BEAM-WAVE INTERACTION IN HELIX-TRAVELLING-WAVE TUBES
through

$$\bar{J} = \rho\vec{v};\ \vec{E} = -\nabla V;\ \nabla.\vec{J} + \frac{\partial\rho}{\partial t} = 0;\ \nabla.\vec{E} = \frac{\rho}{\varepsilon}$$

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“Ali said to Kaamil (al-insān al-kāmil):

Knowledge is better than wealth. Knowledge protects you while you have to protect your wealth. Knowledge is a judge, while wealth has to be judged on. Wealth decreases when it is expended, while knowledge purifies when it is given.”

“Ali said in a poem:

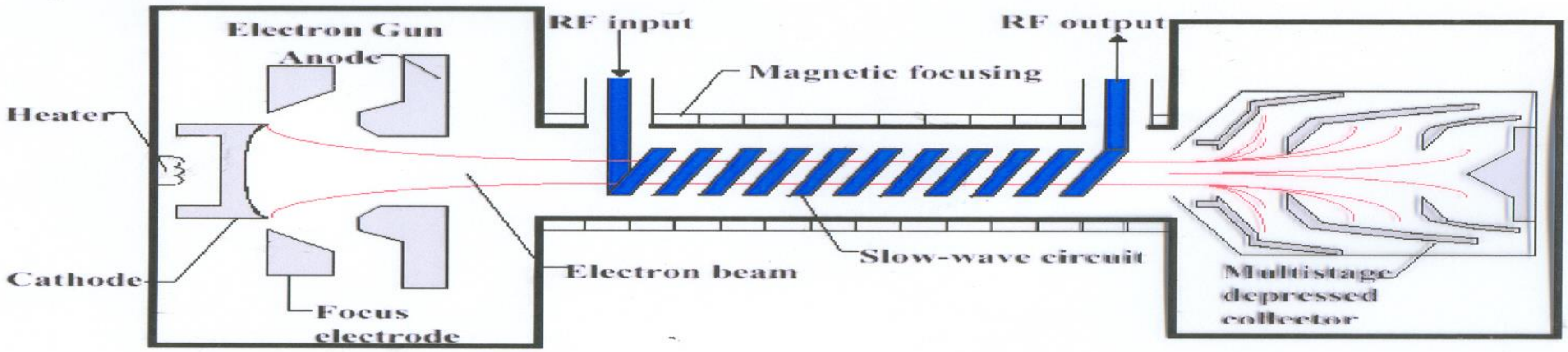
Succeed with knowledge and live energetically forever; men are all dead, only the possessors of knowledge are truly alive."

“What man ‘learns’ is really what he discovers by taking the cover off his own soul, which is a mine of infinite knowledge.”

Dedication

Professor N. B. Chakrabarty, who at the Indian Institute of Technology, Kharagpur, India, mentored my doctoral research in the area of nonlinear Eulerian hydrodynamic analysis of double-stream amplifier (Haeff tube) and beam-plasma amplifier.

Professor Alexander Scott Gilmour, Jr., who responded to my request and authored a paper entitled “An overview of my efforts to bridge the gap in the microwave tube area between what universities provide and what the industry needs” in the Special Issue on “Microwave Tubes and Applications” in the Journal of Electromagnetic Waves and Applications (Taylor and Francis) (issue 17, vol. 31, 2017), which I guest-edited. Dr. Gilmour is the recipient of J. R. Pierce award in 2018.



Travelling-Wave Tube

- Two basic constituents:
  - Electron beam
  - Electromagnetic interaction structure

- Principal parts:
  - Electron gun: beam formation
  - Focusing structure: beam confinement
  - Collector: collection of spent beam
  - Slow-wave structure (SWS)
  - RF input and output couplers
  - Attenuator

“We learn from history that we do not learn from history.” — GWF Hegel

From the historical time line we know
who invented what.

“Success has many fathers, but failure is an orphan.”

Who did invent transmission and reception of radio waves?

(a) G. Marconi

(b) A. S. Popov

(c) J. C. Bose

(d) S. N. Bose

Answer: (c) J. C. Bose

Who did invent travelling-wave tube?

(a) R. Kompfner

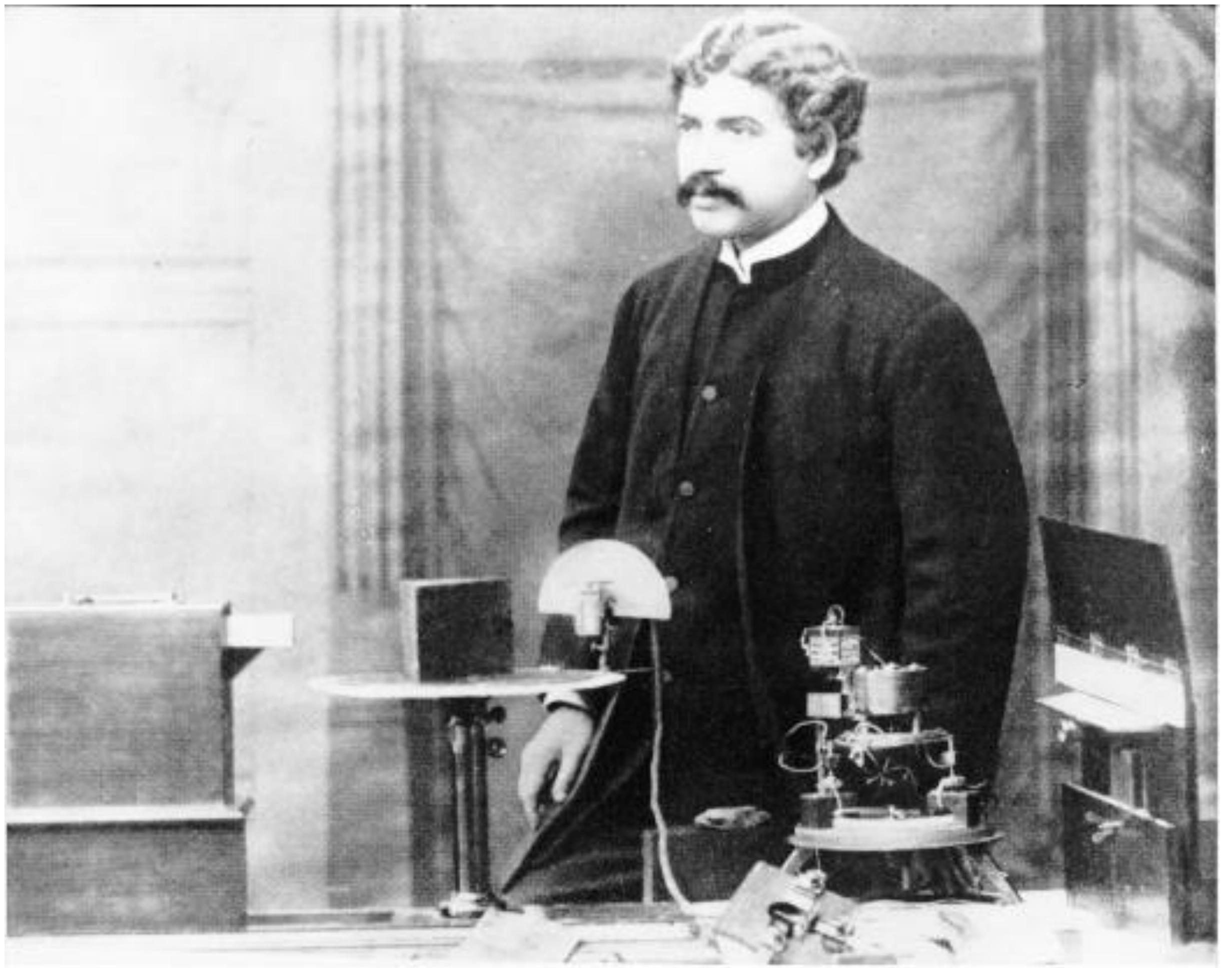
(b) N. E. Lindenblad

(c) J. R. Pierce

(d) A. Haeff

Answer: (d) A. Haeff

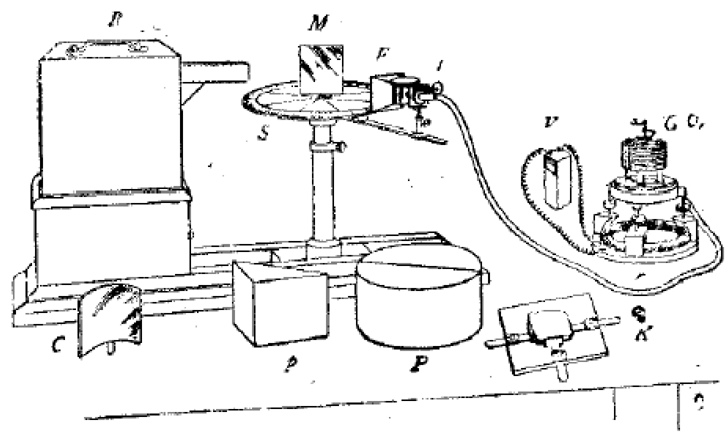
J.C. Bose (1858-1937) at the Royal Institution, London, 1897



J C Bose

IEEE Milestone Plaque for Sir JC Bose

By 1895, Sir. J. C. Bose made the first public demonstration of radio waves in the Kolkata town hall. Details of the apparatus used are vague, but at a distance of 75 feet, he remotely rang an electric bell and ignited a small charge of gunpowder. He called it *Adrisya Alok,* or "Invisible Light". The frequency of operation is nearly 60 GHz. He termed horn antenna as collecting funnel.



R, radiator ; S, spectrometer-circle ; M, plane mirror ; C, cylindrical mirror ; *p*, totally reflecting prism ; P, semi-cylinders ; K, crystal-holder ; F, collecting funnel attached to the spiral spring receiver ; *t*, tangent screw, by which the receiver is rotated ; V, voltaic cell ; *r*, circular rheostat ; G, galvanometer.

Courtesy: C Subhradeep (CEERI)

IEEE Milestone Plaque

IEEE MILESTONE IN ELECTRICAL ENGINEERING AND COMPUTING

First Millimeter-Wave Communication Equipment by JC Bose, 1894-1896

Sir Jagadish Chadra Bose, in 1895, first demonstrated at Presidency College, Calcutta, India, transmission and reception of electromagnetic waves at 60 GHz over a distance of 23 meters, through two intercepting walls by remotely ringing a bell and detonating gunpowder. For this communication system, Bose developed entire millimetre-wave components such as: a spark transmitter, coherer, dielectric lens, polarizer, horn antenna and cylindrical diffraction grating.

September 2012

IEEE Monogram

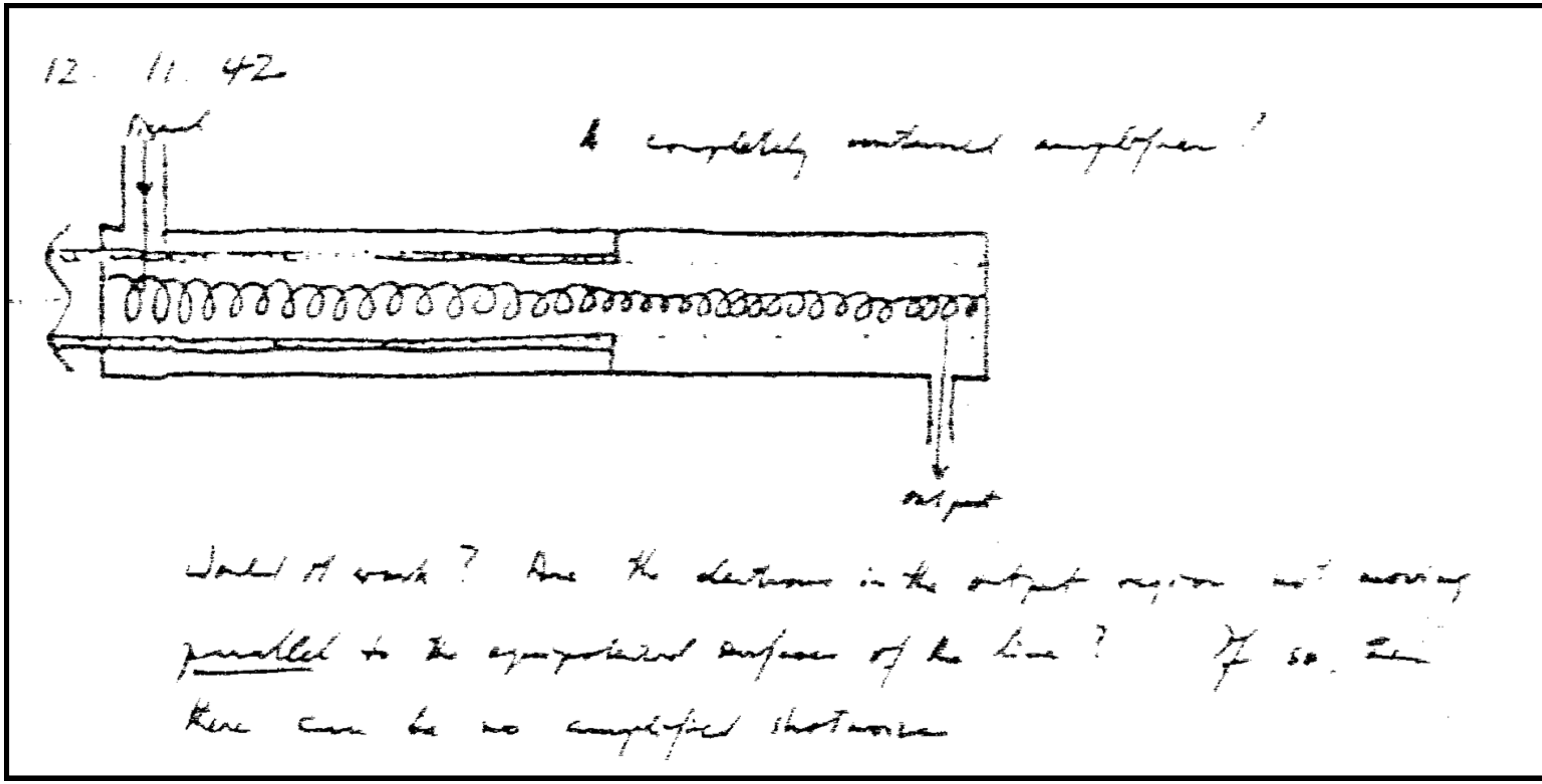
Courtesy: Subhradeep (CEERI)

“In 1895 Bose gave his first public demonstration of electromagnetic waves, using them to ring a bell remotely and to explode some gunpowder. In 1896 the Daily Chronicle of England reported: *"The inventor (J.C. Bose) has transmitted signals to a distance of nearly a mile and herein lies the first and obvious and exceedingly valuable application of this new theoretical marvel."*

