text{(to be recalled)}$$

Substituting

$$V_1(0) = \left(1 + 4QC/\delta_1^2\right)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)V_{in}$$ (recalled)

in

$$V_{\text{out}} = V_{\text{in}}(1 + 4QC/\delta_1^2)(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}) \times \exp - j\beta_e(1 - Cy_1)l \times \exp(\beta_e Cx_1 l)$$ (recalled)

We obtain

$$V_{\text{out}} = V_{\text{in}}(1 + 4QC/\delta_1^2)(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}) \times \exp - j\beta_e(1 - Cy_1)l \times \exp(\beta_e Cx_1 l)$$

$$G = 20\log_{10}\left|\frac{V_{out}}{V_{in}}\right| = A + 20\log_{10}(\exp(\beta_e Cx_1 l))$$

$$A = 20\log_{10}\left|\left(1 + \frac{4QC}{\delta_1^2}\right)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)\right|$$

$$\beta_e l = 2\pi N$$

$$G = A + 20\log_{10}(\exp(\beta_e Cx_1 l)) = A + 20\log_e(\exp(\beta_e Cx_1 l)(\log_{10} e))$$

$$G = A + 20Cx_1\beta_e l = A + 40C\pi N x_1 = A + 40\pi(\log_{10} e)x_1 CN$$

$$G = A + 20Cx_1\beta_e l = A + 40C\pi N x_1 = A + 40\pi(\log_{10} e)x_1 CN$$ (rewritten)

$$\boxed{B = 40\pi(\log_{10} e)x_1 \approx 54.6x_1}$$

$$\boxed{G = A + BCN}$$

$$\boxed{\left.\begin{array}{l} G = A + BCN \\ A = 20\log_{10}\left|\left(1 + \dfrac{4QC}{\delta_1^2}\right)\left(\dfrac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)\right| \\ B = 40\pi(\log_{10} e)x_1 \approx 54.6x_1 \end{array}\right\}}$$

## Cubic dispersion equation:

$$(\delta^2 + 4QC)(j\delta + jd - b) = 1$$

$$\left.\begin{array}{l} b = 0 \\ d = 0 \\ QC = 0 \end{array}\right\} \text{(special case)}$$

$$\delta^3 = -j$$

$$\left.\begin{array}{l} \delta_1 = \sqrt{3}/2 - j(1/2) \\ \delta_2 = -\sqrt{3}/2 - j(1/2) \\ \delta_3 = j \end{array}\right\} \Rightarrow \left.\begin{array}{l} \delta_1 = x_1 + jy_1 \\ \delta_2 = x_2 + jy_2 \\ \delta_3 = x_3 + jy_3 \end{array}\right\} \Rightarrow x_1 = \sqrt{3}/2$$

$$B = 40\pi(\log_{10} e)x_1 \approx 54.6x_1 \cong 47.3$$

$$A = 20\log_{10}\left|\left(1 + 4QC/\delta_1^2\right)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)\right| = 20\log_{10}(1/3) \cong -9.54$$

$$G = -9.54 + 47.3\,CN \Leftarrow G = A + BCN \Leftarrow \left.\begin{array}{l} A = -9.54 \\ B = 47.3 \end{array}\right\}$$

$$(b = 0, d = 0, QC = 0)$$

## Hot Attenuation

• Reflections take place at the input and output ends of a slow-wave structure due to imperfect matching

• Reflection at the input end sets up three forward waves one of which spatially grows and travels to the output end where it is reflected again, and the repetition of the process would eventually cause a regenerative oscillation in the device.

• In the presence of the attenuator, the RF modulation on the beam remains, so that, in the forward direction, the spatially growing wave is excited again beyond the attenuator region. In the reverse direction, however, the growing wave interaction does not take place, and, consequently, the circuit field in the wave traveling in the reverse direction is reduced to zero beyond the attenuator and does not spatially grow further in this direction to reach the input end of the structure.

In order to prevent oscillations in the device, therefore, a lossy section (attenuator) is placed along the slow-wave structure roughly 1/3 to 1/2 way down the structure in the form of

• an absorbing layer on the dielectric supports for the helix of a helix-TWT called the *attenuator*,
• a lossy ceramic button, loading a spacer cavity wall in the structure of a coupled-cavity

## Extension of Pierce's theory to estimate ‘hot’ attenuation

- One attenuator section is added per about $20$ dB gain of the device.
- We are going to estimate 'hot' attenuation for infinite 'cold' attenuation.
- We assume that beyond the attenuator, the circuit voltage $= 0$ (corresponding to ‘cold' attenuation $= \infty$ ).
- RF modulation on the beam however remains beyond the attenuator region.

Attenuator length is negligibly small. The superscripts a and b refer to the quantities immediately preceding and following the attenuator length in the following expressions:

$$v_1^{b} + v_2^{b} + v_3^{b} = v_1^{a} + v_2^{a} + v_3^{a} \Longleftarrow v_1 \propto \frac{V}{\delta} \Longleftarrow v_1 \propto \frac{V'}{\delta} \Longleftarrow QC = 0$$

(recalled)

$$V' = \frac{V}{1 + \frac{4QC}{\delta^2}}$$

(recalled)

$$\Downarrow$$

$$\frac{V_1^{b}}{\delta_1} + \frac{V_2^{b}}{\delta_2} + \frac{V_3^{b}}{\delta_3} = \frac{V_1^{a}}{\delta_1} + \frac{V_2^{a}}{\delta_2} + \frac{V_3^{a}}{\delta_3}$$

$$J_1 \propto \frac{V'}{\delta^2}$$

(recalled)

$$J_1^{b} + J_2^{b} + J_3^{b} = J_1^{a} + J_2^{a} + J_3^{a} \Longleftarrow J_1 \propto \frac{V}{\delta^2} \Longleftarrow J_1 \propto \frac{V'}{\delta^2} \Longleftarrow QC = 0$$

(recalled)

$$\Downarrow$$

$$\frac{V_1^{b}}{\delta_1^2} + \frac{V_2^{b}}{\delta_2^2} + \frac{V_3^{b}}{\delta_3^2} = \frac{V_1^{a}}{\delta_1^2} + \frac{V_2^{a}}{\delta_2^2} + \frac{V_3^{a}}{\delta_3^2}$$

$$V_{out} = V_{in}(1 + 4QC/\delta_1^2)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right) \times \exp - j\beta_e(1 - Cy_1)l \times \exp(\beta_e C x_1 l)$$

$$QC = 0$$

$$V_{out} = V_{in}\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right) \times \exp - j\beta_e(1 - Cy_1)l \times \exp(\beta_e C x_1 l)$$

