# **Some broadbanding aspects in slow-wave and fast-wave travelling-wave tubes**

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**"***For whatever may be said about the importance of aiming at depth rather than width in our studies, and however strong the demand of the present age may be for specialists, there will be work, not only for those who build up particular sciences and write monographs on them, but for those who open up such communications between the different groups of builders as will facilitate a healthy interaction between them.***"** ⎯ **James Clerk Maxwell**

*My presentation of two lectures is dedicated to Professor Dr. Dr. h. c. Manfred Thumm known as "Gyrotron Man" in India*

*I am thankful to Professor Dr. John Jelonnek, the Institute Director, and all the teachers, scientists, and students of the Institute for giving me this prestigious 'platform of learning' for interacting with and learning from you* 

*What man 'learns' is really what he discovers by taking the cover off his own soul, which is a mine of infinite knowledge*

⎯ **Swami Vivekananda** 

Some broadbanding aspects in slow-wave and fast-wave travelling-wave tubes

# **Some broadbanding aspects in**

## ■ **Slow-wave travelling-wave tubes**

- (i) Conventional TWT: Cerenkov radiation, longitudinal spacecharge-wave interaction, linear beam device
- (ii) Slow-wave cyclotron amplifier (SWCA): Bremsstrahlung radiation, Weibel instability based, cyclotron-mode interaction, gyro-device

### ■ **Fast-wave travelling-wave tubes**

- (i) Gyro-TWT: Bremsstrahlung radiation, CRM instability based, cyclotron-mode interaction, gyro-device
- (ii) Cyclotron auto-resonance maser (CARM): Bremsstrahlung radiation, both CRM instability and Weibel instability based, cyclotron-mode interaction, gyro-device

### **Travelling-wave tube (TWT) is a slow-wave device**



□ TWT is also known as Kompfner tube named after the inventor, though **Lindenblad** invented the device earlier than Kompfner

❑ Slow-wave structure is a periodic structure, typically, helical

❑ Electron beam is non-periodic and is formed, typically, by Pierce gun

❑ TWT is a Cerenkov device



**Sketch of the travelling-wave tube from R. Kompfner's note book**



### **N. E. Lindenblad's travelling-wave tube amplification at 390 MHz over a 30 MHz band (U. S. Patent 2,300,052, filed on May 4, 1940 issued on October 27, 1942)**

Helix wound around the outside the glass envelope. Signal applied to the grid of the electron gun (also applied to the helix in other experiments). Series of permanent magnets (non-periodic). Pitch tapered for velocity re-synchronization

## **Gyro-TWT is a fast-wave device**



### **http://www.insight-product.com/Gyro-TWTv1.html**

❑ Interaction structure is usually non-periodic, typically, a hollow-pipe waveguide □ Electron beam is periodic and is formed, typically, by magnetic injection gun ❑ Gyro-TWT is a bremsstrahlung device

# **Millimetre-wave consideration in conventional microwave tubes**

- Reduction of structure size
- Reduction of beam radius
- •Larger magnetic field for beam confinement for
	- Smaller beam radius
		- Larger beam current
		- Smaller beam voltage
		- Larger beam perveance
	- Heavy solenoids or advanced magnetic materials are required
- Larger cathode emission densities entailing the risk of cathode life

Higher beam voltage can increase the beam power and also reduce the required magnetic field but is associated with a reduced beam pervenace making it difficult to contain thermal electrons, and has limitation arising from backward-wave oscillation in wideband helix TWTs.

Lower beam current can reduce magnetic field but it reduces the beam power and is associated with a reduced beam pervenace making it difficult to contain thermal electrons.