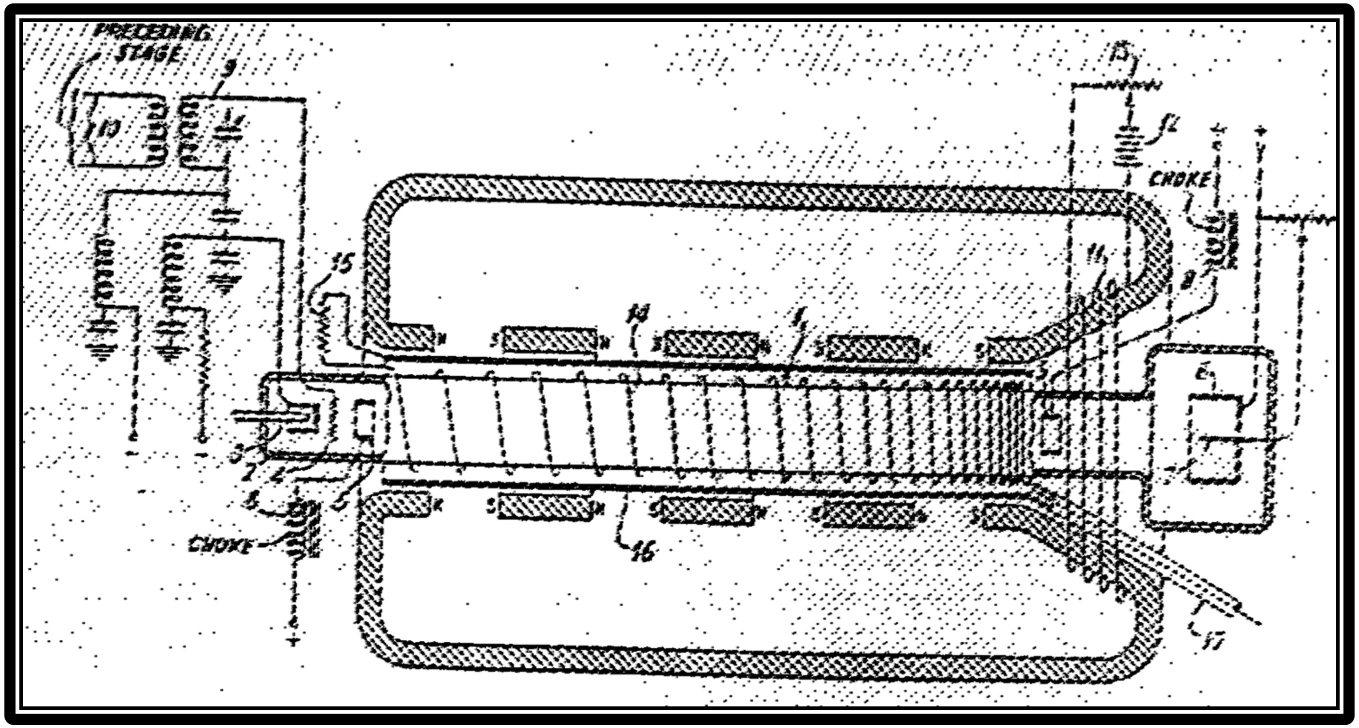
“Popov in Russia was doing similar experiments, but had written in December 1895 that he was still entertaining the hope of remote signaling with radio waves.”

“The first successful wireless signaling experiment by Marconi on Salisbury Plain in England was not until May 1897.”

Source: D. T. Emerson, “The work of Jagadis Chunder Bose: 100 years of mm-wave research,” *IEEE Trans. Microwave Th. Tech*. December 1997, 45, No. 12 (2267-2273)



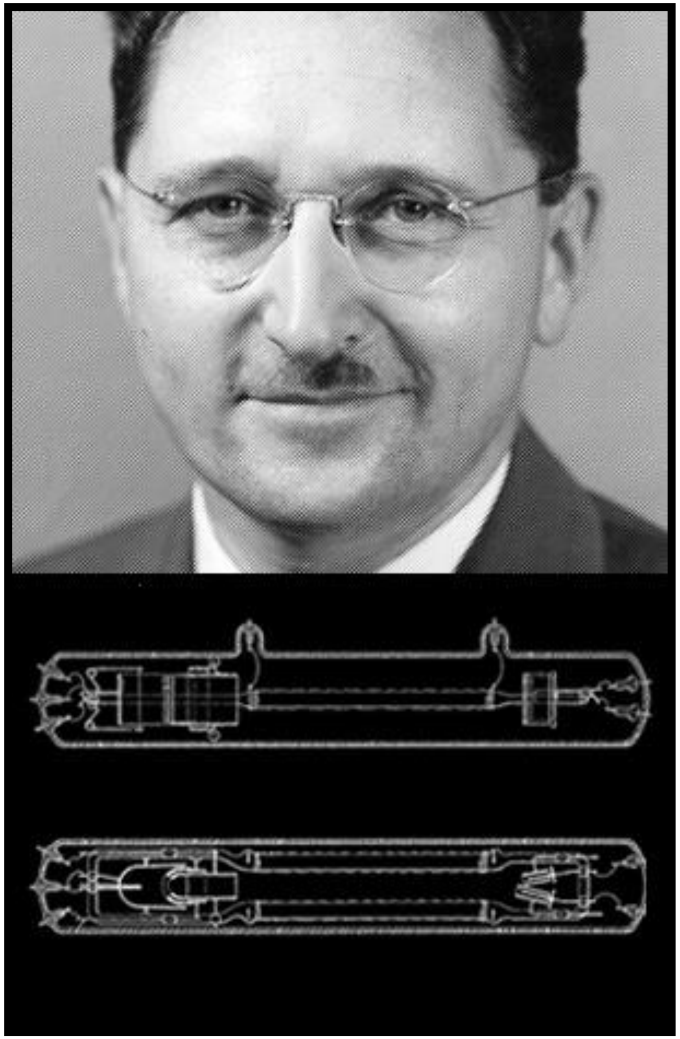
Sketch of the travelling-wave tube from R. Kompfner's note book (1942) (Fig. 12.2 of the book: A.S. Gilmour, "Klystrons, Traveling Wave Tubes, Magnetrons, Crossed-Field Amplifiers, and Gyrotrons," (Artech House, Norwood, 2011))



N. E. Lindenblad's travelling-wave tube amplification at 390 MHz over a 30 MHz band (U. S. Patent 2,300,052, filed on May 4, 1940.

(Fig. 12.1 of the book: A.S. Gilmour, "Klystrons, Traveling Wave Tubes, Magnetrons, Crossed-Field Amplifiers, and Gyrotrons," (Artech House, Norwood, 2011))

Helix wound around the outside the glass envelope. Signal applied to the grid of the electron gun (also applied to the helix in other experiments). Series of permanent magnets (non-periodic). Pitch tapered for velocity re-synchronization



“The patent Andrei Haeff filed in 1933 for a primitive type of traveling-wave tube has been largely ignored.”

Courtesy SK Datta

- Haeff invented TWT in 1933.

(Haeff also invented the double-stream amplifier (Haeff tube), in which two electron beams with slightly different DC velocities are intimately mixed such that the slow space-charge wave of the faster beam couples to the fast space-charge wave of the slower beam resulting in growing waves).

- Lindenblad invented TWT in 1940.

- Kompfner invented TWT, however, not before 1943.

- Pierce and Field significantly contributed to the development of the TWT, however, not before 1947.

Scope

- Historical timeline of the invention of the travelling-wave tube
- Electron bunching and requirement of near-synchronism
- Space-charge waves and coupling to structure wave
- Gain equation
- Hot attenuation
- Johnson’s start oscillation condition
- Some broadbanding aspects

Prerequisite

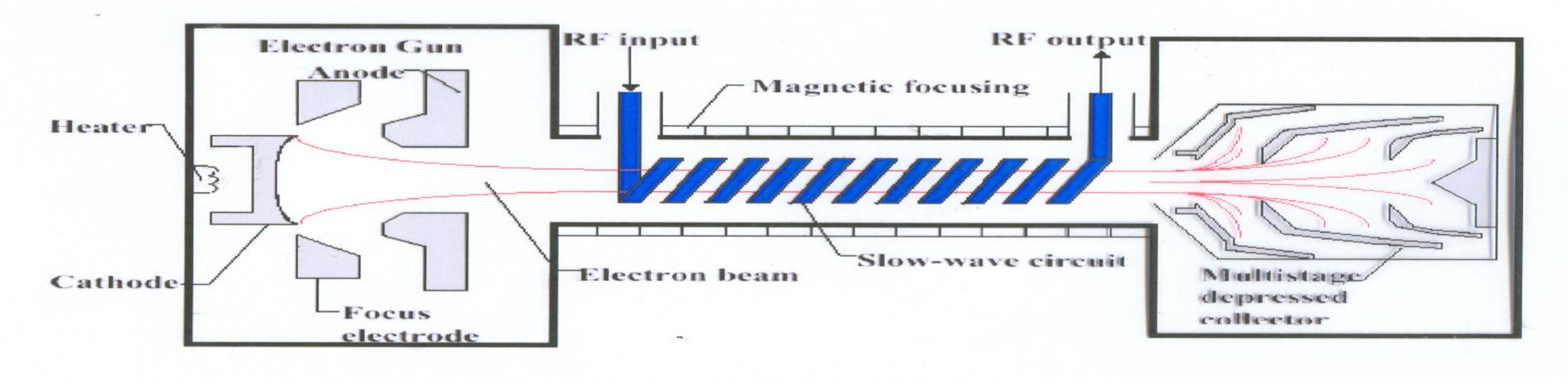
$$\bar{J} = \rho\vec{v};\ \vec{E} = -\nabla V;\ \nabla.\vec{J} + \frac{\partial \rho}{\partial t} = 0;\ \nabla.\vec{E} = \frac{\rho}{\varepsilon}$$

Gain equation of a TWT given in any text book in microwave engineering

Amplification of space-charge waves

An electron beam supports space-charge waves. **Amplification of space-charge waves takes place in**

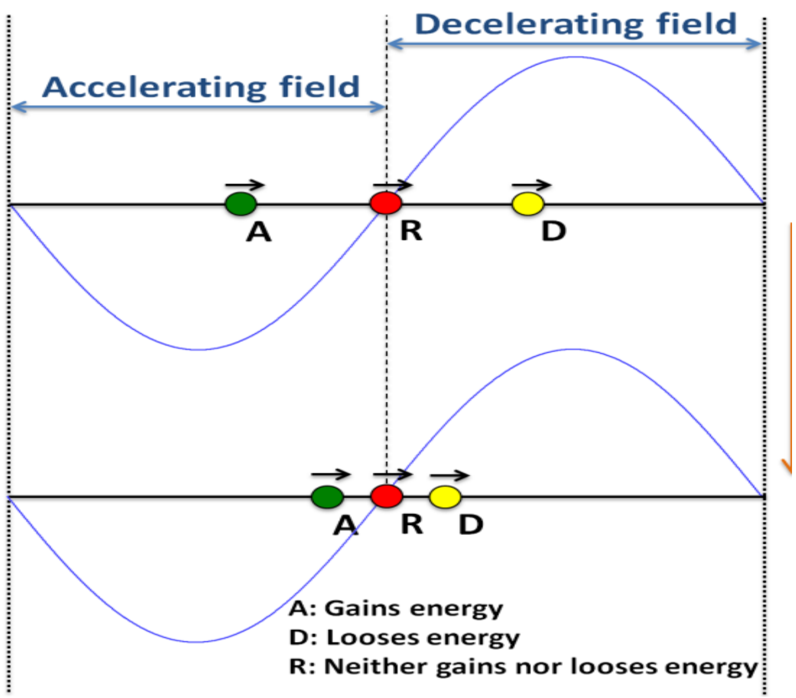
- an electron beam of uniform diameter in a resistive-wall cylindrical waveguide.
- an electron beam in a rippled-wall (varying diameter) conducting-wall cylindrical waveguide.
- an electron beam of varying diameter in a conducting-wall smooth cylindrical waveguide.
- an electron beam mixed with another beam of a slightly different DC electron beam velocity (two-stream amplifier/Haeff tube).
- an electron beam penetrating through a plasma (beam-plasma amplifier)
- **an electron beam interacting with RF waves supported by a slow-wave structure (TWT).**



The TWT is a growing-wave device (tube). It is an amplifier.
It is a slow-wave tube in which the interaction structure is a slow-wave structure (such as helix) that supports RF waves of phase velocity less than the speed *c* of light.
The applied DC magnetic field confines the electron beam in the device; it does not take place in beam-wave interaction.

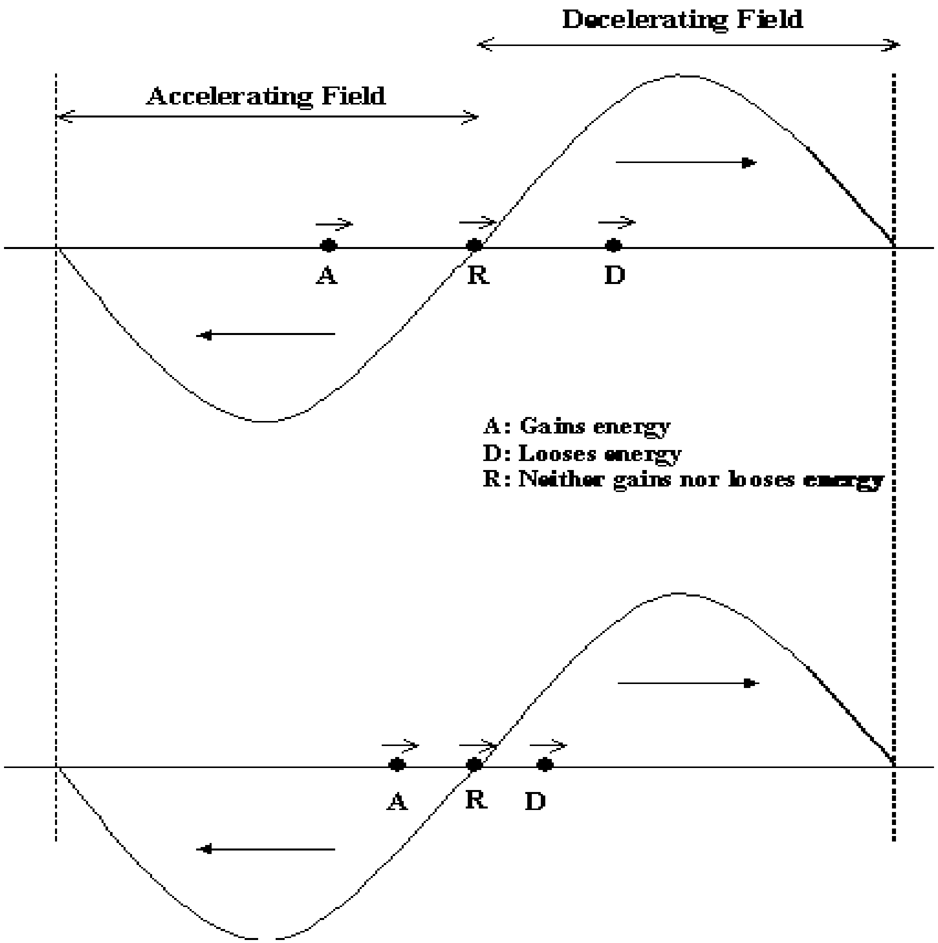
It belongs to the class of linear beam, O-type, Cerenkov radiation type of vacuum electron devices/ microwave tubes. In this tube, the bunched electrons transfer their axial kinetic energy to RF waves.