For the contributions from all the three forward wave components, and taking $l_1$ as the distance where the attenuator begins

$$V_1^a = V_{in}\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)\exp(\beta_e C x_1 l_1)\exp - j\beta_e(1 - Cy_1)l_1$$

$$V_2^a = V_{in}\left(\frac{1}{(1-\delta_3/\delta_2)(1-\delta_1/\delta_2)}\right)\exp(\beta_e C x_2 l_1)\exp - j\beta_e(1 - Cy_2)l_1$$

$$V_3^a = V_{in}\left(\frac{1}{(1-\delta_1/\delta_3)(1-\delta_2/\delta_3)}\right)\exp(\beta_e C x_3 l_1)\exp - j\beta_e(1 - Cy_3)l_1$$

$$\left.\begin{aligned} x_1 &= \sqrt{3}/2, y_1 = -1/2 \\ x_2 &= -\sqrt{3}/2, y_2 = -1/2 \\ x_3 &= 0, y_3 = 1 \end{aligned}\right\} \Leftarrow \left.\begin{aligned} \delta_1 &= x_1 + jy_1 = \sqrt{3}/2 - j(1/2) \\ \delta_2 &= x_2 + jy_2 = -\sqrt{3}/2 - j(1/2) \\ \delta_3 &= x_3 + jy_3 = j \end{aligned}\right\} \Leftarrow b = QC = d = 0$$

$\Downarrow$

$$V_1^a = \frac{V_{in}}{3}\exp(\beta_e C x_1 l_1)\exp - j\beta_e(1 - Cy_1)l_1$$

$$V_2^a = \frac{V_{in}}{3}\exp(\beta_e C x_2 l_1)\exp - j\beta_e(1 - Cy_2)l_1$$

$$V_3^a = \frac{V_{in}}{3}\exp(\beta_e C x_3 l_1)\exp - j\beta_e(1 - Cy_3)l_1$$

$\Rightarrow$

$V_1^b + V_2^b + V_3^b = 0$ ('cold' attenuation = $\infty$)

$$\frac{V_1^b}{\delta_1} + \frac{V_2^b}{\delta_2} + \frac{V_3^b}{\delta_3} = \frac{V_1^a}{\delta_1} + \frac{V_2^a}{\delta_2} + \frac{V_3^a}{\delta_3}$$ (recalled)

$$\frac{V_1^b}{\delta_1^2} + \frac{V_2^b}{\delta_2^2} + \frac{V_3^b}{\delta_3^2} = \frac{V_1^a}{\delta_1^2} + \frac{V_2^a}{\delta_2^2} + \frac{V_3^a}{\delta_3^2}$$ (recalled)

Solving for $V_1^b$ $\Downarrow$ $\beta_e l = 2\pi N$ (recalled)

$$V_1^b = \frac{V_{in}}{3}\exp - (j2\pi N_1)\left[\frac{2}{3}\exp(2\pi C N_1(x_1 + jy_1)) - \frac{1}{3}\exp(2\pi C N_1(x_2 + jy_2)) - \frac{1}{3}\exp(2\pi C N_1(x_3 + jy_3))\right]$$

$$V_1^b = \frac{V_{in}}{3}\exp-(j2\pi N_1)[\frac{2}{3}\exp(2\pi CN_1(x_1+jy_1)) - \frac{1}{3}\exp(2\pi CN_1(x_2+jy_2))$$

$$-\frac{1}{3}\exp(2\pi CN_1(x_3+jy_3))]$$ (rewritten)

$$(x_1=\sqrt{3}/2, y_1=-1/2; x_2=\sqrt{3}/2, y_2=-1/2; x_3=0, y_3=1)$$

$$V_1^a = \frac{V_{in}}{3}\exp(\beta_e Cx_1 l_1)\exp-j\beta_e(1-Cy_1)l_1$$ (recalled) ⟸ $\beta_e l = 2\pi N$

⇓

$$V_1^a = \frac{V_{in}}{3}\exp(-j2\pi N_1)\exp(2\pi CN_1(x_1+jy_1))$$

$$(x_1=\sqrt{3}/2, y_1=-1/2; x_2=\sqrt{3}/2, y_2=-1/2; x_3=0, y_3=1)$$

$$\left|\frac{V_1^b}{V_1^a}\right| = \left|\frac{2}{3}+\frac{1}{3}\exp-(2\pi CN_1\sqrt{3})+\frac{1}{3}\exp-2\pi CN_1(\frac{\sqrt{3}}{2}-j\frac{3}{2})\right|$$

Taking $CN_1 > 0.2$ (typical practical values)

$$\left|\frac{V_1^b}{V_1^a}\right| \cong \frac{2}{3}$$

'Hot' Attenuation ~ $20\log_{10}3/2 = 3.52$ dB (typically, though 'Cold' Attenuation = ∞ !

## Johnson's  Start-Oscillation Condition

Johnson's start-oscillation condition
(H. R. Johnson, "Backward-wave oscillators, " *Proc. IRE*, June 1955, pp. 684-694)

Backward-wave mode: The phase velocity $v_p$ of the slow-wave structure is positive and its group velocity $v_g$ is negative.

Let us recall the following:

$\Gamma_0 = j\beta_0 =$ cold circuit propagation constant

$\beta_0 = \beta_e(1+bC)$ ($b$ =velocity synchronization parameter)

$\Gamma_0 = j\beta_0 = j\beta_e(1+bC)$ (in the absence of circuit loss)

$\Gamma_0 = \beta_e Cd + j\beta_e(1+bC)$ ($d$ = structure loss parameter)

$$\frac{-\Gamma\Gamma_0 K}{(\Gamma+\Gamma_0)(\Gamma-\Gamma_0)} = \frac{2V_0}{I_0}\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}$$ (TWT dispersion relation)

$$(\delta^2+4QC)(j\delta+jd-b)=1$$ (TWT cubic dispersion relation)

$$\frac{-\Gamma\Gamma_0 K}{(\Gamma+\Gamma_0)(\Gamma-\Gamma_0)} = \frac{2V_0}{I_0}\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}$$

(TWT dispersion relation) (recalled)

For power flow in the opposite direction (backward-wave mode) $K$ has to be interpreted with a change of sign

$$\frac{+\Gamma\Gamma_0 K}{(\Gamma+\Gamma_0)(\Gamma-\Gamma_0)} = \frac{2V_0}{I_0}\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}$$

(plus sign in the left hand side)

Consequent change in the sign of the right hand side of the TWT cubic dispersion relation: $(\delta^2+4QC)(j\delta+jd-b)=1$

$$\boxed{(\delta^2+4QC)(j\delta+jd-b)=-1}$$

(cubic dispersion relation corresponding to the backward-wave mode) (minus sign in the right hand side)

$$(\delta^2 + 4QC)(j\delta + jd - b) = -1$$

(cubic dispersion relation corresponding to the backward-wave mode)

(to be recalled)

Output voltage for backward-wave mode:

Contribution from the growing-wave component:

$$V_{\text{out}} = V_{\text{in}}(1 + 4QC/\delta_1^2)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right) \times \exp(\beta_e C x_1 l)\exp - j\beta_e(1 - Cy_1)l$$