Tight tolerances required for tiny interaction structures

Thermal management becomes difficult

Pressure fitting, instead of more effective than brazed-helix technology that would be difficult to implement

Special thermally conducting materials, like Type II-A diamond, for dielectric helix-supports

Plasma spraying of beryllia on the surface of the helix

## COMMERCIALLY AVAILABLE MILLIMETER-WAVE TWT'S

### Communication



## Space



### **Pulsed Radar And ECM**



### **CW Radar and ECM**



## **Millimetre-wave technology gap**

- ❑ Difficulty of operating conventional microwave tubes at higher frequencies arising from interaction structures becoming tiny, etc.
- ❑ Difficulty of operating quantum mechanical devices like the laser in view of
	- Decrease of quantum energy at lower frequencies according to the relation  $E = h \nu$
	- Difficulty to retain population inversion at lower frequencies

### **Fast-wave tubes come to rescue!**

## **Size of interaction structures**



TE<sub>01</sub> waveguide radius is  $(0.61\lambda/0.025 \lambda)$  ~25 times helix radius TE<sub>02</sub> waveguide radius is  $(1.12\lambda/0.025 \lambda)$  ~45 times helix radius

**The mechanism of interaction of a slow-wave device such as the slow-wave TWT is based on the property of an electron beam to support**  *space-charge waves*

**The mechanism of interaction of a fast-wave device such as the fast-wave gyro-TWT is based on the property of an electron beam to support** *cyclotron waves*

## **Space-charge and cyclotron waves**

### Space-charge waves Space-charge waves

$$
J = \rho v
$$
 (Current density equation)

$$
\frac{\partial J_1}{\partial z} + \frac{\partial \rho_1}{\partial t} = 0
$$
 (Continuity equation)

$$
\frac{dv_1}{dt} = \frac{e}{m} E_s = \eta E_s
$$
 (Force equation)  

$$
\frac{\partial E_s}{\partial z} = \frac{\rho_1}{\varepsilon_0}
$$
 (Poisson's equation)

RF quantities vary as  $\exp j(\omega t - \beta z)$ 

$$
\begin{aligned}\n\downarrow \\
\beta &= \beta_e \mp \beta_p \\
\hline\n\omega - \beta v_0 \mp \omega_p &= 0 \\
v_p &= \frac{\omega}{\omega \mp \omega_p} v_0\n\end{aligned}
$$

 $\mathbf{H}$ 

 $\begin{array}{cc} e & \ & \mathcal{V}_0 \end{array}$  $\beta_{e}=\frac{\omega}{\tau}$  ${\color{black} \mathcal{V}_0}$ *p p*  $\omega$  $\overline{\beta}_{{}_B} =$ *p v*  $\beta=\frac{\omega}{\tau}$ 1/ 2 0  $(\frac{|{\bf 7}||{\bf 70}||}{\sqrt{2}})$  $\mathcal E$  $|\eta||\rho$  $\omega_{_p}$  =

Force equations along *x* and *y* for a magnetic field along *z*

$$
m\frac{dv_{1x}}{dt} = e(\vec{v} \times \vec{B})_x
$$

$$
m\frac{dv_{1y}}{dt} = e(\vec{v} \times \vec{B})_y
$$

RF quantities vary as  $\exp j(\omega t - \beta z)$ 

$$
\boxed{\boldsymbol{\omega} - \boldsymbol{\beta} \boldsymbol{v}_0 \mp \boldsymbol{\omega}_c = 0} \quad \boxed{\boldsymbol{\omega}_c = -\eta \, \boldsymbol{B} = |\eta| \boldsymbol{B}}
$$

Upper sign for the fast wave and lower sign for the slow wave

# **Space-charge waves**

$$
J = \rho v \qquad J = J_0 + J_1 \qquad \rho = \rho_0 + \rho_1 \qquad v = v_0 + v_1
$$
  
\n
$$
J_0 = \rho_0 v_0
$$
  
\n
$$
J_1 = \rho_0 v_1 + v_0 \rho_1 \qquad \frac{\partial J_1}{\partial z} = \rho_0 \frac{\partial v_1}{\partial z} + v_0 \frac{\partial \rho_1}{\partial z} \qquad \frac{\partial J_1}{\partial z} + \frac{\partial \rho_1}{\partial t} = 0
$$
  