Axial Bunching in a Travelling-Wave Tube



Bunching of typically two electrons ‘A’ and ‘D’ subjected to the accelerating and decelerating RF electric fields, respectively, in the interaction region of a TWT around a reference electron ‘R’ that experiences no such fields.

Electrons are *bunched* though there is *no net energy transfer* from the electron beam to RF waves.



$$v_0 = v_{ph}$$

Exact synchronism: no net energy transfer between the beam to RF waves

$$v_0 \gtrsim v_{ph}$$

Near-synchronism: net energy transfer from the beam to RF waves

Millimetre-Wave Consideration in Conventional Microwave tubes

B. N. Basu, *Electromagnetic Theory and Applications in Beam-wave Electronics* (World Scientific, Singapore,1996)

$$B^2_{\text{Brillouin}} = \frac{\sqrt{2} I_0}{\pi \varepsilon_0 |\eta|^{3/2} V_0^{1/2} b^2}$$

- Reduction of structure size
- Reduction of beam radius
- Larger magnetic field for beam confinement for
  - ✓ Smaller beam radius b
    Larger beam current I_0
    Smaller beam voltage V_0
    Larger beam perveance $I_0 / (V_0)^{3/2}$
- Heavy solenoids or advanced magnetic materials are required
- Larger cathode emission densities entailing the risk of cathode life

• Higher beam voltage can increase the beam power and also reduce the required magnetic field but is associated with a reduced beam perveance making it difficult to contain thermal electrons, and has limitation arising from backward-wave oscillation in wideband helix TWTs.

• Lower beam current can reduce magnetic field but it reduces the beam power and is associated with a reduced beam perveance making it difficult to contain thermal electrons.

•Tight tolerances required for tiny interaction structures

•Thermal management becomes difficult

Pressure fitting, instead of more effective than brazed-helix technology that would be difficult to implement

Special thermally conducting materials, like Type II-A diamond, for

dielectric helix-supports

Plasma spraying of beryllia on the surface of the helix

A.S. Gilmour, Jr., *Microwave Tubes (Artech House, Washington, 1986).*

COMMERCIALLY AVAILABLE MILLIMETER-WAVE TWT'S

Communication

Type	Frequency Range	Power Output	Duty Cycle	Saturated Gain
814H	91.0-96.0 GH	0.10 kW	CW	25 dB

Space

Type	Frequency Range	Power Output	Duty Cycle	Saturated Gain
944H	42.0-42.5 GHz	100 W	CW	44 dB

Pulsed Radar And ECM

Type	Frequency Range	Power Output	Duty Cycle	Saturated Gain
982H	93.0-95.0 GHz	12kW	0.5	50dB

CW Radar and ECM

Type	Frequency Range	Power Output	Duty Cycle	Saturated Gain
920H	59.7-60.3 GHz	0.05 kW	CW	35 dB

The mechanism of interaction of a slow-wave device such as the TWT is based on the property of an electron beam to support space-charge waves.

SPACE-CHARGE WAVES

Space-Charge Waves

$$\bar{J} = \rho \vec{v}$$

$$J = \rho v \qquad J = J_0 + J_1 \qquad \rho = \rho_0 + \rho_1 \qquad v = v_0 + v_1$$

$$J_0 = v_0 \rho_0$$

$$\frac{\partial J_1}{\partial z} = \rho_0 \frac{\partial v_1}{\partial z} + v_0 \frac{\partial \rho_1}{\partial z} \qquad \frac{\partial J_1}{\partial z} + \frac{\partial \rho_1}{\partial t} = 0$$

$$J_1 = \rho_0 v_1 + v_0 \rho_1$$

$$-\frac{\partial \rho_1}{\partial t} = \rho_0 \frac{\partial v_1}{\partial z} + v_0 \frac{\partial \rho_1}{\partial z} \qquad \frac{\partial \rho_1}{\partial t} + v_0 \frac{\partial \rho_1}{\partial z} = -\rho_0 \frac{\partial v_1}{\partial z}$$

$$D\rho_1 = -\rho_0 \frac{\partial v_1}{\partial z} \qquad D = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}$$

$$\frac{dv_1}{dt} = \frac{e}{m} E_s = \eta E_s \qquad \frac{dv_1}{dt} = \frac{\partial v_1}{\partial t} + \frac{dz}{dt}\frac{\partial v_1}{\partial z} = \frac{\partial v_1}{\partial t} + v \frac{\partial v_1}{\partial z} = \frac{\partial v_1}{\partial t} + (v_0 + v_1)\frac{\partial v_1}{\partial z} = \frac{\partial v_1}{\partial t} + v_0 \frac{\partial v_1}{\partial z}$$

$$\frac{\partial v_1}{\partial t} + v_0 \frac{\partial v_1}{\partial z} = \eta E_s \qquad D v_1 = \eta E_s \qquad [\frac{d}{dt} = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} = D]$$

$$D\rho_1 = -\rho_0 \frac{\partial v_1}{\partial z}$$

$$D = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}$$

$$Dv_1 = \eta E_s$$

$$D^2 \rho_1 = -\rho_0 D \frac{\partial v_1}{\partial z} = -\rho_0 \frac{\partial}{\partial z} Dv_1 = -\rho_0 \frac{\partial}{\partial z} \eta E_s = -\eta \rho_0 \frac{\partial E_s}{\partial z}$$

$$[\frac{d}{dt} = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} = D]$$

$$\nabla . \vec{E} = \frac{\rho}{\varepsilon}$$

$$D^2 \rho_1 = -\eta \rho_0 \frac{\partial E_s}{\partial z} \qquad \frac{\partial E_s}{\partial z} = \frac{\rho_1}{\varepsilon_0}$$

RF quantities vary as *j(ωt-βz)*

$$D^2 \rho_1 = -\eta \rho_0 \frac{\rho_1}{\varepsilon_0} = \frac{-\eta \rho_0}{\varepsilon_0} \rho_1 = \frac{-|\eta||\rho_0|}{\varepsilon_0} \rho_1 = -\omega_p{}^2 \rho_1$$

$$D = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} = j\omega - j\beta v_0 = j(\omega - \beta v_0)$$

$$D^2 = -\omega_p{}^2 \qquad [\frac{|\eta||\rho_0|}{\varepsilon_0} = \omega_p{}^2]$$

$$D = \pm j\omega_p$$

$$[\beta_e = \frac{\omega}{v_0}; \beta_p = \frac{\omega_p}{v_0}]$$

$$\omega - \beta v_0 \mp \omega_p = 0 \qquad \frac{\omega}{v_0} - \beta \mp \frac{\omega_p}{v_0} = 0$$

$$\omega - \beta v_0 \mp \omega_p = 0 \qquad \beta_e - \beta \mp \beta_p = 0 \qquad \beta = \beta_e \mp \beta_p$$

(dispersion relation for space-charge waves)

Space-Charge Waves

$$\omega - \beta v_0 \mp \omega_p = 0$$

$$\beta = \beta_e \mp \beta_p$$

$$\beta = \frac{\omega}{v_p} = \frac{\omega}{v_0} \mp \frac{\omega_p}{v_0} = \frac{\omega \mp \omega_p}{v_0}$$

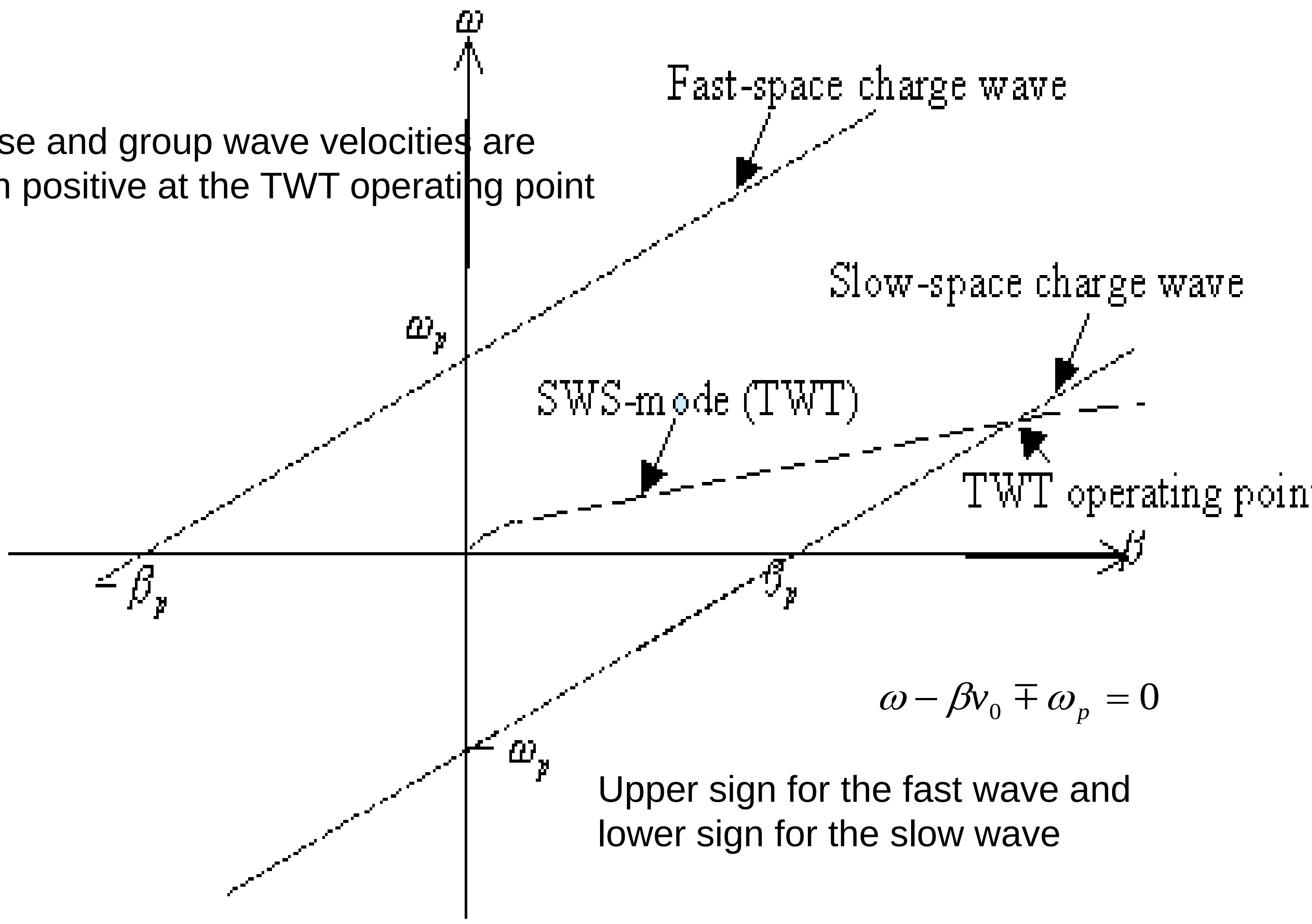
$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega \mp \omega_p} v_0$$

$$\beta_e = \frac{\omega}{v_0} \qquad \beta_p = \frac{\omega_p}{v_0} \qquad \omega_p = \left(\frac{|\eta|\rho_0}{\varepsilon_0}\right)^{1/2}$$

Upper sign for the fast wave and lower sign for the slow wave

Intersection between slow space-charge and circuit/structure waves at the TWT operating point in the dispersion plot

Phase and group wave velocities are each positive at the TWT operating point



$$\omega - \beta v_0 \mp \omega_p = 0$$

Upper sign for the fast wave and lower sign for the slow wave

Some TWT features

- Cerenkov radiation type
- Magnetic field for beam confinement
- Larger magnetic field at higher frequencies for beam confinement
- Conversion of axial beam kinetic energy
- Axial non-relativistic electron bunching
- Near-synchronism between DC beam velocity and circuit phase velocity
- Electron beam velocity to be slightly greater than RF phase velocity
- Slow space-charge wave on electron beam to couple to forward circuit wave
- Space-charge-limited operation
- Pierce gun
- Smaller structure sizes at higher frequencies
- BWO absolute instability above the pi-point frequency

Pierce's theory for TWT dispersion relation and gain

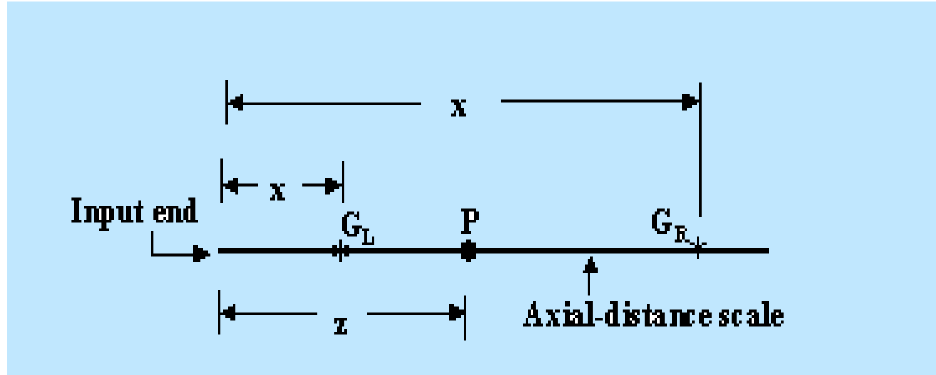
Ratio of circuit voltage V to beam current I found as circuit equation and the same ratio also found as electronic equation

$$\frac{V}{I} = X \quad \text{(Circuit equation)}$$

$$\frac{V}{I} = Y \quad \text{(Electronic equation)}$$

TWT dispersion relation is obtained as: $X = Y$ (dispersion relation)

Circuit equation



Axial distances of the circuit (z) and the left (G_L) and right (G_R) generator points (x)

At a point on the circuit:

Infinitesimal current generator $\equiv$ Effect of a modulated beam element

$dE_R =$ Circuit field amplitude of the wave traveling to the left due to an infinitesimal current generator G_R to the right of the point

$dE_L =$ Circuit field amplitude of the wave traveling to the right due to an infinitesimal current generator G_L to the left of the point

$E_i =$ Input Circuit field amplitude (at z=0)

$$E(z) = E_i \exp(-j\beta_0 z) + \int_0^z dE_R \exp - j\beta_0 (z - x) + \int_{x=z}^{x=\ell} dE_L \exp - j\beta_0 (x - z)$$

$dE_L = \zeta_L\{x\}dx$ $\quad$ $dE_R = \zeta_R\{x\}dx$, say

$$E(z) = E_i \exp(-j\beta_0 z) + \int_0^z \zeta_R\{x\} \exp - j\beta_0 (z - x) dx + \int_{x=z}^{x=\ell} \zeta_L\{x\} \exp - j\beta_0 (x - z) dx$$

