Johnson's Start-Oscillation Condition

B. N. Basu
SKFGI
Mankundu
West Bengal-712 139
India

Former affiliation:
Centre of Research in Microwave Tubes
Department of Electronics Engineering
Institute of Technology
Banaras Hindu University
Varanasi-221 005
India

Prerequisite:

Section 8.2 of B. N. Basu, Electromagnetic Theory and Applications in Beam-Wave Electronics (World Scientific; 1996; Singapore, New Jersey, London, Hong Kong)

Johnson's start-oscillation condition

(H. R. Johnson, "Backward-wave oscillators, " *Proc. IRE*, June 1955, pp. 684-694)

Backward-wave mode: **The slow-wave structure phase velocity v_p is positive and group velocity v_g is negative.**

Let us recall the following:

See Section 8.2 of B. N. Basu, Electromagnetic Theory and Applications in Beam-Wave Electronics (World Scientific; 1996; Singapore, New Jersey, London, Hong Kong)

$\Gamma_0 = j\beta_0 =$ **cold circuit propagation constant**

$\beta_0 = \beta_e(1+bC)$ **(**$b =$ **velocity synchronization parameter)**

$\Gamma_0 = j\beta_0 = j\beta_e(1+bC)$ **(in the absence of circuit loss)**

$\Gamma_0 = \beta_e Cd + j\beta_e(1+bC)$ **(**$d =$ **structure loss parameter)**

$$\frac{-\Gamma\Gamma_0 K}{(\Gamma+\Gamma_0)(\Gamma-\Gamma_0)} = \frac{2V_0}{I_0}\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}$$ **(TWT dispersion relation)**

$$(\delta^2+4QC)(j\delta+jd-b)=1$$ **(TWT cubic dispersion relation)**

$$\frac{-\Gamma\Gamma_0 K}{(\Gamma+\Gamma_0)(\Gamma-\Gamma_0)} = \frac{2V_0}{I_0}\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}$$

(TWT dispersion relation)

For power flow in the opposite direction (backward-wave mode) K **has to be interpreted with a change of sign**

$$\frac{+\Gamma\Gamma_0 K}{(\Gamma+\Gamma_0)(\Gamma-\Gamma_0)} = \frac{2V_0}{I_0}\frac{(j\beta_e-\Gamma)^2+\beta_p^2}{j\beta_e\Gamma}$$

Consequent change in the sign of the right hand side of the TWT cubic dispersion relation: $(\delta^2+4QC)(j\delta+jd-b)=1$

$$\boxed{(\delta^2+4QC)(j\delta+jd-b)=-1}$$

(cubic dispersion relation corresponding to the backward-wave mode)

$$\boxed{(\delta^2 + 4QC)(j\delta + jd - b) = -1}$$

(cubic dispersion relation corresponding to the backward-wave mode)

(to be recalled)

Output voltage for backward-wave mode:

Contribution from the growing-wave component:

$$V_{\text{out}} = V_{\text{in}}(1 + 4QC/\delta_1^{\;2})\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)$$

$$\times \exp(\beta_e C x_1 l)\exp - j\beta_e(1 - Cy_1)l$$

(recalled)

Contribution from the growing-wave component:

$$V_{\text{out}} = V_{\text{in}}(1+4QC/\delta_1^{\ 2})(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)})$$
$$\times \exp(\beta_e C x_1 l)\exp - j\beta_e(1-Cy_1)l$$

(rewritten)

Contributions from all the three wave components:

$$V_{\text{out}} = V_{\text{in}}(1+4QC/\delta_1^{\ 2})(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)})$$
$$\times \exp(\beta_e C x_1 l)\exp - j\beta_e(1-Cy_1)l$$

$$+V_{\text{in}}(1+4QC/\delta_2^{\ 2})(\frac{1}{(1-\delta_3/\delta_2)(1-\delta_1/\delta_2)})$$
$$\times \exp(\beta_e C x_2 l)\exp - j\beta_e(1-Cy_2)l$$

$$+V_{\text{in}}(1+4QC/\delta_3^{\ 2})(\frac{1}{(1-\delta_1/\delta_3)(1-\delta_2/\delta_3)})$$
$$\times \exp(\beta_e C x_3 l)\exp - j\beta_e(1-Cy_3)l$$

$$V_{\text{out}} = V_{\text{in}}(1+4QC/\delta_1^{\,2})\left(\frac{1}{(1-\delta_2/\delta_1)(1-\delta_3/\delta_1)}\right)$$

$$\times \exp(\beta_e C x_1 l)\exp - j\beta_e(1-Cy_1)l$$

$$+V_{\text{in}}(1+4QC/\delta_2^{\,2})\left(\frac{1}{(1-\delta_3/\delta_2)(1-\delta_1/\delta_2)}\right)$$