\n
$$
-\frac{\partial \rho_1}{\partial t} = \rho_0 \frac{\partial v_1}{\partial z} + v_0 \frac{\partial \rho_1}{\partial z} \qquad \frac{\partial \rho_1}{\partial t} + v_0 \frac{\partial \rho_1}{\partial z} = -\rho_0 \frac{\partial v_1}{\partial z}
$$
  
\n
$$
D \rho_1 = -\rho_0 \frac{\partial v_1}{\partial z} \qquad D = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}
$$
  
\n
$$
\frac{dv_1}{dt} = \frac{e}{m} E_s = \eta E_s \qquad \frac{dv_1}{dt} = \frac{\partial v_1}{\partial t} + \frac{dz}{dt} \frac{\partial v_1}{\partial z} = \frac{\partial v_1}{\partial t} + v_0 \frac{\partial v_1}{\partial z} = \frac{\partial v_1}{\partial t} + (v_0 + v_1) \frac{\partial v_1}{\partial z} = \frac{\partial v_1}{\partial t}
$$
  
\n
$$
\frac{\partial v_1}{\partial t} + v_0 \frac{\partial v_1}{\partial z} = \eta E_s \qquad Dv_1 = \eta E_s \qquad Dv_1 = \eta E_s \qquad [\frac{d}{dt} = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} = D]
$$

*z*

*v*

 $\widehat{o}$  $+\nu_{\circ} \frac{\nu v_1}{\nu}$ 

 $v_{\rm o}$   $\overline{\stackrel{\frown}{\partial}}$ 

0

$$
D\rho_1 = -\rho_0 \frac{\partial v_1}{\partial z}
$$

$$
D^2 \rho_1 = -\rho_0 D \frac{\partial v_1}{\partial z} = -\rho_0 \frac{\partial}{\partial z} D v_1 = -\rho_0 \frac{\partial}{\partial z} \eta E_s = -\eta \rho_0 \frac{\partial E_s}{\partial z}
$$

 $\widehat{O}$ 

*Es*

1

$$
Dv_1 = \eta E_s
$$

$$
\left[\frac{d}{dt} = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} = D\right]
$$

RF quantities vary as  $\exp j(\omega t - \beta z)$ 

$$
D = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} = j\omega - j\beta v_0 = j(\omega - \beta v_0)
$$



*E*

 $\widehat{o}$ 

$$
D=\pm j\omega_p
$$

$$
\omega - \beta v_0 = \pm \omega_p
$$

$$
\omega - \beta v_0 \mp \omega_p = 0
$$

Dispersion relation for space-charge waves

# **Cyclotron waves**

$$
m \frac{dv_{1x}}{dt} = e(\vec{v} \times \vec{B})_x \qquad m \frac{dv_{1y}}{dt} = e(\vec{v} \times \vec{B})_y
$$
  
\n
$$
B_x = 0, B_y = 0, B_z = B \qquad \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}
$$
  
\n
$$
\frac{dv_{1x}}{dt} = \frac{e}{m} (\vec{v} \times \vec{B})_x = \eta (\vec{v} \times \vec{B})_x
$$
  
\nRF quantities vary as exp  $j(\omega t - \beta z)$   
\n
$$
Dv_{1y} = -\eta (\vec{v} \times \vec{B})_y
$$
  
\n
$$
Dv_{1x} = \eta Dv_y = -\omega_c v_y
$$
  
\n
$$
\frac{[d}{dt} = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} = D]
$$
  
\n
$$
Dv_{1y} = -\eta Dv_x = \omega_c v_{1x}
$$
  
\n
$$
D^2 v_{1x} = -\omega_c Dv_{1y} = -\omega_c (\omega_c v_{1x}) = -\omega_c^2 v_{1x}
$$

 $D^2 = - \omega_c^2$  $\omega - \beta v_0 \pm \omega_c = 0$  Dispersion relation for cyclotron waves