An infinitesimal current generator sees identical halves of the matched transmission line both to its left and to its right

$$dE_R = \zeta_R\{x\}\,dx \qquad dE_L = \zeta_L\{x\}\,dx \qquad dE_R = dE_L = dE$$

$$\zeta_R\{x\} = \zeta_L\{x\} = \zeta\{x\} \qquad dE_R = dE_L = dE = \zeta\{x\}\,dx$$

$$E(z) = E_i \exp(-j\beta_0 z) + \int_{\text{at } x=0}^{\text{at } x=z} \zeta_R\{x\} \exp - j\beta_0 (z-x)\,dx$$

$$+ \int_{\text{at } x=z}^{\text{at } x=\ell} \zeta_L\{x\} \exp - j\beta_0 (x-z)\,dx$$

↓ In view of $\zeta_R\{x\} = \zeta_L\{x\} = \zeta\{x\}$

$$E(z) = E_i \exp(-j\beta_0 z) + \int_{\text{at } x=0}^{\text{at } x=z} \zeta\{x\} \exp - j\beta_0 (z-x)\,dx$$

$$+ \int_{\text{at } x=z}^{\text{at } x=\ell} \zeta\{x\} \exp - j\beta_0 (x-z)\,dx$$

$$I_1 = \int_0^z \zeta\{x\} \exp - j\beta_0 (z-x)\,dx, \qquad I_2 = \int_z^\ell \zeta\{x\} \exp - j\beta_0 (x-z)\,dx$$

$$E(z) = E_i \exp(-j\beta_0 z) + I_1 + I_2 \qquad \frac{dE}{dz} = -j\beta_0 E_i \exp(-j\beta_0 z) + \frac{dI_1}{dz} + \frac{dI_2}{dz}$$

$$I_1 = \int_0^z \zeta\{x\} \exp - j\beta_0 (z - x) dx, \qquad I_2 = \int_z^\ell \zeta\{x\} \exp - j\beta_0 (x - z) dx$$

$$E(z) = E_i \exp(-j\beta_0 z) + I_1 + I_2$$

$$\frac{dE}{dz} = -j\beta_0 E_i \exp(-j\beta_0 z) + \frac{dI_1}{dz} + \frac{dI_2}{dz}$$

In view of $\frac{dI_1}{dz} = -j\beta_0 I_1 + \zeta\{z\}, \ \frac{dI_2}{dz} = j\beta_0 I_1 - \zeta\{z\}$

$$\Rightarrow \frac{d^2E}{dz^2} = -\beta_0^2 [E_i \exp(-j\beta_0 z) - (I_1 + I_2)] - 2j\beta_0 \zeta(z)$$

$$E(z) = E_i \exp(-j\beta_0 z) + I_1 + I_2$$

$$\Rightarrow \frac{d^2E}{dz^2} = -\beta_0^2 E - 2j\beta_0 \zeta(z)$$

$(dE_R = dE_L =) dE = \zeta\{x\} dx \Downarrow$

$$\Uparrow \Leftarrow \zeta\{z\} = \frac{dE}{dz} \qquad \Leftarrow \qquad dE = \zeta\{z\} dz$$

$$\Rightarrow \quad \frac{d^2E}{dz^2} = -\beta_0^2 E \ - 2j\beta_0 \frac{dE}{dz}$$

In view of $\frac{dI_1}{dz} = -j\beta_0 I_1 + \zeta\{z\},\ \frac{dI_2}{dz} = j\beta_0 I_1 - \zeta\{z\}$

$$\Rightarrow \frac{d^2E}{dz^2} = -\beta_0^2[E_i \exp(-j\beta_0 z) - (I_1 + I_2)] - 2j\beta_0\zeta(z)$$

In view of $E(z) = E_i \exp(-j\beta_0 z) + I_1 + I_2$

$$\Rightarrow \frac{d^2E}{dz^2} = -\beta_0^2 E - 2j\beta_0\zeta(z) \qquad (dE_R = dE_L =) dE = \zeta\{x\} dx \Downarrow$$

$$\Uparrow \Leftarrow \zeta\{z\} = \frac{dE}{dz} \qquad \Leftarrow \qquad dE = \zeta\{z\} dz$$

$$\Rightarrow \frac{d^2E}{dz^2} = -\beta_0^2 E - 2j\beta_0 \frac{dE}{dz}$$

dE/dz in terms of beam current i and interaction impedance K:

$$K = \frac{E^2}{2\beta^2 P} \qquad \Leftarrow K = \frac{|V|^2}{2P}, \qquad V = \text{axial voltage}$$

$$K = \frac{E^2}{2\beta^2 P} \qquad \Rightarrow \qquad E^2 = 2\beta^2 KP$$

$$\Rightarrow 2EdE = 2\beta^2 KdP \Rightarrow \qquad dP = \frac{EdE}{\beta^2 K}$$

In the present context, $dP_R = \dfrac{E_R dE_R}{\beta^2 K}, \quad dP_L = \dfrac{E_L dE_L}{\beta^2 K}$

$$dP = dP_R + dP_L = \frac{E_R\, dE_R}{\beta^2 K} + \frac{E_L\, dE_L}{\beta^2 K} \quad \Leftarrow dE_R = dE_L = dE$$

$$\Rightarrow dP = \frac{E_R + E_L}{\beta^2 K} dE$$

dP = Increment of circuit power at a point due to a modulated beam element of length *d*z

dP_R, dP_L = Increments of circuit power due to two waves sent by the infinitesimal current generators to the left and to the right of the point, respectively

$$dP = \frac{E_R + E_L}{\beta^2 K} dE$$ (Re-written)

$dP =$ Power lost by the beam element of length dz

$=$(Power lost by half beam element of length $dz/2$ experiencing E_L)

+ (Power lost by half beam element of length $dz/2$ experiencing E_R)

$$= (-eE_L v_1)(n\alpha\, dz/2) + (-eE_R v_1)(n\alpha\, dz/2)$$

$$= -e(E_R + E_L)v_1(n\alpha\, dz/2)$$

(n = electron concentration, α = beam cross-sectional area)

In view of the relation $J_1 = nev_1$ and $i = J_1\alpha$

$$dP = -i(E_R + E_L)\frac{dz}{2}$$

We obtained the same quantity dP earlier as

$$dP = \frac{E_R + E_L}{\beta^2 K} dE$$

So by comparing,

$$\frac{dE}{dz} = -\frac{\beta_0{}^2 K\, i}{2}$$

We can substitute the above in the following equation already obtained.

$$\frac{d^2E}{dz^2} = -\beta_0^2 E \;\; - 2\, j\beta_0 \frac{dE}{dz}$$ (already obtained)

$$\Leftarrow \frac{dE}{dz} = -\frac{\beta_0{}^2 Ki}{2}$$

$$\frac{d^2E}{dz^2} = -\beta_0^2 E + j\beta_0^3 Ki$$

RF quantities vary as exp(-Γz), that is, $d/dz = -\Gamma$, $d^2/dz^2 = \Gamma^2$

$E = -\partial V/\partial z = \Gamma V$ (quasi-static assumption)

$$\Rightarrow \frac{V}{i} = \frac{j\beta_0^3 K}{(\Gamma^2 + \beta_0^2)\Gamma} \qquad \Leftarrow \Gamma_0 = j\beta_0, \Gamma \approx \Gamma_0$$

$$\Rightarrow \frac{V}{i} = -\frac{\Gamma\Gamma_0 K}{(\Gamma^2 - \Gamma_0^2)}$$ (circuit equation)

Electronic equation

Electronic motion in the presence of the circuit electric field intensity E plus the space-charge electric field intensity E_s.

$$\frac{\partial v_1}{\partial t}+\frac{dz}{dt}\frac{\partial v_1}{\partial z}=\eta(E+E_s) \;\Rightarrow\; \frac{\partial v_1}{\partial t}+v\frac{\partial v_1}{\partial z}=\eta(E+E_s)$$

$$\Rightarrow \frac{\partial v_1}{\partial t}+(v_0+v_1)\frac{\partial v_1}{\partial z}=\eta(E+E_s)$$

$$\Rightarrow \frac{\partial v_1}{\partial t}+v_0\frac{\partial v_1}{\partial z}+v_1\frac{\partial v_1}{\partial z}=\eta(E+E_s)$$

$$\Rightarrow \frac{\partial v_1}{\partial t}+v_0\frac{\partial v_1}{\partial z}=\eta(E+E_s) \text{ (ignoring } v_1\frac{\partial v_1}{\partial z}) \quad \Leftarrow \frac{\partial}{\partial t}+v_0\frac{\partial}{\partial z}=j\omega-v_0\Gamma$$

[RF quantities vary as $\exp(j\omega t-\Gamma z)$]

$$\Rightarrow (j\omega-v_0\Gamma)v_1=\eta(E+E_s)$$

$$\Rightarrow \quad v_1=\frac{\eta(E+E_s)}{j\omega-v_0\Gamma}$$

$$-\Gamma E_s = \frac{\rho_1}{\varepsilon_0} \qquad \Leftarrow \frac{\partial E_s}{\partial z} = \frac{\rho_1}{\varepsilon_0} \text{ (Poisson's equation)}$$

$$\nabla . \vec{E} = \frac{\rho}{\varepsilon}$$

[RF quantities vary as $\exp(j\omega t - \Gamma z)$]

$$\Rightarrow E_s = -\frac{\rho_1}{\Gamma \varepsilon_0}$$

$$\nabla . \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$-\Gamma J_1 = -j\omega \rho_1 \qquad \Leftarrow \frac{\partial J_1}{\partial z} = -\frac{\partial \rho_1}{\partial t} \text{ (Continuity equation)}$$

[RF quantities vary as $\exp(j\omega t - \Gamma z)$]

$$\Rightarrow \rho_1 = \frac{\Gamma J_1}{j\omega}$$

$$E_s = -\frac{\rho_1}{\Gamma \varepsilon_0} \text{ (obtained earlier)} \quad \Leftarrow \rho_1 = \frac{\Gamma J_1}{j\omega}$$

$$\Rightarrow E_s = -\frac{J_1}{j\omega\varepsilon_0}$$

$$v_1 = \frac{\eta(E + E_s)}{j\omega - v_0\Gamma} \qquad \Leftarrow E_s = -\frac{J_1}{j\omega\varepsilon_0}$$

$$\Rightarrow v_1 = \frac{\eta\left(E - \frac{J_1}{j\omega\varepsilon_0}\right)}{j\omega - v_0\Gamma}$$

$\rho_1 = \frac{\Gamma J_1}{j\omega}$ (already obtained)

$$J_1 = \rho_0 v_1 + v_0 \rho_1$$

$$\Rightarrow \; J_1 = \frac{\rho_0 \eta (E - \frac{J_1}{j\omega\varepsilon_0})}{j\omega - v_0\Gamma} + v_0 \frac{\Gamma J_1}{j\omega} \qquad \Leftarrow \omega_p^2 = \eta\rho_0 / \varepsilon_0 = |\eta||\rho_0| / \varepsilon_0$$

$\vec{E} = -\nabla V$

$$J_1((j\omega - v_0\Gamma)^2 + \omega_p^2)) = j\omega\eta\rho_0 E \qquad \Leftarrow J_0 = \rho_0 v_0,\ \beta_e = \omega / v_0,\ E = \Gamma V$$

$$\Rightarrow J_1((j\omega - v_0\Gamma)^2 + \omega_p^2) = j\beta_e \eta J_0 \Gamma V$$ $\Leftarrow$ Multiply by α

$(J_1\alpha = i,\ J_0\alpha = i_0)$

$$\Rightarrow \frac{V}{i} = \frac{(j\omega - v_0\Gamma)^2 + \omega_p^2}{j\beta_e \eta i_0 \Gamma} \qquad \Leftarrow \beta_e = \omega / v_0, \quad \beta_p = \omega_p / v_0$$

$$\Rightarrow \frac{V}{i} = \frac{((j\beta_e - \Gamma)^2 + \beta_p^2) v_0^2}{j\beta_e \eta i_0 \Gamma} \Leftarrow \eta = -|\eta|,\ i_0 = -|i_0| = -I_0,\ {v_0}^2 = 2|\eta| V_0$$

(V_0. I_0 = Beam voltage, Beam current)

$$\frac{V}{i} = \left(\frac{2V_0}{I_0}\right)\left(\frac{(j\beta_e - \Gamma)^2 + \beta_p^2}{j\beta_e \Gamma}\right)$$

(Electronic equation)

Dispersion relation and its solution for the growth parameter

$$\frac{V}{i} = -\frac{\Gamma\Gamma_0 K}{(\Gamma^2 - \Gamma_0^2)\Gamma}$$ (Circuit equation)

$$\frac{V}{i} = \left(\frac{2V_0}{I_0}\right)\left(\frac{(j\beta_e - \Gamma)^2 + \beta_p^2}{j\beta_e \Gamma}\right)$$ (Electronic equation)

$$\Rightarrow \frac{-\Gamma\Gamma_0 K}{\Gamma^2 - \Gamma_0^2} = \frac{2V_0}{I_0} \frac{(j\beta_e - \Gamma)^2 + \beta_p^2}{j\beta_e \Gamma}$$

$$\Rightarrow \frac{-\Gamma\Gamma_0 K}{(\Gamma + \Gamma_0)(\Gamma - \Gamma_0)} = \frac{2V_0}{I_0} \frac{(j\beta_e - \Gamma)^2 + \beta_p^2}{j\beta_e \Gamma}$$ (Dispersion relation of a TWT)

Four solutions — three forward waves and one backward wave

Three forward wave solutions:

$-\Gamma = -j\beta_e + \beta_e C\delta$ $\;(C\delta << 1)$ (C, δ are dimensionless quantities) $(\Gamma \approx j\beta_e)$

$\Gamma_0 = j\beta_0 =$ Cold circuit axial propagation constant

$\beta_0 = \beta_e(1 + bC)$ (b = Velocity synchronization parameter)

$$b = \frac{\beta_0 - \beta_e}{\beta_e C} = \frac{v_0 - v_p}{v_p C}$$ (v_p = Circuit phase velocity)

$\Gamma_0 = \beta_e Cd + j\beta_0$ $\;(Cd << 1)$ (d = Circuit loss parameter)

$\Rightarrow \Gamma_0 = \beta_e Cd + j\beta_e(1 + bC)$ $\;(bC << 1)$ $\quad \Gamma_0 \approx j\beta_e$

$\Rightarrow$

$j\beta_e - \Gamma = \beta_e C\delta$

$\Gamma + \Gamma_0 = j\beta_e + j\beta_e(1 + bC) + \beta_e(Cd - C\delta) \approx 2j\beta_e$