(recalled)

Contribution from the growing-wave component:

$$V_{out} = V_{in}(1+4QC/\delta_1^2)(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)})$$
$$\times \exp(\beta_e C x_1 l)\exp - j\beta_e(1-Cy_1)l$$

(rewritten)

Combing the contributions from all the three wave components, we obtain:

$$V_{out} = V_{in}(1+4QC/\delta_1^2)(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)})$$
$$\times \exp(\beta_e C x_1 l)\exp - j\beta_e(1-Cy_1)l$$
$$+V_{in}(1+4QC/\delta_2^2)(\frac{1}{(1-\delta_3/\delta_2)(1-\delta_1/\delta_2)})$$
$$\times \exp(\beta_e C x_2 l)\exp - j\beta_e(1-Cy_2)l$$
$$+V_{in}(1+4QC/\delta_3^2)(\frac{1}{(1-\delta_1/\delta_3)(1-\delta_2/\delta_3)})$$
$$\times \exp(\beta_e C x_3 l)\exp - j\beta_e(1-Cy_3)l$$

$$V_{out} = V_{in}(1+4QC/\delta_1^2)(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)})$$
$$\times \exp(\beta_e C x_1 l) \exp - j\beta_e(1-Cy_1)l$$
$$+V_{in}(1+4QC/\delta_2^2)(\frac{1}{(1-\delta_3/\delta_2)(1-\delta_1/\delta_2)})$$
$$\times \exp(\beta_e C x_2 l) \exp - j\beta_e(1-Cy_2)l$$
$$+V_{in}(1+4QC/\delta_3^2)(\frac{1}{(1-\delta_1/\delta_3)(1-\delta_2/\delta_3)})$$
$$\times \exp(\beta_e C x_3 l) \exp - j\beta_e(1-Cy_3)l$$

(rewritten)

$$\beta_e l = 2\pi N$$

$$e^{j2\pi N}\frac{V_{out}}{V_{in}} = \left(\frac{\delta_1^2 + 4QC}{(\delta_1-\delta_2)(\delta_1-\delta_3)}\right)e^{2\pi CN\delta_1}$$
$$+\left(\frac{\delta_2^2 + 4QC}{(\delta_2-\delta_3)(\delta_2-\delta_1)}\right)e^{2\pi CN\delta_2}$$
$$+\left(\frac{\delta_3^2 + 4QC}{(\delta_3-\delta_1)(\delta_3-\delta_2)}\right)e^{2\pi CN\delta_3}$$

$$e^{j2\pi N}\frac{V_{out}}{V_{in}} = \left(\frac{\delta_1^{\,2}+4QC}{(\delta_1-\delta_2)(\delta_1-\delta_3)}\right)e^{2\pi CN\delta_1} + \left(\frac{\delta_2^{\,2}+4QC}{(\delta_2-\delta_3)(\delta_2-\delta_1)}\right)e^{2\pi CN\delta_2} + \left(\frac{\delta_3^{\,2}+4QC}{(\delta_3-\delta_1)(\delta_3-\delta_2)}\right)e^{2\pi CN\delta_3}$$

(rewritten)

$$\frac{V_{out}}{V_{in}} = 0$$

(oscillation condition)

(infinite voltage at the input end)

$$\left(\frac{\delta_1^{\,2}+4QC}{(\delta_1-\delta_2)(\delta_1-\delta_3)}\right)e^{2\pi CN\delta_1} + \left(\frac{\delta_2^{\,2}+4QC}{(\delta_2-\delta_3)(\delta_2-\delta_1)}\right)e^{2\pi CN\delta_2} + \left(\frac{\delta_3^{\,2}+4QC}{(\delta_3-\delta_1)(\delta_3-\delta_2)}\right)e^{2\pi CN\delta_3} = 0$$

(to be recalled)

The following parameters are relevant to finding the parameter $CN$

$$QC = (\frac{Q}{N})(CN)$$

$$\boxed{\frac{Q}{N} = \frac{QC}{CN} = \frac{2|\eta| V_0}{\varepsilon_0 r_b^2 \omega^3 K l}}$$

(independent of beam current)

$\omega$ has to be interpreted as the frequency where the phase velocity of the forward-wave mode of the SWS becomes equal to that of the backward-wave mode. $K$ has to be taken as the interaction impedance at this frequency.

$$C^3 = \frac{KI_0}{4V_0}$$

$$QC = \frac{1}{4}(\frac{\beta_p}{\beta_e C})^2 = \frac{1}{4}(\frac{\omega_p / v_0}{\omega / v_0 C})^2$$

$$\omega_p^2 = \frac{|\eta||\rho_0|}{\varepsilon_0}$$

$$J_0 = \rho_0 v_0$$

$$|J_0| = \frac{I_0}{\pi r_b^2}$$

$r_b =$ beam radius

$$\beta_e l = 2\pi N$$

One can simultaneously solve the following two equations for $CN$ :

(i)

$$(\delta^2 + 4(\frac{Q}{N})(CN))(j\delta - jd - b) = -1 \quad \longleftarrow \quad (\delta^2 + 4QC)(j\delta - jd - b) = -1$$ (recalled)

$$QC = (\frac{Q}{N})(CN)$$

(ii)

$$\left(\frac{\delta_1^{\ 2} + 4QC}{(\delta_1 - \delta_2)(\delta_1 - \delta_3)}\right)e^{2\pi CN\delta_1} + \left(\frac{\delta_2^{\ 2} + 4QC}{(\delta_2 - \delta_3)(\delta_2 - \delta_1)}\right)e^{2\pi CN\delta_2} + \left(\frac{\delta_3^{\ 2} + 4QC}{(\delta_3 - \delta_1)(\delta_3 - \delta_2)}\right)e^{2\pi CN\delta_3} = 0$$ (recalled)

The solution for $CN$ thus obtained may be interpreted as the start-oscillation current $I_0$ while making use of the relations:

$C = (KI_0/(4V_0))^{1/3}$ and $\beta_e l = 2\pi N$

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## Some concepts in widening helix-TWTs

Zero-to-slightly-negative-dispersion structure for wideband performance
Anisotropically loaded helix:
  Metal vane/ segment loaded envelope
Inhomogeneously loaded helix:
  Helix with tapered geometry dielectric supports such as half-moon-shaped and T-shaped supports

Negative dispersion ensures constancy of Pierce's velocity synchronization parameter *b* with frequency

Multi-dispersion structures for wideband performance

Constancy of $b$ with frequency with negative dispersion

$$b = \frac{v_0 - v_p}{v_p C} = \frac{v_0 - v_p}{v_p (KI_0 / 4V_0)^{1/3}} = \frac{\dfrac{v_0 - v_p}{v_p}}{K^{1/3}} \frac{1}{(I_0 / 4V_0)^{1/3}}$$