### **Space-charge and cyclotron waves**

 $v_p$   $v_0$   $v_0$   $v_0$ *p p p*  $\omega$   $\omega$   $\omega$ <sub>n</sub>  $\omega$  +  $\omega$  $\beta$ ∓  $=$   $\frac{w}{r}$   $=$   $\frac{w}{r}$   $\frac{w}{r}$   $\frac{w}{r}$   $\frac{w}{r}$   $\frac{w}{r}$   $\frac{w}{r}$  $v_p = -\frac{1}{2} = -\frac{v_0}{2}$ *p p*  $\beta$   $\omega \mp \omega$  $\omega$  $\beta$  $\omega$ ∓  $=$   $=$  $\beta = \beta_{_e} \mp \beta_{_p}$  $\beta = \beta_{\scriptscriptstyle e} \mp \beta_{\scriptscriptstyle c}$ Space-charge waves Cyclotron waves  $\omega - \beta v_{0} \mp \omega_{p} = 0$  $\omega - \beta v_0 \mp \omega_c = 0$  $v_p$   $v_0$   $v_0$   $v_0$ *c c p*  $\beta = \frac{\omega}{\omega} = \frac{\omega}{\omega} + \frac{\omega_c}{\omega} = \frac{\omega + \omega_c}{\omega}$ Ŧ  $=$   $\frac{16}{10}$   $=$   $\frac{16}{10}$   $\frac{16}{10}$   $=$   $\frac{16}{10}$   $\frac{16}{10}$   $=$  $v_{p} = \frac{v_{p}}{v_{p}}$ *c p*  $\omega \mp \omega$  $\omega$  $\bar{+}$ =

Upper sign for the fast wave and lower sign for the slow wave

$$
\beta_e = \frac{\omega}{v_0}; \beta_p = \frac{\omega_p}{v_0}; \omega_p = (\frac{|\eta|\rho_0}{\varepsilon_0})^{1/2} \quad \bigg| \qquad \beta_e = \frac{\omega}{v_0}; \beta_c = \frac{\omega_c}{v_0}; \omega_c = |\eta|B
$$

# **Amplification of space-charge waves**

- An electron beam of uniform diameter in a resistive-wall cylindrical waveguide
- An electron beam in a rippled-wall (varying diameter) conducting-wall cylindrical waveguide
- An electron beam of varying diameter in a conducting-wall cylindrical waveguide
- An electron beam mixed with another beam of a slightly different DC electron beam velocity (two-stream amplifier)
- An electron beam penetrating through a plasma (beam-plasma amplifier)
- An electron beam interacting with RF waves supported by a slow-wave structure (TWT)

# **Intersection between slow space-charge and circuit waves**

 $\omega - \beta v_{0} \mp \omega_{p} = 0$ 

**Upper sign for the fast wave and lower sign for the slow wave**



**Intersection between fast-cyclotron and circuit waves**

$$
\omega - \beta v_0 \mp \omega_c = 0
$$

**Upper sign for the fast wave and lower sign for the slow wave**





**Plots of beam-mode and waveguide-mode dispersion characteristics showing the operating point as the intersection between these plots (***Prepared by Vishal Kesari* **<***vishal\_kesari@rediffmail.com***>)**

### **Conditions for grazing intersection or coalescence between the beam-mode and waveguide-mode dispersion characteristics**

Dispersion relation for cyclotron wave

 $\omega - \beta v_0 \mp \omega_c = 0$  (upper sign for the fast cyclotron wave)

Dispersion relation for fast cyclotron wave:

$$
\omega - \beta v_0 - \omega_c = 0 \qquad (v_0 = v_z)
$$

Beam-mode dispersion relation:  $\omega - \beta v = \frac{3\omega_c}{c} = 0$  $\gamma$  $\omega - \beta v_{_7} - \frac{s\omega_c}{\omega}$ *z s v* (for beam harmonic *s* operation)  $(v_0 = v_z)$ 

$$
\gamma = (1 - \frac{v_z^2 + v_t^2}{c^2})^{-1/2} \qquad \gamma = 1 + \frac{|e|V_0}{mc^2}
$$