$\Gamma - \Gamma_0 = -\beta_e C(\delta + d + jb)$

$$j\beta_e - \Gamma = \beta_e C\delta\,,$$

$$\Gamma + \Gamma_0 = j\beta_e + j\beta_e(1 + bC) + \beta_e(Cd - C\delta) \approx 2j\beta_e$$

$$\Gamma - \Gamma_0 = -\beta_e C(\delta + d + jb)$$

$$\Downarrow$$

$$\frac{-\Gamma\Gamma_0 K}{(\Gamma + \Gamma_0)(\Gamma - \Gamma_0)} = \frac{2V_0}{I_0}\frac{(j\beta_e - \Gamma)^2 + \beta_p^2}{j\beta_e\Gamma} \Leftarrow C^3 = \frac{KI_0}{4V_0}\,, QC = \frac{\beta_p^{\ 2}}{4\beta_e^{\ 2}C^2}$$

(QC = Space-charge parameter)

$$(\delta^2 + 4QC)(j\delta + jd - b) = 1$$

(Cubic dispersion relation)

Special case: $b = d = QC = 0$

$$\delta^3 = -j$$

$$\delta_1 = \sqrt{3}/2 - j(1/2),\ \delta_2 = -\sqrt{3}/2 - j(1/2),\ \delta_3 = j,$$

$$-\Gamma = -j\beta_e + \beta_e C\delta$$

$$-\Gamma_1 = \beta_e C\sqrt{3}/2 - j\beta_e(1 + C/2)$$

$$-\Gamma_2 = -\beta_e C\sqrt{3}/2 - j\beta_e(1 + C/2)$$

$$-\Gamma_3 = -j\beta_e(1 - C)$$

The first wave (with Γ_1) grows

Phase velocity $\omega / \beta_e(1 + C/2) = v_0 /(1 + C/2) < v_0$

The second wave (with Γ_2) attenuates with phase velocity

$$\omega / \beta_e(1 + C/2) = v_0 /(1 + C/2) < v_0$$

The third wave neither grows nor decays with phase velocity

$$\omega / \beta_e(1 - C) = v_0 /(1 - C) > v_0$$

Fourth backward wave solution:

$$-\Gamma = j\beta_e + \beta_e C\delta \quad (\text{instead of } -\Gamma = -j\beta_e + \beta_e C\delta)$$

One can solve the dispersion relation (as has been done for the forward wave case) to obtain the fourth solution for the case of *b* = *d* = *QC* = 0:

$$\delta_4 = -\frac{jC^2}{4}$$

$$-\Gamma_4 = j\beta_e + \beta_e C\delta_4 = j\beta_e(1 - C^3/4)$$

The fourth wave (backward wave) neither grows nor attenuates with phase velocity

$$\omega/\beta_e(1 - C^3/4) = v_0/(1 - C^3/4) > v_0$$

If the structure is perfectly matched the fourth wave is not excited to a significant extent.

v_1 in terms of circuit voltage V:

$$J_1((j\omega - v_0\Gamma)^2 + \omega_p^2) = j\beta_e \eta J_0 \Gamma V \text{ (Recalled)}$$

$$\Rightarrow J_1 = \frac{j\beta_e \eta J_0 \Gamma}{(j\omega - v_0\Gamma)^2 + \omega_p^2} V$$

$$\Downarrow \quad \Leftarrow E = \Gamma V,\ \omega_p{}^2 = \eta\,\rho_0/\varepsilon_0,\ \omega/v_0 = \beta_e$$

$$v_1 = \frac{\eta\left(E - \frac{J_1}{j\omega\varepsilon_0}\right)}{j\omega - v_0\Gamma} \text{ (Recalled)}$$

$$\Rightarrow v_1 = \frac{\eta\,\Gamma(j\beta_e - \Gamma)}{v_0((j\beta_e - \Gamma)^2 + \beta_p{}^2)} V \qquad \Leftarrow -\Gamma = -j\beta_e + \beta_e C\delta,\ \Gamma \approx j\beta_e,\ QC = \frac{\beta_p{}^2}{4\beta_e{}^2 C^2}$$

$$\Downarrow$$

$$\Rightarrow v_1 = \frac{j\eta}{v_0 C\delta\left(1 + \frac{4QC}{\delta^2}\right)} V \qquad \Leftarrow V' = \frac{V}{1 + \frac{4QC}{\delta^2}} \text{ (defined as)}$$

$$\Rightarrow v_1 = \frac{j\eta}{v_0 C\delta} V' \qquad \rightarrow \quad v_1 \propto \frac{V'}{\delta}$$

J_1 in terms of circuit voltage V:

$$J_1 = \frac{j\beta_e \eta J_0 \Gamma}{(j\omega - v_0\Gamma)^2 + \omega_p^2} V \qquad \Leftarrow \omega / v_0 = \beta_e \, , \, \omega_p / v_0 = \beta_p$$

$\Rightarrow$

$$J_1 = \frac{j\beta_e \eta J_0 \Gamma}{v_0{}^2 (j\beta_e - \Gamma)^2 + \beta_p^2)} V \qquad \Leftarrow -\Gamma = -j\beta_e + \beta_e C\delta \, , \; \Gamma \approx j\beta_e \, ,$$

$$QC = \frac{\beta_p{}^2}{4\beta_e{}^2 C^2}$$

$$\Rightarrow J_1 = -\frac{\eta J_0}{v_0{}^2 C^2 \delta^2 (1 + \frac{4QC}{\delta^2})} V \quad \Leftarrow \quad V' = \frac{V}{1 + \frac{4QC}{\delta^2}}$$ ✓ (defined as)

$$\Rightarrow J_1 = -\frac{\eta J_0}{v_0{}^2 C^2 \delta^2} V' \rightarrow \quad J_1 \propto \frac{V'}{\delta^2}$$

Circuit voltage:

$$V\{z\} = V_1\{0\}\exp(-\Gamma_1 z) + V_2\{0\}\exp(-\Gamma_2 z) + V_3\{0\}\exp(-\Gamma_3 z)$$

$$\Downarrow\Leftarrow -\Gamma_1 = -j\beta_e + \beta_e C(x_1 + jy_1)\text{, etc.}$$

$$\begin{aligned} V\{z\} = V_1\{0\}\exp(\beta_e C x_1 z)\exp - j\beta_e(1 - Cy_1)z \\ + V_2\{0\}\exp(\beta_e C x_2 z)\exp - j\beta_e(1 - Cy_2)z \\ + V_3\{0\}\exp(\beta_e C x_3 z)\exp - j\beta_e(1 - Cy_3)z \end{aligned}$$

Input ($z = 0$) conditions:

$$v_{1,in} = 0$$

$$\Rightarrow v_1\{0\} + v_2\{0\} + v_3\{0\} = 0 \qquad \Leftarrow v_1 \propto \frac{V'}{\delta}$$

$$\frac{V_1'}{\delta_1} + \frac{V_2'}{\delta_2} + \frac{V_3'}{\delta_3} = 0 \checkmark$$

$$J_{1,in} = 0$$

$$\Rightarrow J_1\{0\} + J_2\{0\} + J_3\{0\} = 0 \qquad \Leftarrow J_1 \propto \frac{V'}{\delta^2}$$

$$\frac{V_1'}{\delta_1^{\ 2}} + \frac{V_2'}{\delta_2^{\ 2}} + \frac{V_3'}{\delta_3^{\ 2}} = 0 \checkmark$$

$$V_{in} = V_1\{0\} + V_2\{0\} + V_3\{0\} \qquad \Leftarrow V' = \frac{V}{1 + \frac{4QC}{\delta^2}}$$

$$V_{in} = (V_1'\{0\} + V_2'\{0\} + V_3'\{0\}) + 4QC\left(\frac{V_1'\{0\}}{\delta_1^{\ 2}} + \frac{V_2'\{0\}}{\delta_2^{\ 2}} + \frac{V_3'\{0\}}{\delta_3^{\ 2}}\right)$$

$$V_{in} = (V_1'\{0\} + V_2'\{0\} + V_3'\{0\}) + 4QC\left(\frac{V_1'\{0\}}{\delta_1^{\ 2}} + \frac{V_2'\{0\}}{\delta_2^{\ 2}} + \frac{V_3'\{0\}}{\delta_3^{\ 2}}\right)$$

In view of the input condition

$$\frac{V_1'}{\delta_1^{\ 2}} + \frac{V_2'}{\delta_2^{\ 2}} + \frac{V_3'}{\delta_3^{\ 2}} = 0$$

$$V_{in} = V_1'\{0\} + V_2'\{0\} + V_3'\{0\}$$

$$V_1'\{0\} + V_2'\{0\} + V_3'\{0\} = V_{in}$$

One can solve the input equations

$$\frac{V_1'}{\delta_1}+\frac{V_2'}{\delta_2}+\frac{V_3'}{\delta_3}=0$$

$$\frac{V_1'}{\delta_1^{\,2}}+\frac{V_2'}{\delta_2^{\,2}}+\frac{V_3'}{\delta_3^{\,2}}=0$$

$$V_1^{'}\{0\}+V_2^{'}\{0\}+V_3^{'}\{0\}=V_{in}$$

Solution:

$$V_1'(0)=\frac{V_{in}}{\left(1-\delta_2/\delta_1\right)\left(1-\delta_3/\delta_1\right)}$$

$$V_2'\{0\}=\frac{V_{in}}{\left(1-\delta_3/\delta_2\right)\left(1-\delta_1/\delta_2\right)}$$

$$V_3'\{0\}=\frac{V_{in}}{\left(1-\delta_1/\delta_3\right)\left(1-\delta_2/\delta_3\right)}$$

In view of the relation

$$V=(1+\frac{4QC}{\delta^2})V' \quad \Leftarrow \quad V'=\frac{V}{1+\dfrac{4QC}{\delta^2}}$$

Growing-wave circuit voltage component at the input

$$V_1(0)=\left(1+\frac{4QC}{\delta_1^2}\right)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)V_{in}$$

V_{out} = Growing wave component at the output

$$V_{out} = V\{l\} = V_1\{0\}\exp(-\Gamma_1 l)$$

$$-\Gamma_1 = -j\beta_e + \beta_e C\delta_1 = -j\beta_e + \beta_e C(x_1 + jy_1)$$

$$V_{out} = V\{l\} = V_1\{0\}\exp(\beta_e C x_1 l)\exp - j\beta_e(1 - Cy_1)l$$

$$V_1(0) = \left(1 + 4QC/\delta_1^2\right)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)V_{in}$$

$$V_{out} = V_{in}(1 + 4QC/\delta_1{}^2)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)$$

$$\times \exp(\beta_e C x_1 l)\exp - j\beta_e(1 - Cy_1)l$$

Gain G in dB:

$$G = 20\log_{10}\left|\frac{V_{out}}{V_{in}}\right| = A + 20\ \log_{10}(\exp(\beta_e C x_1 \ell))$$

$$A = 20\log_{10}\left|\left(1+4QC/\delta_1^2\right)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)\right|$$

$$\Rightarrow = A + 20\ \log_{10}(\exp(\beta_e C x_1 \ell))$$

$$= A + 20\ \log_e(\exp(\beta_e C x_1 \ell))\log_{10} e$$

$$\Rightarrow\ G = A + 20Cx_1(\log_{10} e)(\beta_e l)\ \Leftarrow \beta_e l = 2\pi N \Leftarrow N(2\pi/\beta_e) = l \Leftarrow N\lambda_e = l$$

$$\Rightarrow G = A + 20Cx_1(\log_{10} e)(2\pi N) = A + 40\pi(\log_{10} e)x_1 CN$$

$$G = A + BCN$$

$$B = 40\pi(\log_{10} e)x_1 \approx 54.6x_1$$

$\delta_1, \delta_2\ \delta_3$ are the solutions of $(\delta^2 + 4QC)(j\delta + jd - b) = 1$

x_1 is the real part of $\delta_1 = x_1 + jy_1$

Special case: $b = QC = d = 0$:

$$\delta_1 = \sqrt{3}/2 - j(1/2),\ \delta_2 = -\sqrt{3}/2 - j(1/2),\ \delta_3 = j$$

$$V_1(0) = \left(1 + \frac{4QC}{\delta_1^2}\right)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)V_{in} \qquad \Leftarrow QC = 0$$

$$\Downarrow$$

$$V_1(0) = \frac{V_{in}}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)} = \frac{V_{\mathrm{in}}}{3}$$

Similarly,

$$V_2(0) = \frac{V_{in}}{(1-\delta_3/\delta_2)(1-\delta_1/\delta_2)} = \frac{V_{in}}{3}$$

$$V_3(0) = \frac{V_{in}}{(1-\delta_1/\delta_3)(1-\delta_2/\delta_3)} = \frac{V_{in}}{3}$$

We conclude that the input signal is evenly distributed among the three forward-wave components

Thus for the special case ($b = QC = d = 0$):

$G = A + BCN$

$\delta_1 = \sqrt{3}/2 - j(1/2)$, $\delta_2 = -\sqrt{3}/2 - j(1/2)$, $\delta_3 = j$, $x_1 = \sqrt{3}/2$ and $QC = 0$

$$\Downarrow$$

$$A = 20\log_{10}\left|\left(1 + 4QC/\delta_1^2\right)\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)\right|$$

$$\Rightarrow A = 20\log_{10}(1/3) \cong -9.54$$

$$B = 40\pi(\log_{10} e)x_1 \approx 54.6x_1 \cong 47.3$$

$$G = -9.54 + 47.3\, CN$$

Extension of Pierce's theory to estimate hot attenuation

Lossy section is provided with the slow-wave structure to prevent the device from oscillating due to imperfect matching

- One attenuator section per about 20 dB gain of the device
- Estimate of 'hot' attenuation for infinite 'cold' attenuation
- Beyond the attenuator
  - Circuit voltage = 0 ('Cold' attenuation = ∞)
  - RF modulation on the beam however remains

Superscripts '*a*' and '*b*' represent

Quantities immediately preceding and beyond the attenuator, respectively.