Negative dispersion: $v_p$ increases with frequency

$v_0 - v_p$ decreases with frequency

$\dfrac{v_0 - v_p}{v_p}$ decreases with frequency

$\rightarrow$ Numerator of the expression for $b$ decreases with frequency

$K$ decreases with frequency and hence the
$\rightarrow$ Denominator of the expression for $b$ decreases with frequency

$b$ remains constant with frequency

# Conventional TWTs with multi-dispersion, multi-section structures

Small-signal gain equation $G \sim BCN$

$$N\lambda_e = l$$

$$N\frac{v_0}{f} = l$$

$$C = (KI_0 / 4V_0)^{1/3}$$

$$N = \frac{f\,l}{v_0}$$

$$G \sim B(KI_0 / 4V_0)^{1/3}\frac{f\,l}{v_0}$$

$G$ is proportional to $K^{1/3} f\,l$

$G$ is proportional to $K^{1/3} f\, l$

Gain-frequency response:
Lower gain at lower frequencies as $G$ is proportional to $f$
Lower gain at higher frequencies as $G$ is proportional to $K^{1/3}$, the latter decreasing with frequency

Conventional structure: If you had increased the length $l$, then the gain $G$ would be compensated at lower frequencies $f$. However, then the gain $G$ would become very high at higher frequencies $f$ entailing the risk of oscillation in the device. Therefore, let us arrive at the design of a helical slow-wave structure the effective length of which is large at lower frequencies, which at the same time becomes relatively smaller at higher frequencies. (The design should ensure that the gain is not enhanced at any frequency to a high value causing oscillation in the device). The answer lies in a multi-dispersion, multi-section helix TWTs!

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One positive-dispersion helix section of length $l_1$ synchronous with the beam only at lower frequencies and the other nearly dispersion-free helix section of effective length $l_2$ synchronous with the beam both at lower and higher frequencies.

Effective length increased to $l_1 + l_2$ at lower frequencies
Effective length reduced to $l_2$ at higher frequencies (since the section of length $l_1$ goes out of synchronism at higher frequencies

Gain is proportional to $K^{1/3} f\, l$

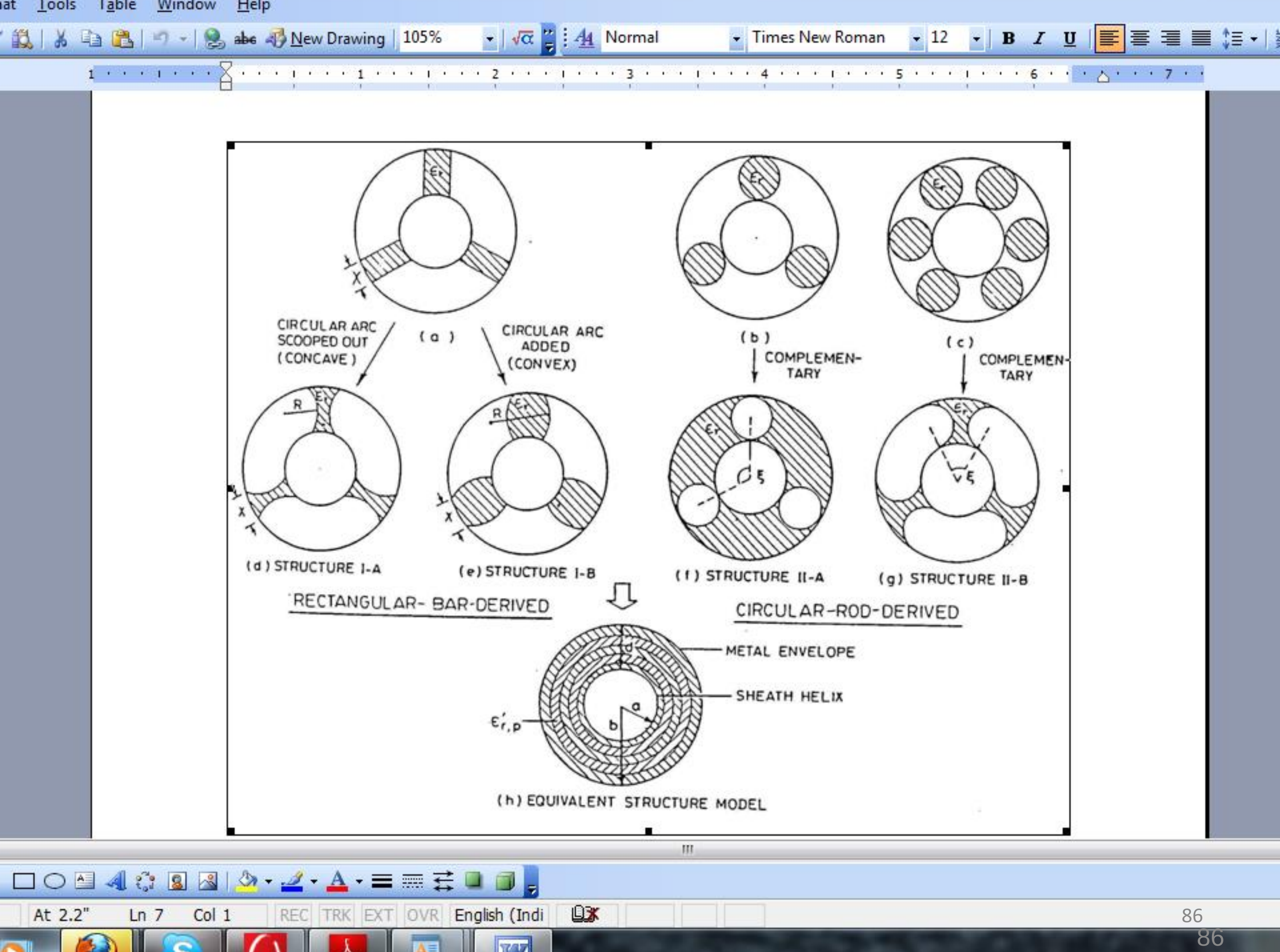
We have to control (i) the nature and the amounts of dispersion of the sections by suitably loading the sections and (ii) the lengths of the two sections