 $\omega_c$  absorbs the relativistic factor  $\gamma$  if it does not appear in the expression

*<sup>e</sup> B* Beam-mode dispersion relation:  $\omega - \beta v_z - \frac{s \omega_c}{s} = 0$ *s*  $[\omega_{c} = \frac{1}{2}$ *v*  $\omega_{c}^{}$   $=$ *z*  $\gamma$ *m* Wavequide-mode dispersion relation  $\omega^2 - \beta^2 c^2 - \omega_m^2$ Waveguide-mode dispersion relation:  $\omega^2 - \beta^2 c^2 - \omega_{cut}^2 = 0$  $\omega$   $\beta c^2$  $\widehat{O}$  $\widehat{O}$  $\beta$  $\omega$   $\beta c^2$ c-line  $\Rightarrow \frac{\partial \omega}{\partial \beta} = \frac{\beta c^2}{\omega} \Rightarrow v_g = \frac{\partial \omega}{\partial \beta} = \frac{\beta c}{\omega}$  $\beta$  $\omega_{\rm cut}$  $\omega$  $2\omega \frac{\partial \omega}{\partial t} - 2\beta c^2 = 0$  $\Rightarrow v_{e} =$  $\beta$   $c$  $v_g = \frac{1}{\partial \beta} =$  $\omega$ Gyro-TWT  $\overline{\partial B}$  =  $\beta$  $\widehat{O}$  $\beta$  $\beta$  $\omega$ dispersion relation  $\beta$   $c^2$ Grazing intersection:  $v_z = v_g$  $\Rightarrow v_z =$  $\theta$  $\omega$  $s\omega_c^{\dagger}$ 2  $\beta\,c^{\,2}$ From  $\omega - \beta v_z - \frac{s \omega_c}{s} = 0$  and  $v_z = \frac{\beta c^2}{s} \implies \omega = \frac{s \omega_c}{s} \gamma_z^2$   $[\gamma_z = (1 - \frac{v_z^2}{s})^{-1/2}]$  $\omega - \beta v_z - \frac{s \omega_c}{\omega}$ *s* 2  $v_{\rm z}$ and  $v_z =$  $-\beta v - \frac{1}{2}$  = 0  $=(1-\frac{z}{2})^{-}$  $\gamma$ *v*  ${\mathcal{Y}}_z$ *z z* 2  $\gamma$  $\gamma$ *c*  $\omega$  $\beta\,c^{\,2}$  $v_z = \frac{\rho c}{\sigma} \implies \omega = \omega_{cut} \gamma_z$ From  $\omega^2 - \beta^2 c^2 - \omega_{cut}^2 = 0$  and  $\omega$  $s\omega_c^{}$  $\omega$ 2  $\omega =$   $-\gamma$  =  $\gamma_{z} = \omega_{\rm cut} \gamma$ **Grazing magnetic flux density**  $B = \frac{\gamma}{\gamma} \frac{m}{\gamma} \frac{1}{\gamma}$ *z cut z*  $B = \frac{\gamma}{\gamma} \frac{m}{\gamma} \frac{1}{\gamma} \omega$  $\Rightarrow$  $\gamma$ *cut*  $\gamma$ *z e s <sup>e</sup> B*  $\omega_{c}^{}$   $=$ *m*

## **Features of conventional TWT and gyro-TWT**

## **Conventional TWT**

- ⚫ Cerenkov radiation type
- ⚫ Magnetic field for beam confinement
- ⚫ Larger magnetic field at higher frequencies for beam confinement
- ⚫ Conversion of axial beam kinetic energy
- ⚫ Axial non-relativistic electron bunching
- ⚫ Near-synchronism between DC beam velocity and circuit phase velocity

