

Johnson's Start-Oscillation Condition

B. N. Basu

SKFGI

Mankundu

West Bengal-712 139

India

Former affiliation:

Centre of Research in Microwave Tubes

Department of Electronics Engineering

Institute of Technology

Banaras Hindu University

Varanasi-221 005

India

Prerequisite:

Section 8.2 of B. N. Basu, Electromagnetic Theory and Applications in Beam-Wave Electronics (World Scientific; 1996; Singapore, New Jersey, London, Hong Kong)

Johnson's start-oscillation condition

(H. R. Johnson, "Backward-wave oscillators, " *Proc. IRE*, June 1955, pp. 684-694)

Backward-wave mode: The slow-wave structure phase velocity v_p is positive and group velocity v_g is negative.

Let us recall the following: See Section 8.2 of B. N. Basu, *Electromagnetic Theory and Applications in Beam-Wave Electronics* (World Scientific; 1996; Singapore, New Jersey, London, Hong Kong)

$$\Gamma_0 = j\beta_0 = \text{cold circuit propagation constant}$$

$$\beta_0 = \beta_e(1+bC) \quad (b = \text{velocity synchronization parameter})$$

$$\Gamma_0 = j\beta_0 = j\beta_e(1+bC) \quad (\text{in the absence of circuit loss})$$

$$\Gamma_0 = \beta_e Cd + j\beta_e(1+bC) \quad (d = \text{structure loss parameter})$$

$$\frac{-\Gamma\Gamma_0 K}{(\Gamma + \Gamma_0)(\Gamma - \Gamma_0)} = \frac{2V_0}{I_0} \frac{(j\beta_e - \Gamma)^2 + \beta_p^2}{j\beta_e \Gamma} \quad (\text{TWT dispersion relation})$$

$$(\delta^2 + 4QC)(j\delta + jd - b) = 1 \quad (\text{TWT cubic dispersion relation})$$

$$\frac{-\Gamma\Gamma_0 K}{(\Gamma + \Gamma_0)(\Gamma - \Gamma_0)} = \frac{2V_0}{I_0} \frac{(j\beta_e - \Gamma)^2 + \beta_p^2}{j\beta_e \Gamma} \quad \text{(TWT dispersion relation)}$$



← For power flow in the opposite direction (backward-wave mode) K has to be interpreted with a change of sign

$$\frac{+\Gamma\Gamma_0 K}{(\Gamma + \Gamma_0)(\Gamma - \Gamma_0)} = \frac{2V_0}{I_0} \frac{(j\beta_e - \Gamma)^2 + \beta_p^2}{j\beta_e \Gamma}$$



← Consequent change in the sign of the right hand side of the TWT cubic dispersion relation: $(\delta^2 + 4QC)(j\delta + jd - b) = 1$

$$\boxed{(\delta^2 + 4QC)(j\delta + jd - b) = -1}$$

(cubic dispersion relation corresponding to the backward-wave mode)

$$\boxed{(\delta^2 + 4QC)(j\delta + jd - b) = -1}$$

(cubic dispersion relation corresponding to the backward-wave mode)

(to be recalled)

Output voltage for backward-wave mode:

Contribution from the growing-wave component:

$$V_{\text{out}} = V_{\text{in}} (1 + 4QC / \delta_1^2) \left(\frac{1}{(1 - \delta_2 / \delta_1)(1 - \delta_3 / \delta_1)} \right) \\ \times \exp(\beta_e C x_1 l) \exp - j\beta_e (1 - C y_1) l$$

(recalled)

Contribution from the growing-wave component:

$$V_{\text{out}} = V_{\text{in}} (1 + 4QC / \delta_1^2) \left(\frac{1}{(1 - \delta_2 / \delta_1)(1 - \delta_3 / \delta_1)} \right) \\ \times \exp(\beta_e Cx_1 l) \exp - j\beta_e (1 - Cy_1) l$$

(rewritten)

Contributions from all the three wave components:

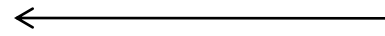
$$V_{\text{out}} = V_{\text{in}} (1 + 4QC / \delta_1^2) \left(\frac{1}{(1 - \delta_2 / \delta_1)(1 - \delta_3 / \delta_1)} \right) \\ \times \exp(\beta_e Cx_1 l) \exp - j\beta_e (1 - Cy_1) l$$
$$+ V_{\text{in}} (1 + 4QC / \delta_2^2) \left(\frac{1}{(1 - \delta_3 / \delta_2)(1 - \delta_1 / \delta_2)} \right) \\ \times \exp(\beta_e Cx_2 l) \exp - j\beta_e (1 - Cy_2) l$$
$$+ V_{\text{in}} (1 + 4QC / \delta_3^2) \left(\frac{1}{(1 - \delta_1 / \delta_3)(1 - \delta_2 / \delta_3)} \right) \\ \times \exp(\beta_e Cx_3 l) \exp - j\beta_e (1 - Cy_3) l$$