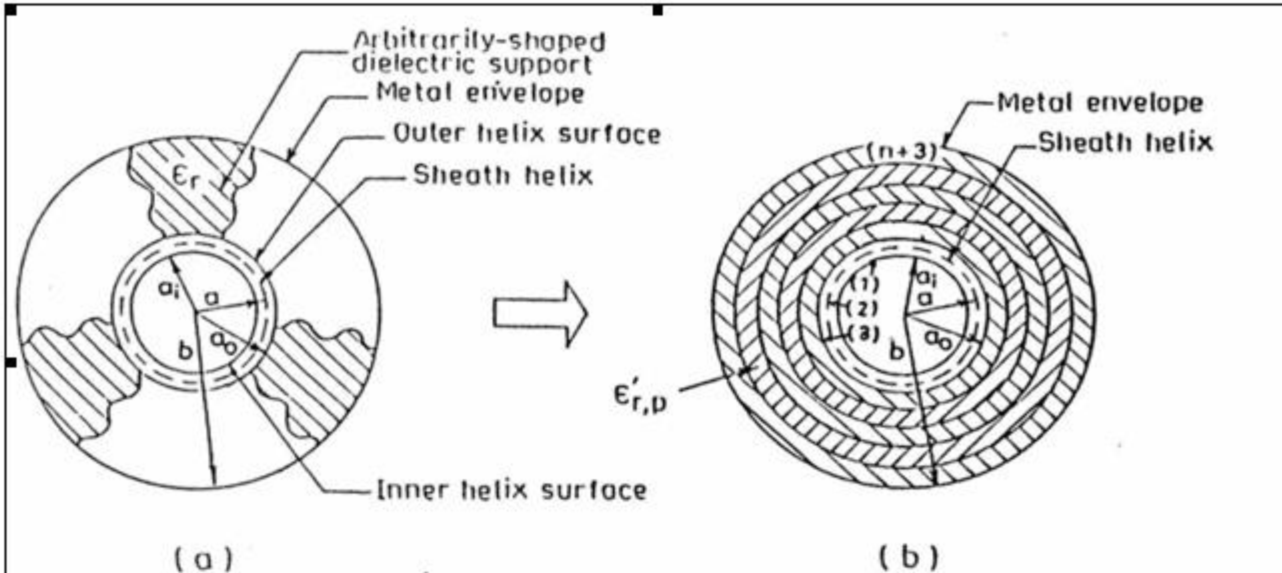
Select structure sections such as segment-loaded helices of controllable dispersion

Analysis should be capable of finding the dispersion and interaction impedance characteristics of the structure sections, say, with metal segment loaded envelopes and their control by structure section parameters like segment dimensions and relative section lengths.

Two-section configuration with one of the sections providing a double-hump peaks in the gain frequency response while the second section providing a single peak between the humps of the first section

Twystron: The first section is a klystron providing a double-hump gain-frequency response. The second section is a TWT providing a peak between the two humps of the first section in the gain-frequency response.


CIRCULAR ARC SCOOPED OUT (CONCAVE)
(a)
CIRCULAR ARC ADDED (CONVEX)
(b)
COMPLEMEN-TARY
(c)
COMPLEMEN-TARY
(d) STRUCTURE I-A
(e) STRUCTURE I-B
(f) STRUCTURE II-A
(g) STRUCTURE II-B
RECTANGULAR-BAR-DERIVED
CIRCULAR-ROD-DERIVED
METAL ENVELOPE
SHEATH HELIX
(h) EQUIVALENT STRUCTURE MODEL




Cross section of a helical slow-wave structure supported by arbitrarily shaped identical dielectric bars **(a)** and its equivalent analytical model **(b)**.

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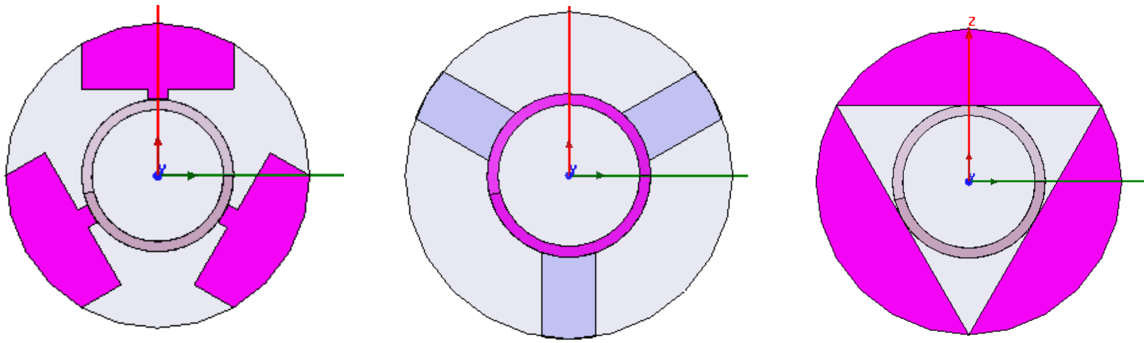
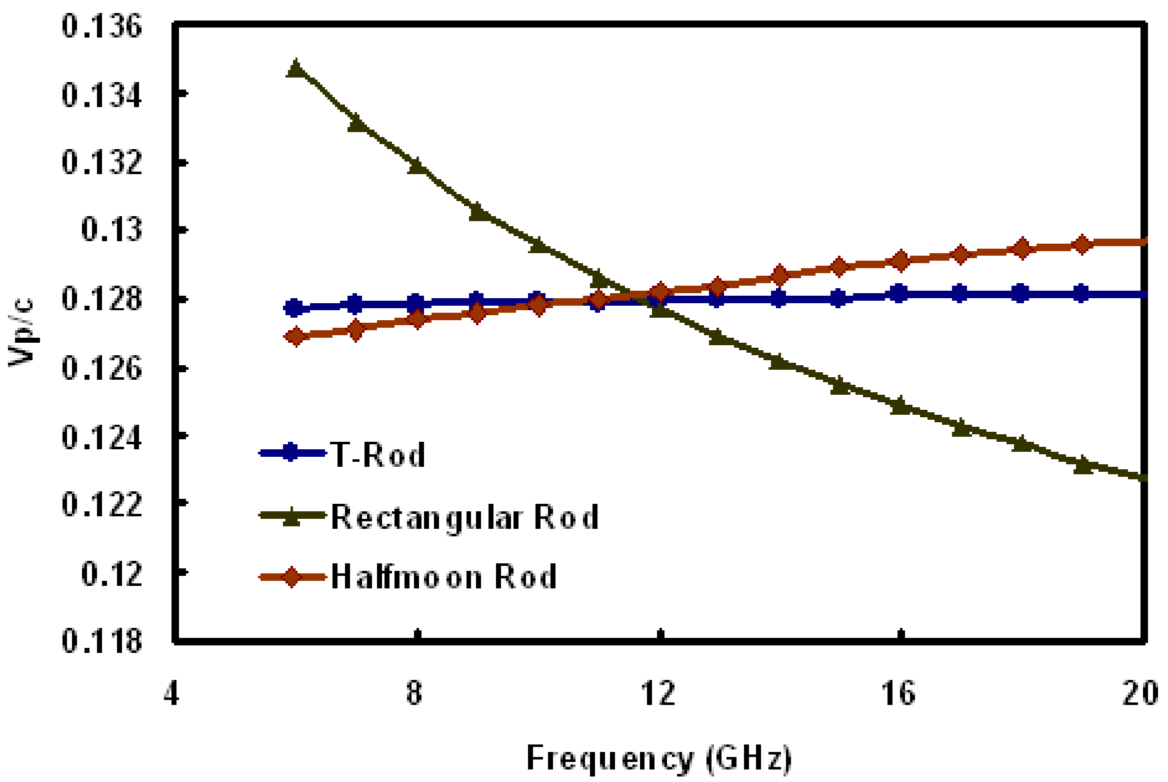
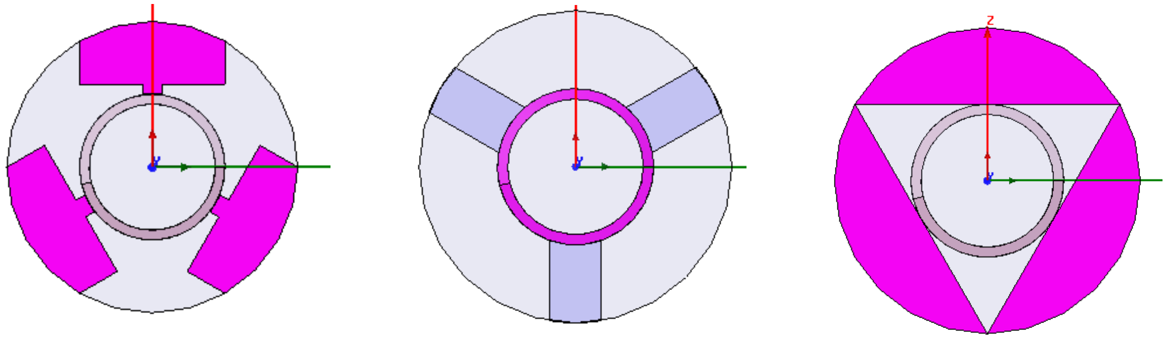