# **Gyro-TWT**

- ⚫ Bremsstrahlung radiation type
- ⚫ Magnetic field for interaction
- ⚫ Larger magnetic field at higher frequencies for cyclotron resonance
- ⚫ Conversion of azimuthal beam kinetic energy
- ⚫ Azimuthal relativistic electron bunching
- ⚫ Near-synchronism between wave frequency and cyclotron frequency

(*Continued*)

## **Conventional TWT**

- Electron beam velocity to be slightly greater than RF phase velocity
- ⚫ Slow space-charge wave on electron beam to couple to forward circuit wave
- Space-charge-limited operation
- Pierce gun
- Smaller structure sizes at higher frequencies
- ⚫ BWO absolute instability above the pipoint frequency

# **Gyro-TWT**

- Wave frequency to be slightly greater than cyclotron frequency
- ⚫ Fast cyclotron wave on electron beam to couple to forward waveguide mode
- ⚫ Temperature-limited operation
- **MIG**
- ⚫ Relatively larger structure sizes at higher frequencies
- ⚫ BWO absolute instability above a threshold current that depends sensitively on the applied magnetic field

**Dispersion relation of a conventional TWT**

$$
(\beta_0^2 - \beta^2)(\beta_p^2 - (\beta_e - \beta)^2) = -2\beta_e^4 C^3
$$
  

$$
C^3 = \frac{K I_0}{4V_0}
$$

**For weak coupling, the right hand side of the dispersion relation is zero.**

 $\beta = \pm \beta_{\rm o} \,$  (Cold circuit forward and backward propagation constants)

 $\beta = \beta_e \mp \beta_p$  (Propagation constants of space-charge waves)

### **Dispersion relation of a gyro-TWT**

$$
(k_0^2 - \beta^2 - k_t^2)(\omega - \beta v_z - \frac{s\omega_c}{\gamma})^2 = \frac{-\mu_0 |e|^2 N_0 \eta_t^2 H_{m, -s}(\omega^2 - \beta^2 c^2)}{\gamma m \pi K_{mn} a^2}
$$
  

$$
K_{mn} = (1 - \frac{m^2}{k_t^2 a^2}) J_m^2 \{k_t a\}
$$
  

$$
H_{m, -s} = J_{s-m}^2 \{k_t r_c\} J_s^2 \{k_t r_L\}
$$

 $r_{c}^{}$  ,  $r_{L}^{}$ : hollow beam radius, Larmor radius  $N_0$ : no. of electrons per unit length

$$
I_0 = N_0 |e| v_z
$$

$$
\eta_t = \frac{v_t}{c}
$$
 *m*: waveguide angular harmonic number  
s: beam harmonic mode number

**For weak coupling, the right hand side of the dispersion relation is zero, giving**

 $k_0^2 - \beta^2 - k_t^2 = 0$  (Cold waveguide-mode dispersion relation)

 $-\beta v$  –  $-\alpha$  = 0  $\gamma$  $\omega - \beta \, v_{_7} - \frac{s \, \omega_c}{\tau}$ *z s*  $v_{\tau} - \frac{\partial \omega_c}{\partial r} = 0$  (Dispersion relation for fast Cyclotron wave)  $\omega - \beta v_0 \mp \omega_c = 0$  taking the upper sign, and interpreting  $v_z = v_0$  and  $s = 1$ 

### **Pierce-type gain expression** *G* =*A* + *BCN*

For the beam-wave coupled system, RF quantities vary  $\exp j(\omega t - \Gamma z)$ as