Subscripts 1, 2, 3 refer to

Three forward waves, respectively

Attenuator length is negligibly small:

$$v_1^b + v_2^b + v_3^b = v_1^a + v_2^a + v_3^a \qquad \Leftarrow v_1 \propto \frac{V}{\delta} \quad \Leftarrow \; v_1 \propto \frac{V'}{\delta} \; \Leftarrow \quad QC = 0$$

$$\Downarrow$$

$$\frac{V_1^b}{\delta_1} + \frac{V_2^b}{\delta_2} + \frac{V_3^b}{\delta_3} = \frac{V_1^a}{\delta_1} + \frac{V_2^a}{\delta_2} + \frac{V_3^a}{\delta_3}$$

Attenuator length is negligibly small:

$$J_1^b + J_2^b + J_3^b = J_1^a + J_2^a + J_3^a \quad \Leftarrow \; J_1 \propto \frac{V}{\delta^2} \qquad \Leftarrow \; J_1 \propto \frac{V'}{\delta^2} \; \Leftarrow \quad QC = 0$$

$$\Downarrow$$

$$\frac{V_1^b}{\delta_1{}^2} + \frac{V_2^b}{\delta_2{}^2} + \frac{V_3^b}{\delta_3{}^2} = \frac{V_1^a}{\delta_1{}^2} + \frac{V_2^a}{\delta_2{}^2} + \frac{V_3^a}{\delta_3{}^2}$$

Recall

$$V_{out} = V_{in}(1+4QC/\delta_1^{\ 2})(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)})$$
$$\times \exp(\beta_e C x_1 l)\exp - j\beta_e(1-Cy_1)l$$

(Obtained considering the contribution from only one (growing wave) component)

Taking the simple case of $QC = 0$

$$V_{out} = V_{in}(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)})\exp(\beta_e C x_1 l)\exp - j\beta_e(1-Cy_1)l$$

For the contributions from all the three forward wave components, and taking l_1 as the distance where the attenuator begins

$$V_1^a = V_{in}(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)})\exp(\beta_e C x_1 l_1)\exp - j\beta_e(1-Cy_1)l_1$$

$$V_2^a = V_{in}(\frac{1}{(1-\delta_3/\delta_2)(1-\delta_1/\delta_2)})\exp(\beta_e C x_2 l_1)\exp - j\beta_e(1-Cy_2)l_1$$

$$V_3^a = V_{in}(\frac{1}{(1-\delta_1/\delta_3)(1-\delta_2/\delta_3)})\exp(\beta_e C x_3 l_1)\exp - j\beta_e(1-Cy_3)l_1$$

For the simple case of $b = QC = d = 0$:

$$\delta_1 = x_1 + jy_1 = \sqrt{3}/2 - j(1/2) \qquad x_1 = \sqrt{3}/2,\ y_1 = -1/2$$

$$\delta_2 = x_2 + jy_2 = -\sqrt{3}/2 - j(1/2) \qquad x_2 = -\sqrt{3}/2,\ y_2 = -1/2$$

$$\delta_3 = x_3 + jy_3 = j \qquad x_3 = 0,\ y_3 = 1$$

$$\Downarrow$$

$$V_1^a = \frac{V_{in}}{3}\exp(\beta_e C x_1 l_1)\exp - j\beta_e(1 - Cy_1)l_1$$

$$V_2^a = \frac{V_{in}}{3}\exp(\beta_e C x_2 l_1)\exp - j\beta_e(1 - Cy_2)l_1$$

$$V_3^a = \frac{V_{in}}{3}\exp(\beta_e C x_3 l_1)\exp - j\beta_e(1 - Cy_3)l_1$$

$$\Downarrow \quad \beta_e l_1 = 2\pi N_1$$

$\Downarrow$ Substitute for V_1^a, V_2^a, V_3^a in the following

$V_1^b + V_2^b + V_3^b = 0$ ('Cold' attenuation = ∞)

$$\frac{V_1^b}{\delta_1} + \frac{V_2^b}{\delta_2} + \frac{V_3^b}{\delta_3} = \frac{V_1^a}{\delta_1} + \frac{V_2^a}{\delta_2} + \frac{V_3^a}{\delta_3}$$

$$\frac{V_1^b}{\delta_1^{\ 2}} + \frac{V_2^b}{\delta_2^{\ 2}} + \frac{V_3^b}{\delta_3^{\ 2}} = \frac{V_1^a}{\delta_1^{\ 2}} + \frac{V_2^a}{\delta_2^{\ 2}} + \frac{V_3^a}{\delta_3^{\ 2}}$$

Solving the three equations for V_1^b (Substituting the values of δ_1, δ_2, δ_3):

$$V_1^b = \frac{V_{in}}{3} \exp-(j2\pi N_1)[\frac{2}{3}\exp(2\pi CN_1(x_1 + jy_1)) - \frac{1}{3}\exp(2\pi CN_1(x_2 + jy_2)) - \frac{1}{3}\exp(2\pi CN_1(x_3 + jy_3))]$$

Recall

$$V_1^a = \frac{V_{in}}{3}\exp(\beta_e C x_1 l_1)\exp - j\beta_e(1 - Cy_1)l_1 \qquad \Leftarrow \beta_e l_1 = 2\pi N_1$$

$\Rightarrow$

$$V_1^a = \frac{V_{in}}{3}\exp(-j2\pi N_1)\exp(2\pi CN_1(x_1 + jy_1))$$

$$V_1^b = \frac{V_{in}}{3}\exp-(j2\pi N_1)[\frac{2}{3}\exp(2\pi CN_1(x_1 + jy_1)) - \frac{1}{3}\exp(2\pi CN_1(x_2 + jy_2))$$

$$-\frac{1}{3}\exp(2\pi CN_1(x_3 + jy_3))]$$

$$(x_1 = \sqrt{3}/2,\ y_1 = -1/2,\quad x_2 = -\sqrt{3}/2,\ y_2 = -1/2 \qquad x_3 = 0,\ y_3 = 1)$$

$$\left|\frac{V_1^b}{V_1^a}\right| = \left|\frac{2}{3} + \frac{1}{3}\exp-(2\pi CN_1\sqrt{3}) + \frac{1}{3}\exp-2\pi CN_1(\frac{\sqrt{3}}{2} - j\frac{3}{2})\right|$$

Taking $CN_1 >$ 0.2 (practical values)

$$\left|\frac{V_1^b}{V_1^a}\right| \cong \frac{2}{3}$$

'Hot' attenuation ~ 20 $\log_{10}3/2$ = 3.52 dB, though 'Cold' attenuation = ∞ !

♋**Extension of Pierce's theory for Johnson's start-oscillation condition**

(H. R. Johnson, "Backward-wave oscillators, " *Proc. IRE*, June 1955, pp. 684-694)

Backward-wave mode: v_p is positive and v_g is negative

Recall:

$$\Gamma_0 = j\beta_0 = \text{Cold circuit axial propagation constant}$$

Ignoring circuit loss

$$\beta_0 = \beta_e(1+bC) \text{ (b = Velocity synchronization parameter)}$$

$$\Gamma_0 = j\beta_e(1+bC)$$

In the presence of circuit loss

$$\Gamma_0 = \beta_e Cd + j\beta_e(1+bC) \text{ } (d = \text{loss paramemter})$$

♓ Dispersion relation of a TWT (forward-wave mode)

$$\frac{-\Gamma\Gamma_0 K}{(\Gamma+\Gamma_0)(\Gamma-\Gamma_0)} = \frac{2V_0}{I_0}\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma} \qquad \Downarrow$$

♓ Dispersion relation for the backward-wave mode

For power flow in the opposite direction (backward-wave mode) K has to be interpreted with a change of sign

$$\frac{+\Gamma\Gamma_0 K}{(\Gamma+\Gamma_0)(\Gamma-\Gamma_0)} = \frac{2V_0}{I_0}\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}$$

$$\Downarrow$$

$$(\delta^2+4QC)(j\delta+jd-b)=-1$$

That circuit voltage in the presence of loss would have to be less at the input (gun) end than in the absence of loss has to be interpreted with a change in the sign of d

$$(\delta^2+4QC)(j\delta-jd-b)=-1 \qquad (\delta = x+jy)$$

♓ Output voltage for backward-wave mode

Contribution from the growing-wave component

$$V_{out} = V_{in}(1 + 4QC/\delta_1^{\,2})\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right) \times \exp(\beta_e C x_1 l)\exp - j\beta_e(1 - Cy_1)l$$

Contributions from all the three wave components

$$\begin{aligned} V_{out} &= V_{in}(1 + 4QC/\delta_1^{\,2})\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right) \times \exp(\beta_e C x_1 l)\exp - j\beta_e(1 - Cy_1)l \\ &+ V_{in}(1 + 4QC/\delta_2^{\,2})\left(\frac{1}{(1-\delta_3/\delta_2)(1-\delta_1/\delta_2)}\right) \times \exp(\beta_e C x_2 l)\exp - j\beta_e(1 - Cy_2)l \\ &+ V_{in}(1 + 4QC/\delta_3^{\,2})\left(\frac{1}{(1-\delta_1/\delta_3)(1-\delta_2/\delta_3)}\right) \times \exp(\beta_e C x_3 l)\exp - j\beta_e(1 - Cy_3)l \end{aligned}$$

$\Downarrow \quad \Leftarrow \beta_e l = 2\pi N$

$\Rightarrow$

$$e^{j2\pi N}\frac{V_{out}}{V_{in}} = \left(\frac{\delta_1{}^2 + 4QC}{(\delta_1-\delta_2)(\delta_1-\delta_3)}\right)e^{2\pi CN\delta_1} + \left(\frac{\delta_2{}^2 + 4QC}{(\delta_2-\delta_3)(\delta_2-\delta_1)}\right)e^{2\pi CN\delta_2} + \left(\frac{\delta_3{}^2 + 4QC}{(\delta_3-\delta_1)(\delta_3-\delta_2)}\right)e^{2\pi CN\delta_3}$$

Oscillation condition:

$$\frac{V_{out}}{V_{in}} = 0$$

$$\Downarrow$$

$$\left(\frac{\delta_1{}^2 + 4QC}{(\delta_1-\delta_2)(\delta_1-\delta_3)}\right)e^{2\pi CN\delta_1} + \left(\frac{\delta_2{}^2 + 4QC}{(\delta_2-\delta_3)(\delta_2-\delta_1)}\right)e^{2\pi CN\delta_2} + \left(\frac{\delta_3{}^2 + 4QC}{(\delta_3-\delta_1)(\delta_3-\delta_2)}\right)e^{2\pi CN\delta_3} = 0$$

$\delta_1,\ \delta_2,\ \delta_3$ are the solutions of $(\delta^2 + 4QC)(j\delta - jd - b) = -1$

The parameters are d, b and QC

The parameter QC may be interpreted as

$$QC = \left(\frac{Q}{N}\right)(CN)$$

$\frac{Q}{N}$ is a parameter defining a particular TWT:

$$QC = \frac{1}{4}\left(\frac{\beta_p}{\beta_e C}\right)^2 = \frac{1}{4}\left(\frac{\omega_p / v_0}{\omega / v_0 C}\right)^2 \quad \text{(Recalled)}$$

$$C^3 = \frac{KI_0}{4V_0}, \qquad \beta_e l = 2\pi N,$$

$$\omega_p{}^2 = \frac{|\eta||\rho_0|}{\varepsilon_0}, \qquad J_0 = \rho_0 v_0, \qquad |J_0| = \frac{I_0}{\pi r_b{}^2}$$

(r_b = Beam radius)

$$\Downarrow$$

$$\frac{Q}{N} = \frac{QC}{CN} = \frac{2|\eta|\, V_0}{\varepsilon_0\, r_b{}^2 \omega^3\, K\, l} \quad \text{(Independent of beam current)}$$

One can simultaneously solve the following two equations for *CN*:

(i)

$$(\delta^2 + 4QC)(j\delta - jd - b) = -1 \text{ (Parameters: } d,\ b,\ QC = (\frac{Q}{N})(CN))$$

(ii)

$$\left(\frac{\delta_1{}^2 + 4QC}{(\delta_1 - \delta_2)(\delta_1 - \delta_3)}\right)e^{2\pi CN\delta_1} + \left(\frac{\delta_2{}^2 + 4QC}{(\delta_2 - \delta_3)(\delta_2 - \delta_1)}\right)e^{2\pi CN\delta_2} + \left(\frac{\delta_3{}^2 + 4QC}{(\delta_3 - \delta_1)(\delta_3 - \delta_2)}\right)e^{2\pi CN\delta_3} = 0$$

The solution for *CN* thus obtained may be interpreted as

Start-oscillation current I_0

(in view of the relations $C = (KI_0/4V_0)^{1/3}$ and $\beta_e\, l = 2\pi N$)

Some concepts in widening slow-wave TWTs

Zero-to-slightly-negative-dispersion structure for wideband performance

Anisotropically loaded helix:

Metal vane/ segment loaded envelope

Inhomogeneously loaded helix:

Helix with tapered geometry dielectric supports such as

half-moon-shaped and T-shaped supports

Negative dispersion ensures constancy of Pierce's velocity synchronization parameter *b* with frequency

Multi-dispersion structures for wideband performance

Constancy of b with frequency with negative dispersion

$$b = \frac{v_0 - v_p}{v_p C} = \frac{v_0 - v_p}{v_p (KI_0 / 4V_0)^{1/3}} = \frac{\frac{v_0 - v_p}{v_p}}{K^{1/3}} \frac{1}{(I_0 / 4V_0)^{1/3}}$$

Negative dispersion: v_p increases with frequency

$v_0 - v_p$ decreases with frequency

$\frac{v_0 - v_p}{v_p}$ decreases with frequency

$\rightarrow$ Numerator of the expression for b decreases with frequency

K decreases with frequency and hence the

$\rightarrow$ Denominator of the expression for b decreases with frequency

b remains constant with frequency

Conventional TWTs with multi-dispersion, multi-section structures

Small-signal gain equation $G \sim BCN$

$$N\lambda_e = l$$

$$N\frac{v_0}{f} = l$$

$$N = \frac{f\,l}{v_0}$$

$$C = (KI_0 / 4V_0)^{1/3}$$

$$G \sim B(KI_0 / 4V_0)^{1/3}\frac{f\,l}{v_0}$$

G is proportional to $K^{1/3} f\, l$

G is proportional to $K^{1/3} f\, l$

Gain-frequency response:

Lower gain at lower frequencies as *G* is proportional to *f*

Lower gain at higher frequencies as *G* is proportional to $K^{1/3}$, the latter decreasing with frequency

Conventional structure: If you had increased the length *l*, then the gain *G* would be compensated at lower frequencies *f*. However, then the gain *G* would become very high at higher frequencies *f* entailing the risk of oscillation in the device.

Therefore, let us arrive at the design of a helical slow-wave structure the **effective length** of which is **large at lower frequencies,** which at the same time becomes relatively **smaller at higher frequencies**. (The design should ensure that the gain is not enhanced at any frequency to a high value causing oscillation in the device).

The answer lies in a multi-dispersion, multi-section helix TWTs!

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One positive-dispersion helix section of length l_1 synchronous with the beam only at lower frequencies and the other nearly dispersion-free helix section of effective length length l_2 synchronous with the beam both at lower and higher frequencies.