$$\begin{aligned}
V_{\text{out}} = & V_{\text{in}} (1 + 4QC / \delta_1^2) \left(\frac{1}{(1 - \delta_2 / \delta_1)(1 - \delta_3 / \delta_1)} \right) \\
& \times \exp(\beta_e Cx_1 l) \exp - j\beta_e (1 - Cy_1) l \\
& + V_{\text{in}} (1 + 4QC / \delta_2^2) \left(\frac{1}{(1 - \delta_3 / \delta_2)(1 - \delta_1 / \delta_2)} \right) \\
& \times \exp(\beta_e Cx_2 l) \exp - j\beta_e (1 - Cy_2) l \\
& + V_{\text{in}} (1 + 4QC / \delta_3^2) \left(\frac{1}{(1 - \delta_1 / \delta_3)(1 - \delta_2 / \delta_3)} \right) \\
& \times \exp(\beta_e Cx_3 l) \exp - j\beta_e (1 - Cy_3) l
\end{aligned}$$

$$\leftarrow \beta_e l = 2\pi N$$

$$\begin{aligned}
e^{j2\pi N} \frac{V_{\text{out}}}{V_{\text{in}}} = & \left(\frac{\delta_1^2 + 4QC}{(\delta_1 - \delta_2)(\delta_1 - \delta_3)} \right) e^{2\pi CN\delta_1} \\
& + \left(\frac{\delta_2^2 + 4QC}{(\delta_2 - \delta_3)(\delta_2 - \delta_1)} \right) e^{2\pi CN\delta_2} \\
& + \left(\frac{\delta_3^2 + 4QC}{(\delta_3 - \delta_1)(\delta_3 - \delta_2)} \right) e^{2\pi CN\delta_3}
\end{aligned}$$

$$e^{j2\pi N} \frac{V_{out}}{V_{in}} = \left(\frac{\delta_1^2 + 4QC}{(\delta_1 - \delta_2)(\delta_1 - \delta_3)} \right) e^{2\pi CN\delta_1}$$



$$\frac{V_{out}}{V_{in}} = 0$$

(oscillation condition)

$$+ \left(\frac{\delta_2^2 + 4QC}{(\delta_2 - \delta_3)(\delta_2 - \delta_1)} \right) e^{2\pi CN\delta_2}$$

$$+ \left(\frac{\delta_3^2 + 4QC}{(\delta_3 - \delta_1)(\delta_3 - \delta_2)} \right) e^{2\pi CN\delta_3}$$

(rewritten)



$$\left(\frac{\delta_1^2 + 4QC}{(\delta_1 - \delta_2)(\delta_1 - \delta_3)} \right) e^{2\pi CN\delta_1}$$

$$+ \left(\frac{\delta_2^2 + 4QC}{(\delta_2 - \delta_3)(\delta_2 - \delta_1)} \right) e^{2\pi CN\delta_2}$$

$$+ \left(\frac{\delta_3^2 + 4QC}{(\delta_3 - \delta_1)(\delta_3 - \delta_2)} \right) e^{2\pi CN\delta_3} = 0$$

(to be recalled)

The following parameters are relevant while finding the parameter CN .

$$QC = \left(\frac{Q}{N}\right)(CN)$$

$$\frac{Q}{N} = \frac{QC}{CN} = \frac{2|\eta| V_0}{\epsilon_0 r_b^2 \omega^3 K l}$$

(independent of beam current)

ω has to be interpreted as the frequency where the phase velocity of the forward-wave mode of the SWS becomes equal to that of the backward-wave mode. K has to be taken as the interaction impedance at this frequency.

$$C^3 = \frac{KI_0}{4V_0}$$

$$QC = \frac{1}{4} \left(\frac{\beta_p}{\beta_e C}\right)^2 = \frac{1}{4} \left(\frac{\omega_p / v_0}{\omega / v_0 C}\right)^2$$

$$\omega_p^2 = \frac{|\eta| |\rho_0|}{\epsilon_0}$$

$$J_0 = \rho_0 v_0$$

$$|J_0| = \frac{I_0}{\pi r_b^2} \quad r_b = \text{beam radius}$$

$$\beta_e l = 2\pi N$$

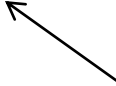
One can simultaneously solve the following two equations for CN :

(i)

$$(\delta^2 + 4(\frac{Q}{N})(CN))(j\delta - jd - b) = -1 \quad \longleftarrow \quad (\delta^2 + 4QC)(j\delta - jd - b) = -1$$

(ii)

$$\left(\frac{\delta_1^2 + 4QC}{(\delta_1 - \delta_2)(\delta_1 - \delta_3)} \right) e^{2\pi CN \delta_1} + \left(\frac{\delta_2^2 + 4QC}{(\delta_2 - \delta_3)(\delta_2 - \delta_1)} \right) e^{2\pi CN \delta_2} + \left(\frac{\delta_3^2 + 4QC}{(\delta_3 - \delta_1)(\delta_3 - \delta_2)} \right) e^{2\pi CN \delta_3} = 0$$

$QC = (\frac{Q}{N})(CN)$


The solution for CN thus obtained may be interpreted as the start-oscillation current I_0 while making use the relations:

$$C = (KI_0 / (4V_0))^{1/3} \quad \text{and} \quad \beta_e l = 2\pi N.$$