$$\times \exp(\beta_e C x_2 l)\exp - j\beta_e(1-Cy_2)l$$

$$+V_{\text{in}}(1+4QC/\delta_3^{\,2})\left(\frac{1}{(1-\delta_1/\delta_3)(1-\delta_2/\delta_3)}\right)$$

$$\times \exp(\beta_e C x_3 l)\exp - j\beta_e(1-Cy_3)l$$

$$\longleftarrow \quad \beta_e l = 2\pi N$$

$$\downarrow$$

$$e^{j2\pi N}\frac{V_{out}}{V_{in}} = \left(\frac{\delta_1^{\,2}+4QC}{(\delta_1-\delta_2)(\delta_1-\delta_3)}\right)e^{2\pi CN\delta_1}$$

$$+\left(\frac{\delta_2^{\,2}+4QC}{(\delta_2-\delta_3)(\delta_2-\delta_1)}\right)e^{2\pi CN\delta_2}$$

$$+\left(\frac{\delta_3^{\,2}+4QC}{(\delta_3-\delta_1)(\delta_3-\delta_2)}\right)e^{2\pi CN\delta_3}$$

$$e^{j2\pi N}\frac{V_{out}}{V_{in}} = \left(\frac{\delta_1{}^2+4QC}{(\delta_1-\delta_2)(\delta_1-\delta_3)}\right)e^{2\pi CN\delta_1} + \left(\frac{\delta_2{}^2+4QC}{(\delta_2-\delta_3)(\delta_2-\delta_1)}\right)e^{2\pi CN\delta_2} + \left(\frac{\delta_3{}^2+4QC}{(\delta_3-\delta_1)(\delta_3-\delta_2)}\right)e^{2\pi CN\delta_3}$$

(rewritten)

$$\boxed{\frac{V_{out}}{V_{in}} = 0}$$

(oscillation condition)

$$\boxed{\left(\frac{\delta_1{}^2+4QC}{(\delta_1-\delta_2)(\delta_1-\delta_3)}\right)e^{2\pi CN\delta_1} + \left(\frac{\delta_2{}^2+4QC}{(\delta_2-\delta_3)(\delta_2-\delta_1)}\right)e^{2\pi CN\delta_2} + \left(\frac{\delta_3{}^2+4QC}{(\delta_3-\delta_1)(\delta_3-\delta_2)}\right)e^{2\pi CN\delta_3} = 0}$$

(to be recalled)

The following parameters are relevant while finding the parameter CN .

$$QC = (\frac{Q}{N})(CN)$$

$$\frac{Q}{N} = \frac{QC}{CN} = \frac{2|\eta|\,V_0}{\varepsilon_0\, r_b^{\,2}\omega^3 K\, l}$$

(independent of beam current)

ω has to be interpreted as the frequency where the phase velocity of the forward-wave mode of the SWS becomes equal to that of the backward-wave mode. K has to be taken as the interaction impedance at this frequency.

$$C^3 = \frac{KI_0}{4V_0}$$

$$QC = \frac{1}{4}(\frac{\beta_p}{\beta_e C})^2 = \frac{1}{4}(\frac{\omega_p / v_0}{\omega / v_0 C})^2$$

$$\omega_p^{\,2} = \frac{|\eta||\rho_0|}{\varepsilon_0}$$

$$J_0 = \rho_0 v_0$$

$$|J_0| = \frac{I_0}{\pi r_b^{\,2}}$$

$r_b =$ beam radius

$$\beta_e l = 2\pi N$$

One can simultaneously solve the following two equations for CN:

(i)

$$(\delta^2 + 4(\frac{Q}{N})(CN))(j\delta - jd - b) = -1 \longleftarrow (\delta^2 + 4QC)(j\delta - jd - b) = -1$$

$$QC = (\frac{Q}{N})(CN)$$

(ii)

$$\left(\frac{\delta_1{}^2 + 4QC}{(\delta_1 - \delta_2)(\delta_1 - \delta_3)}\right) e^{2\pi CN\delta_1} + \left(\frac{\delta_2{}^2 + 4QC}{(\delta_2 - \delta_3)(\delta_2 - \delta_1)}\right) e^{2\pi CN\delta_2} + \left(\frac{\delta_3{}^2 + 4QC}{(\delta_3 - \delta_1)(\delta_3 - \delta_2)}\right) e^{2\pi CN\delta_3} = 0$$

The solution for CN thus obtained may be interpreted as the start-oscillation current I_0 while making use the relations:

$C = (KI_0/(4V_0))^{1/3}$ **and** $\beta_e l = 2\pi N$.