*Source: MTRDC (DRDO)*





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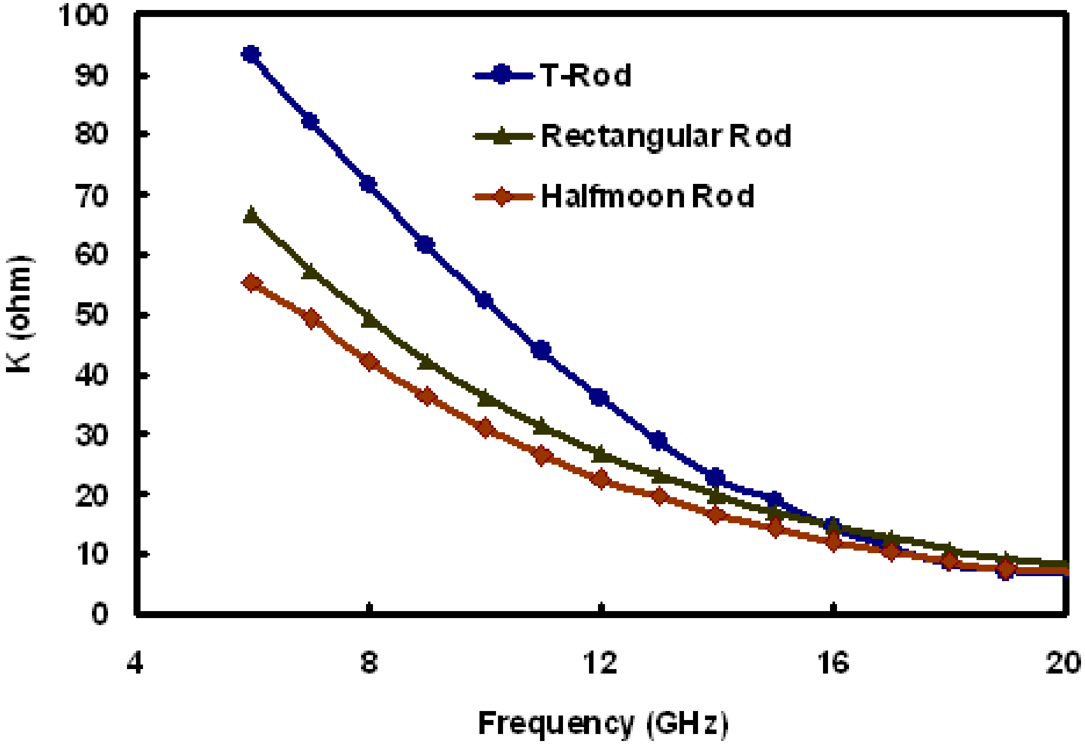