Effective length increased to $l_1 + l_2$ at lower frequencies

Effective length reduced to l_2 at higher frequencies (since the section of length l_1 goes out of synchronism at higher frequencies

Gain is proportional to $K^{1/3} f\, l$

We have to control (i) the nature and the amounts of dispersion of of the sections by suitably loading the sections and (ii) the lengths of the two sections

Select structure sections such as segment loaded helices of controllable dispersion

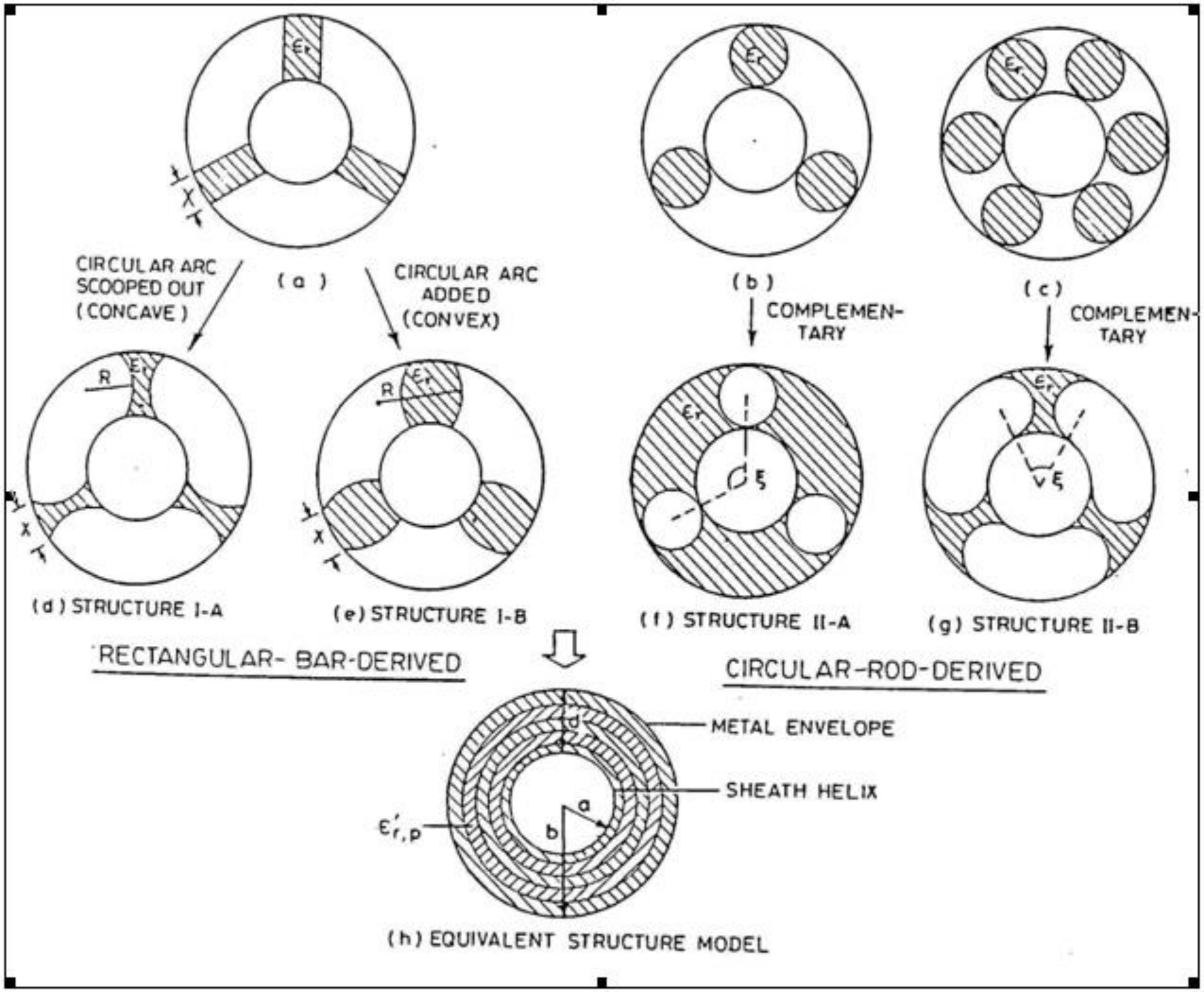
Analysis should be capable of finding the dispersion and interaction impedance characteristics of the structure sections, say, with metal segment loaded envelopes and their control by structure section parameters like segment dimensions and relative section lengths.

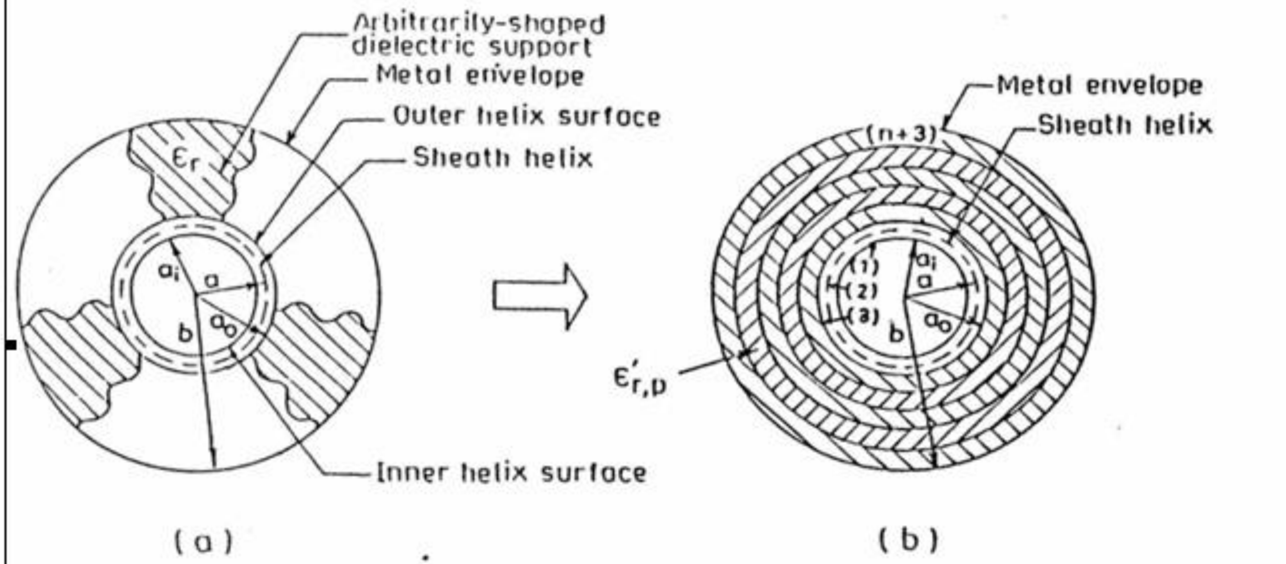
Two-section configuration with one of the sections providing a double-hump in the gain-frequency response

One of the seconds provides a double-hump peaks while the second section provides a single peak between the humps in the gain frequency response

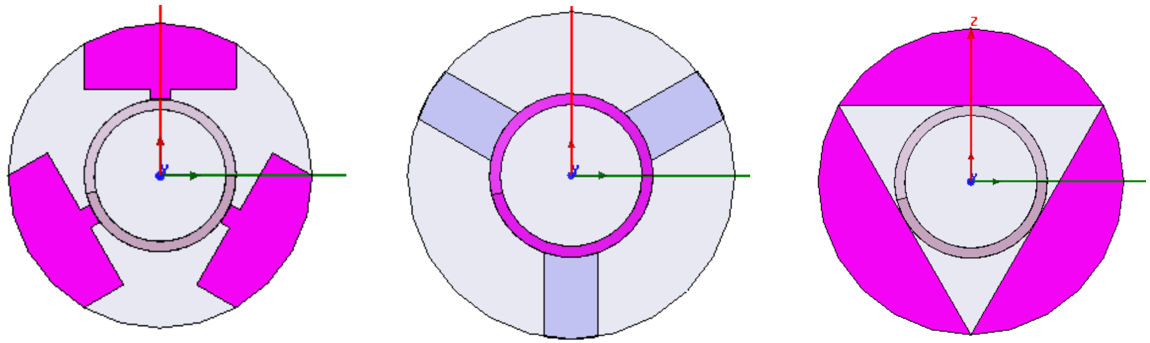
Twystron

The first section is a klystron providing a double-hump gain-frequency response. The second section is a TWT providing a peak between the two humps of the first section in the gain-frequency response.


εr
X
CIRCULAR ARC
SCOOPED OUT
(CONCAVE)
(a)
CIRCULAR ARC
ADDED
(CONVEX)
(b)
COMPLEMEN-
TARY
(c)
COMPLEMEN-
TARY
R
X
(d) STRUCTURE I-A
(e) STRUCTURE I-B
ξ
(f) STRUCTURE II-A
(g) STRUCTURE II-B
RECTANGULAR- BAR-DERIVED
CIRCULAR-ROD-DERIVED
METAL ENVELOPE
SHEATH HELIX
a
b
d
ε'r,p
(h) EQUIVALENT STRUCTURE MODEL



Cross section of a helical slow-wave structure supported by arbitrarily shaped identical dielectric bars **(a)** and its equivalent analytical model **(b)**.



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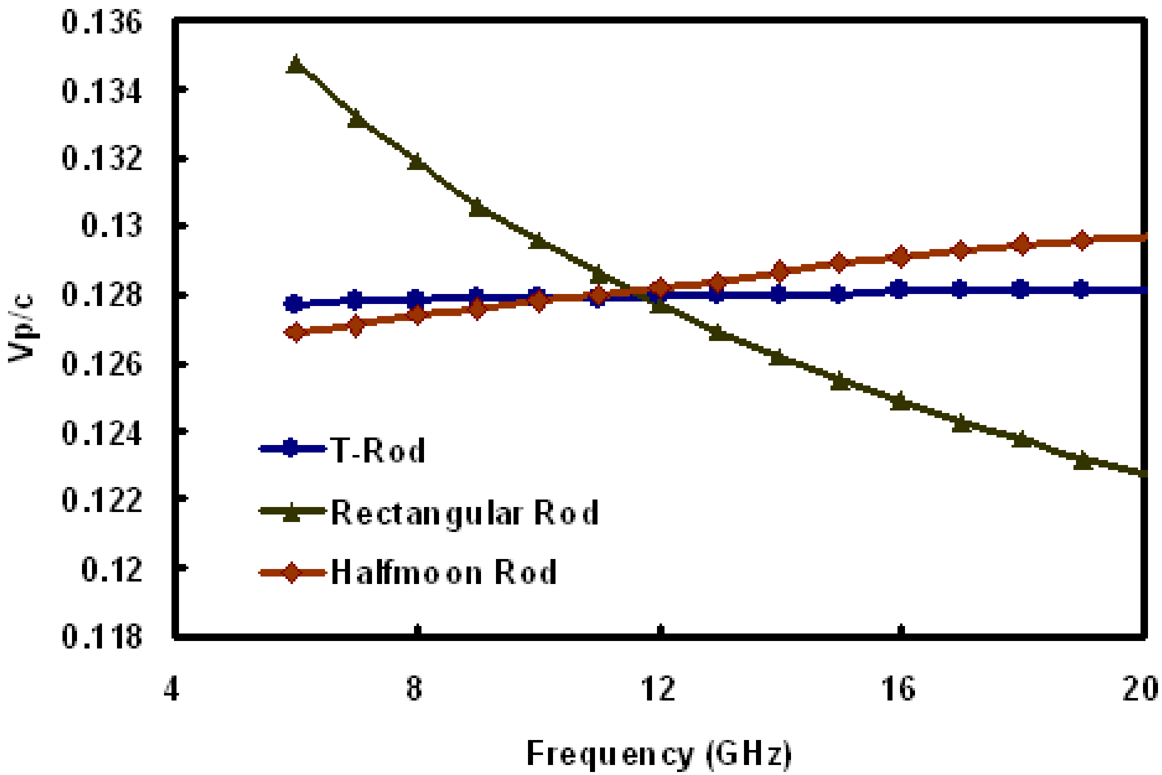

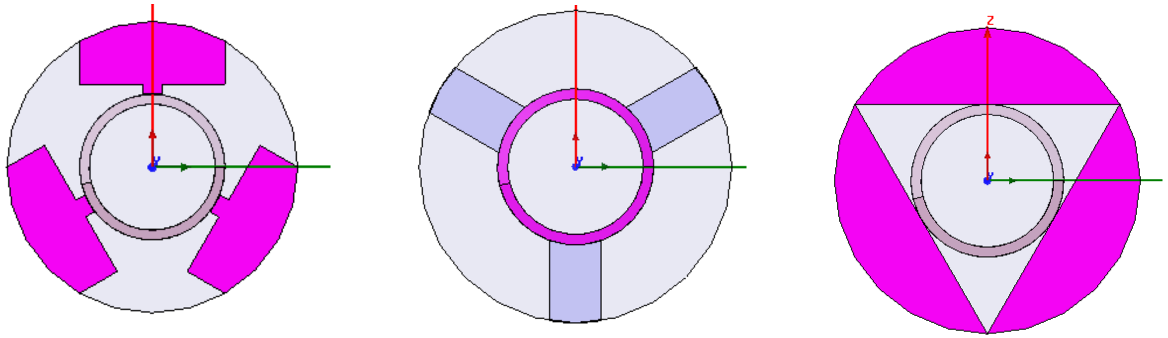

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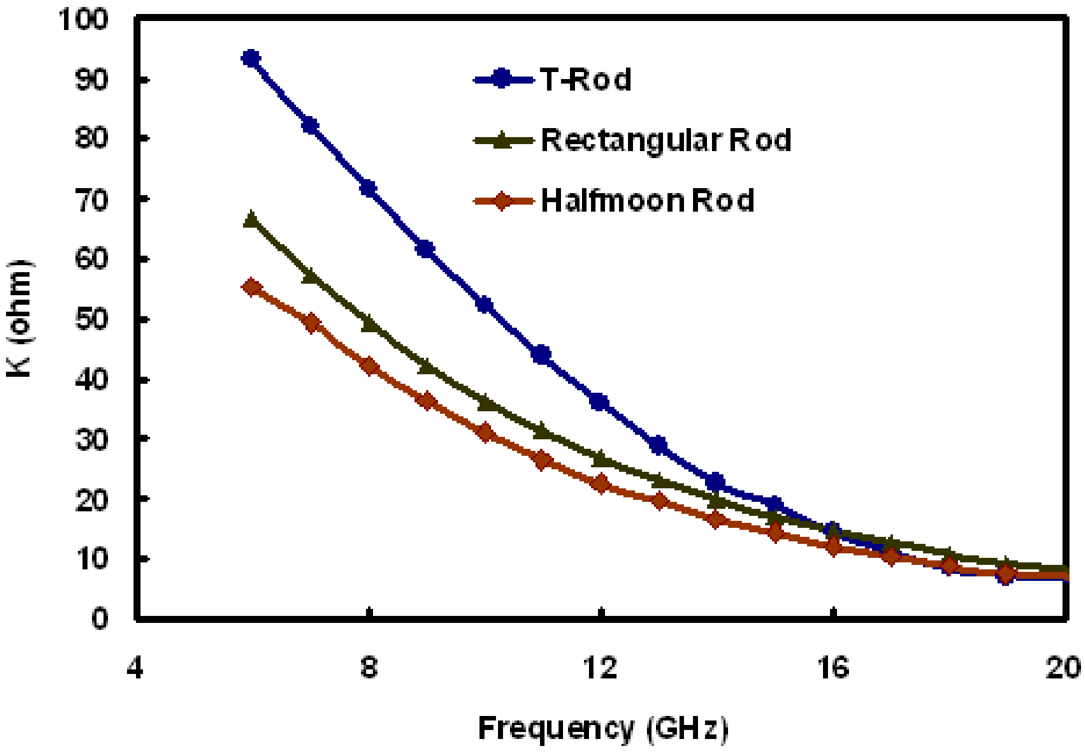