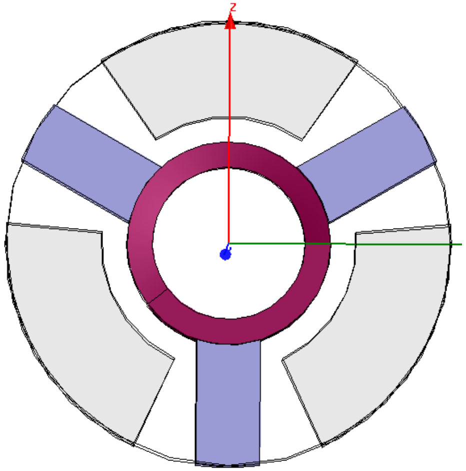


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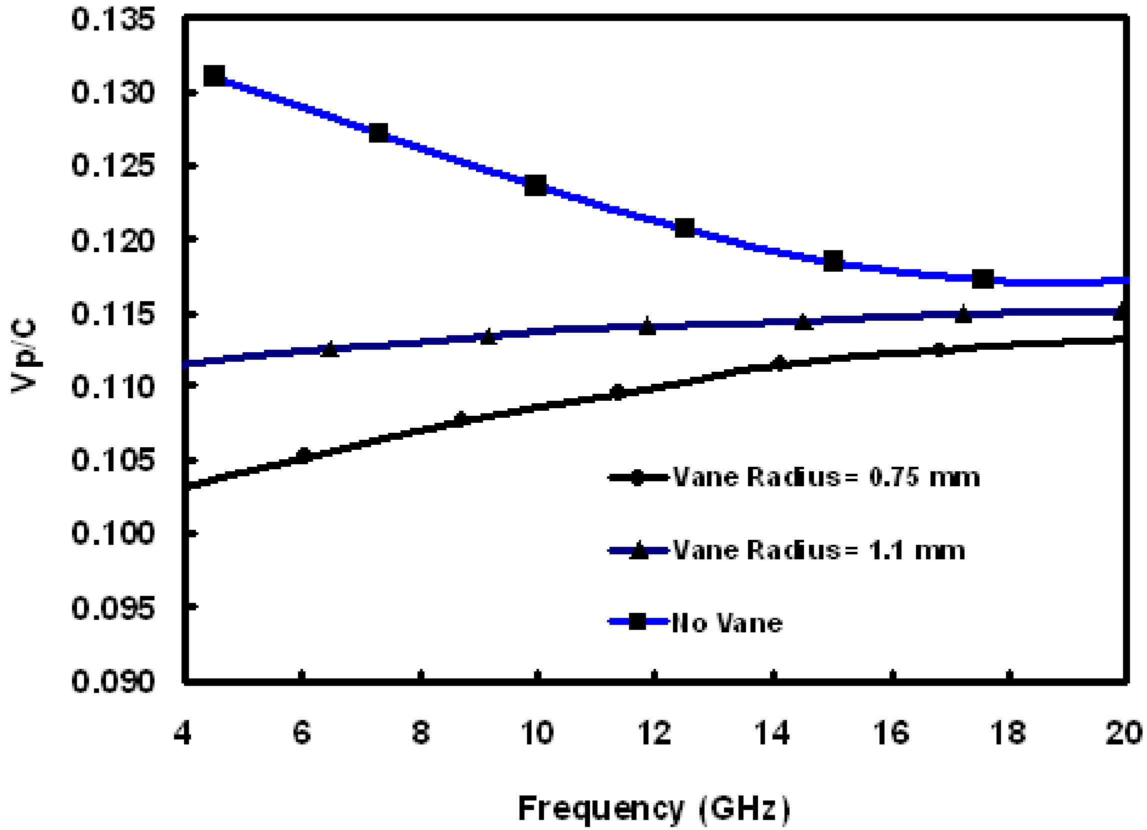
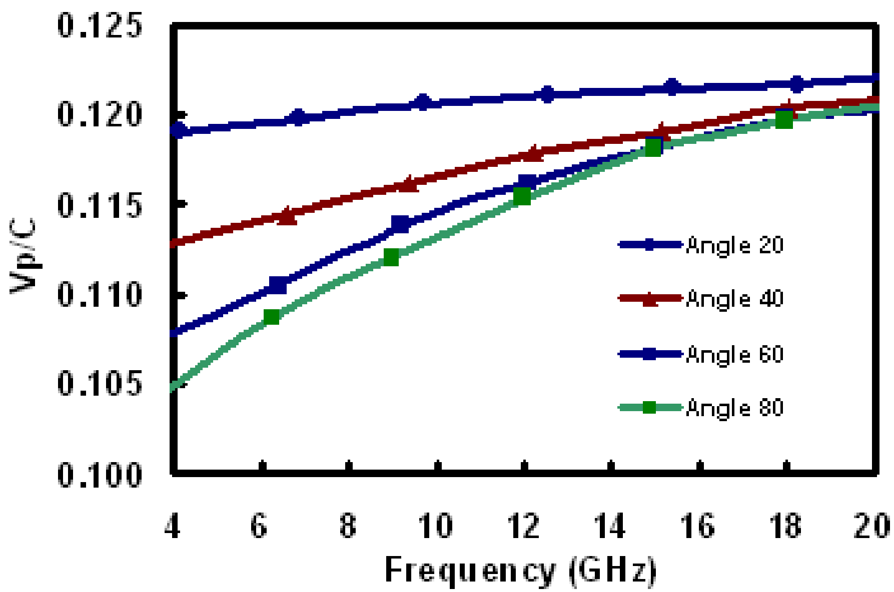
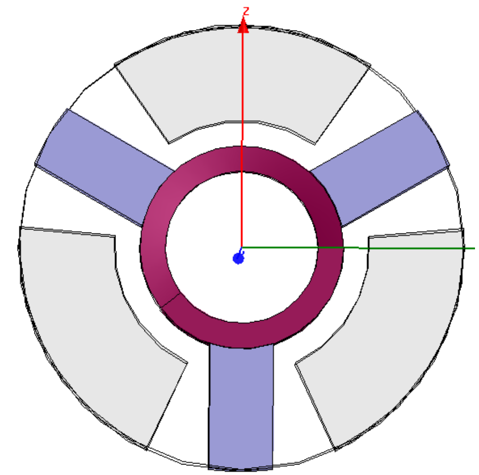
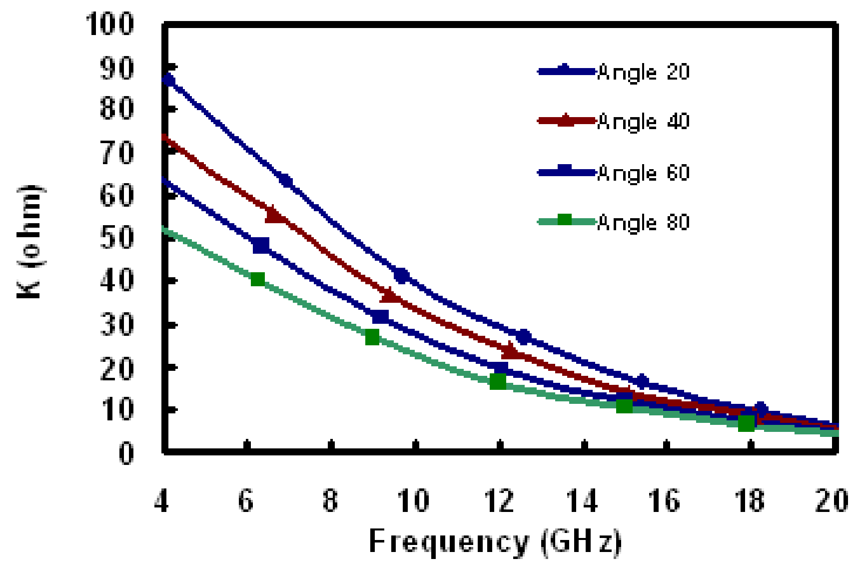
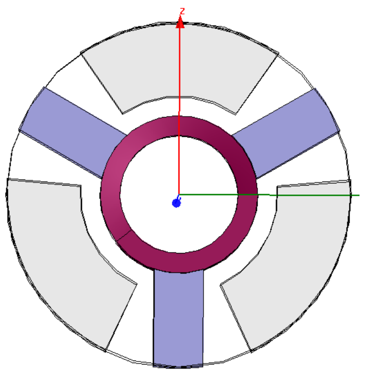




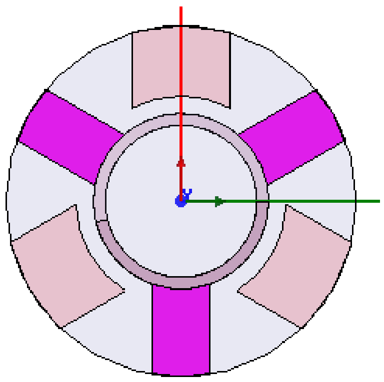




*Source: MTRDC (DRDO)*

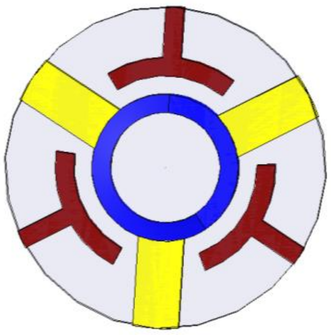




Angular segment

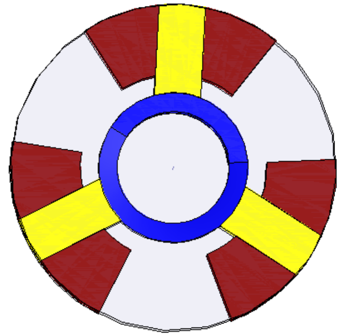


Straight segment



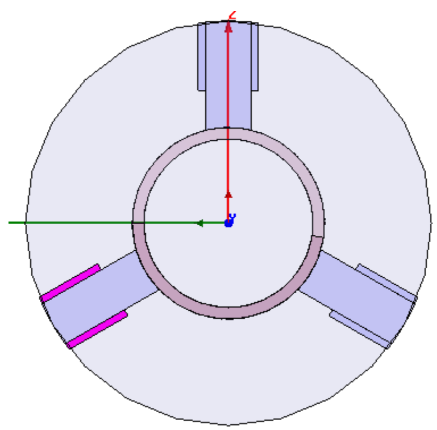
T- segment

*Source: MTRDC (DRDO)*

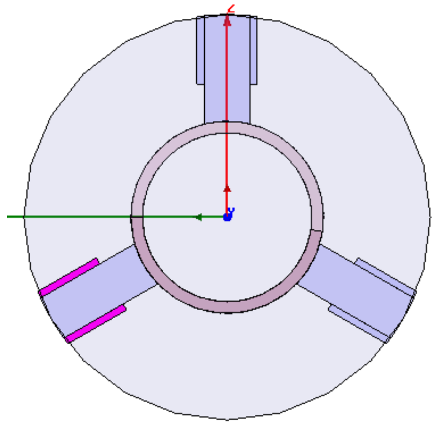


Helix-support-rod embedded
in the metal vane

*Source: MTRDC (DRDO)*

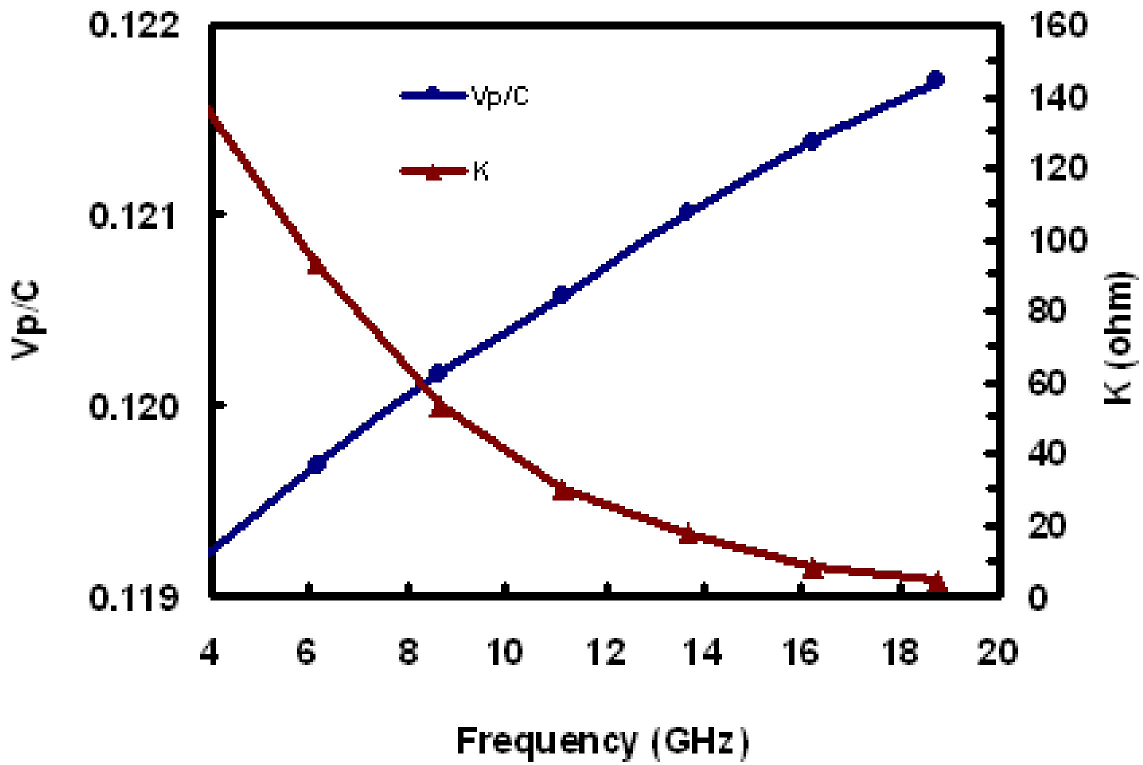


Metal-coated dielectric helix-support rods



Provides negative dispersion with quite high interaction impedance compared to other segment variants

*Source: MTRDC (DRDO)*



## Broadbanding Techniques

- ❑ Homogeneous dielectric loading cannot shape structure dispersion for wide device bandwidths.

- ❑ Inhomogeneous dielectric loading by tapered-cross-section helix supports can shape structure dispersion for wide device bandwidths.

- ❑ Anisotropic loading by azimuthally periodic metal vanes provided with the metal envelope can shape structure dispersion for wide device bandwidths.

- ❑ Axially periodic disc loading cannot shape structure dispersion and cannot widen device bandwidth but can enhance device gain.

The bandwidth of a TWT can be widened by

- providing azimuthally periodic metal vanes/segments with the metal envelope of the helical slow-wave structure and optimizing the structure parameters, thereby obtaining the desired dispersion characteristics of the structure for wide device bandwidths
- appropriately shaping the cross-sectional geometry of dielectric helix supports, thereby obtaining the desired dispersion characteristics of the structure for wide device bandwidths
- using multi-dispersion, multi-section helical structures

In the area of helix-travelling-wave tubes, I worked with Drs.

- S. S. S. Agarwala, S. N. Joshi, R. K. Gupta, R. S. Raju, Lalit Kumar
  (*CSIR-CEERI, Pilani*)

- S. K. Datta, V. Kesari, M. D. Raj Narayan
  (*DRDO-MTRDC, Bengaluru*)

- Ashok K Sinha, P. K. Jain, Sanjay K Ghosh, M.V. Kartikeyan, S. J. Rao,
  U. Tiwari, M. P. Sinha, V. N. Singh, V. V. P. Singh, K. V. R. Murthy
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- R. G. Carter, Ping Wang
  (*Lancaster University*)

- Gun-Sik Park, B. Jia, A. V. Soukhov, S. S. Jung, C. W. Baik, Y. D. Joo, S. T. Han,
  H. S. Kim, H. S. Uhm
  (*Seoul National University*)

- Zhaoun Duan
  (University of Electronic Science and Technology of China)

- Raktim Guha, A. K. Varshney
  (*Supreme Knowledge Foundation Group of Institution, Mankundu*)

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*Courtesy: Mr. Uttam Goswami*


Thank you!