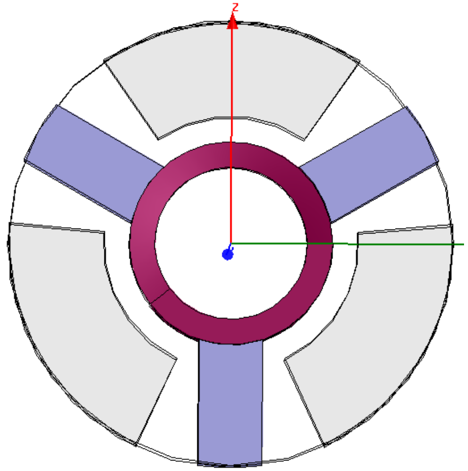

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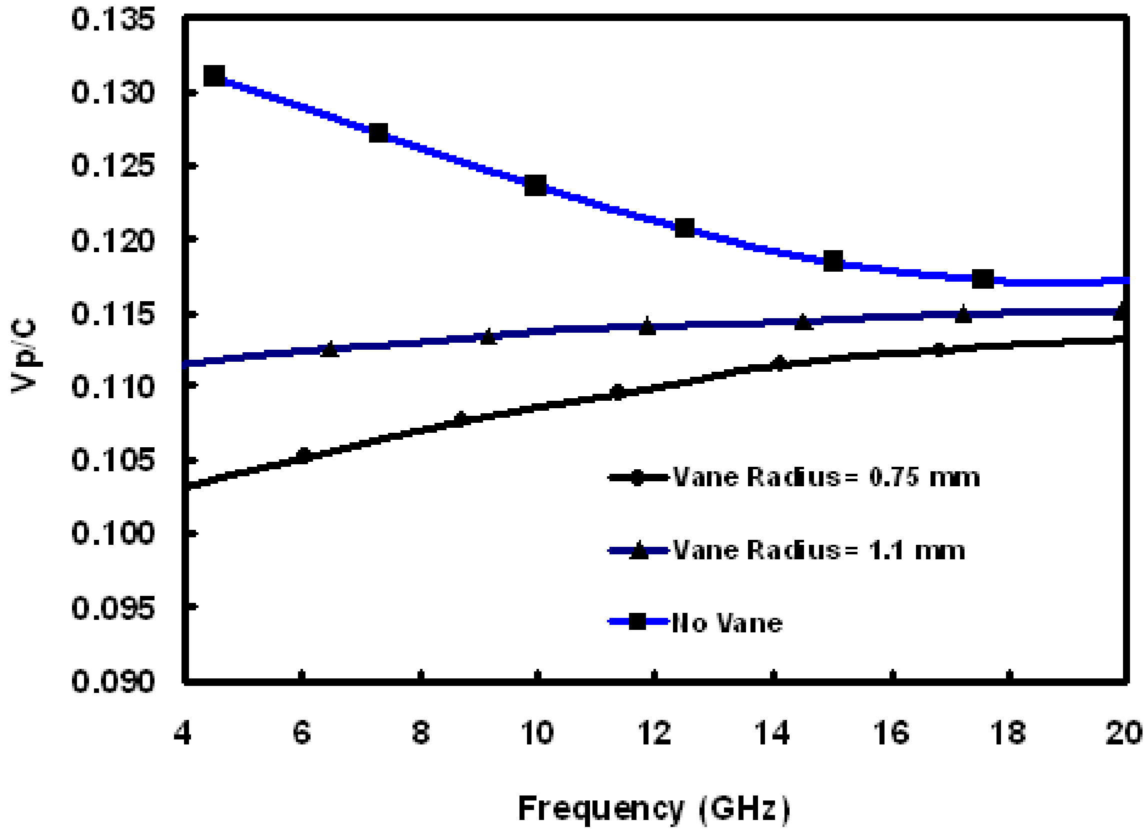

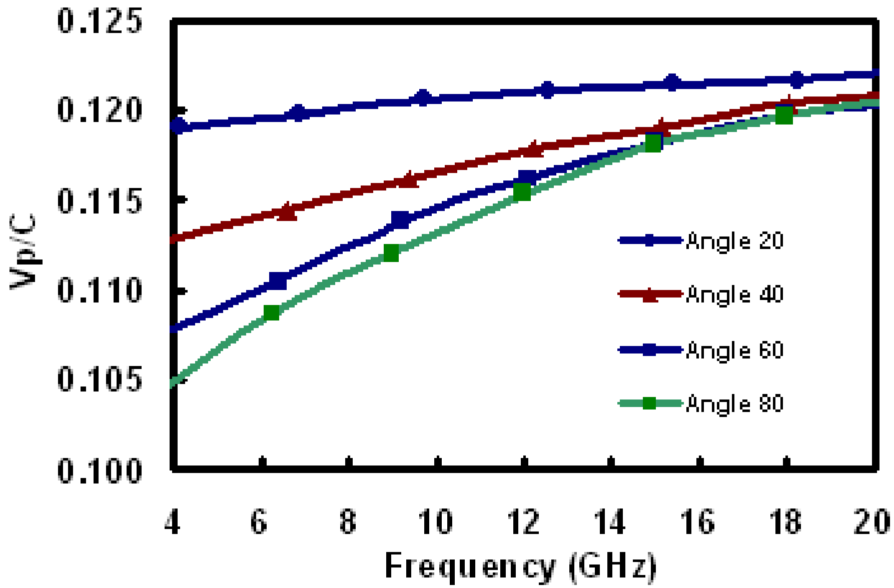

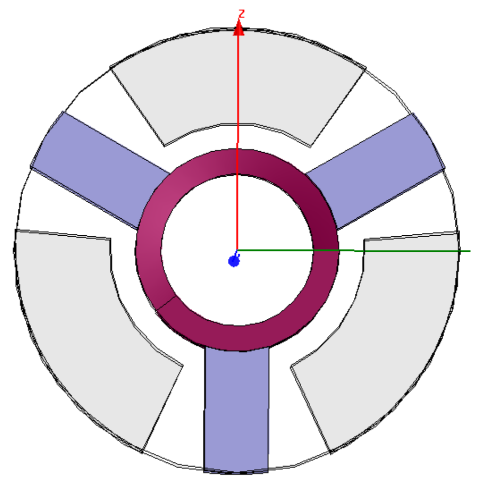

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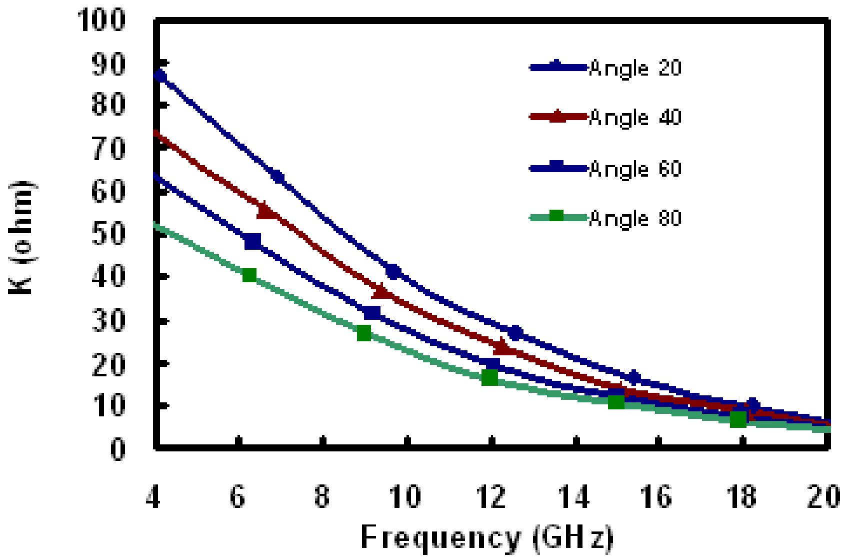
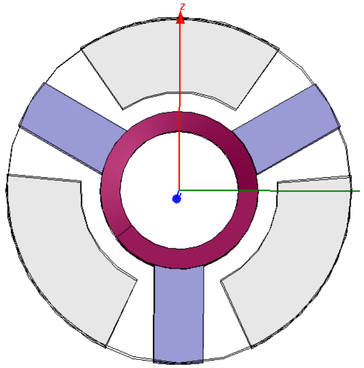
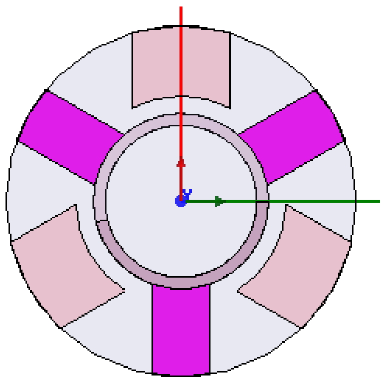
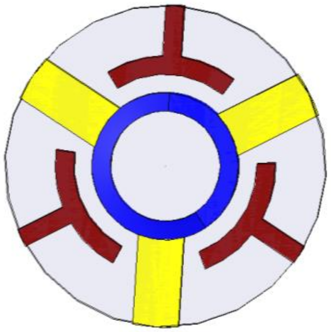




Angular segment

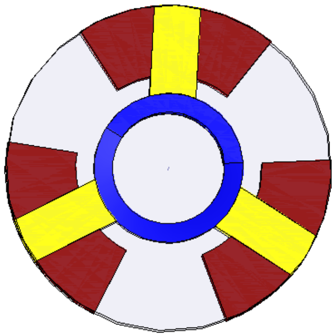


Straight segment



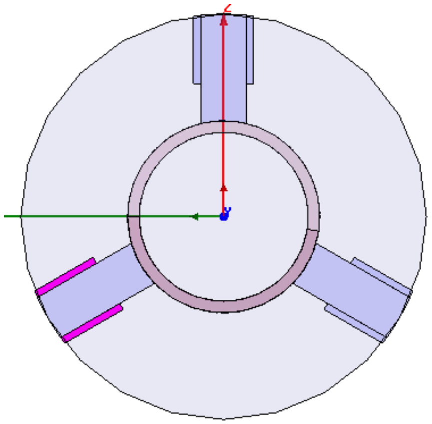
T- segment

Source: MTRDC (DRDO)

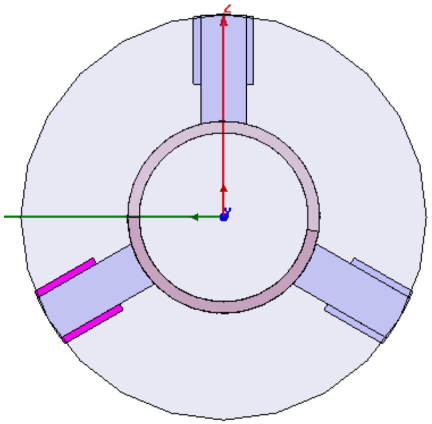


Helix-support rod embedded
in the metal vane

Source: MTRDC (DRDO)

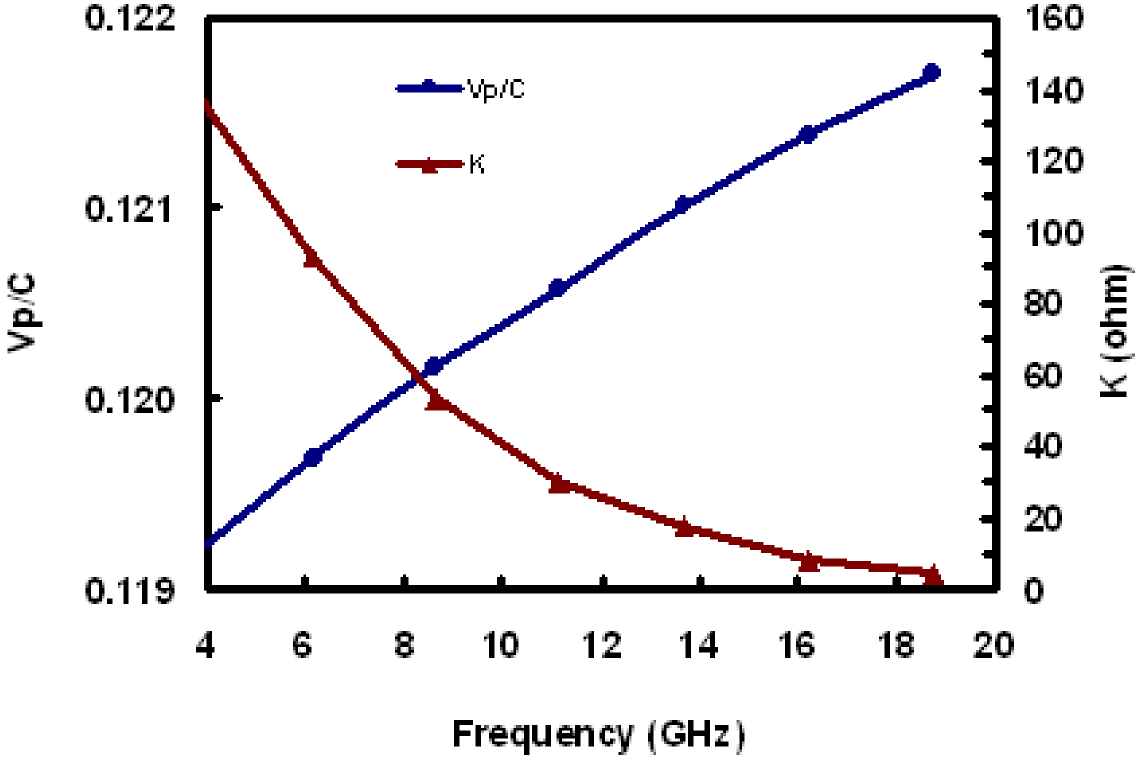


Metal-coated dielectric helix-support rods



Provides negative dispersion with quite high interaction impedance compared to other segment variants

Source: MTRDC (DRDO)



Broadbanding techniques

- Homogeneous dielectric loading <u>cannot</u> shape structure dispersion for wide device bandwidths.

- Inhomogeneous dielectric loading by tapered-cross-section helix supports <u>can</u> shape structure dispersion for wide device bandwidths.

- Anisotropic loading by azimuthally periodic metal vanes provided with the metal envelope <u>can</u> shape structure dispersion for wide device bandwidths.

- Axially periodic disc loading <u>cannot</u> shape structure dispersion and <u>cannot</u> widen device bandwidth but <u>can</u> enhance device gain.

The bandwidth of a TWT can be widened by

- providing azimuthally periodic metal vanes/segments with the metal envelope of the helical slow-wave structure and optimizing the structure parameters, thereby obtaining the desired dispersion characteristics of the structure for wide device bandwidths

- appropriately shaping the cross-sectional geometry of dielectric helix supports, thereby obtaining the desired dispersion characteristics of the structure for wide device bandwidths

- using multi-dispersion, multi-section helical structures

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Thank You!