### **Conventional TWT Gyro-TWT**  $-\Gamma = -j\beta_e + \beta_e C\delta$  $-\Gamma = -j\beta_{mn} + \beta_e C\delta$  $\begin{array}{cc} e & \vline \end{array}$  $\beta_{_e}=\frac{\omega}{\tau}$ *z c e v*  $\beta_e = \frac{\omega - s \omega_c / \gamma}{\sigma}$  $\beta_{mn}$ : TE<sub>mn</sub> waveguide-mode = propagation constant  $^{2}(\delta + jb) = -j$  $\delta^2(\delta + jb) = -j$  Cubic dispersion relation  $\delta(\delta + jb)^2 = j$  $\delta(\delta + jb)^2$  $\delta = x + jy$  $\delta_1 = x_1 + jy_1$ Solution with the positive real part

Conventional TWT	$G = A + BCN$	<b>Gyro-TWT</b>
$b = \frac{\beta_0 - \beta_e}{\beta_e C}$	$A = 20 \log_{10} \left  \frac{1}{(1 - \frac{\delta_2}{\delta_1})(1 - \frac{\delta_3}{\delta_1})} \right $	$b = \frac{\beta_e - \beta_{mn}}{\beta_{mn}C}$
$N = \frac{\beta_e l}{2\pi}$	$N = \frac{\beta_{mn} l}{2\pi}$	
$B = 40\pi (\log_{10} e)(x_1)$	$C^3 = \frac{K I_0}{4V_0}$	$B = \frac{54.6x_1}{1 - Cy_1}$
$K = \frac{ E ^2}{2\beta^2 P}$	$K = \frac{(\mu_0 / \varepsilon_0)^{1/2} (v_r / c)^2 k_t^2 (1 + \alpha_0) H_{m, -s}}{\pi (1 - \frac{m^2}{k_t^2 a^2}) J_m^2 (k_t a)(v_z / c) \beta_{mn}^4}$	
$\alpha_0 = \frac{v_t}{v_z}$ : Beam pitch factor		

−

## **Some concepts in widening slow-wave TWTs**

## **Zero-to-slightly-negative-dispersion structure for wideband performance**

Anisotropically loaded helix Metal vane/ segment loaded envelope Inhomogeneously loaded helix: Helix with tapered geometry dielectric supports such as half-moon-shaped and T-shaped supports

**Negative dispersion ensures constancy of Pierce's velocity synchronization parameter** *b* **with frequency**

**Multi-dispersion structures for wideband performance**



Negative dispersion:  $v_p$  increases with frequency

 $v_0 - v_p$  decreases with frequency  $v_{\scriptscriptstyle p}$  $v_{0} - v_{p}$ decreases with frequency

 $\rightarrow$  Numerator of the expression for *b* decreases with frequency

*K* decreases with frequency and hence the

 $\rightarrow$  Denominator of the expression for *b* decreases with frequency

*b* remains constant with frequency
**Conventional TWTs with multi-dispersion, multi-section structures** 

*G* <sup>~</sup> *BCN* **Small-signal gain equation**

$$
N\lambda_e = l
$$
  
\n
$$
N\frac{v_0}{f} = l
$$
  
\n
$$
C = (KI_0 / 4V_0)^{1/3}
$$
  
\n
$$
N = \frac{f l}{v_0}
$$

$$
G \sim B(KI_0/4V_0)^{1/3} \frac{f l}{v_0}
$$

*G* is proportional to  $K^{1/3} f l$ 

*G* is proportional to  $K^{1/3}f$  *l* 

Gain-frequency response:

.

Lower gain at lower frequencies as *G* is proportional to *f*

Lower gain at higher frequencies as  $G$  is proportional to  $K^{1/3}$ , the latter decreasing with frequency

Conventional structure: If you had increased the length *l*, then the gain *G* would be compensated at lower frequencies *f*. However, then the gain *G* would become very high at higher frequencies *f*.

Therefore, let us arrive at the design of a helical slow-wave structure the **effective length** of which is **large at lower frequencies,** which at the same time becomes relatively **smaller at higher frequencies**. (The design should ensure that the gain is not enhanced at any frequency to a high value causing oscillation in the device).

**The answer lies in a multi-dispersion, multi-section helix TWTs!** 

One positive-dispersion helix section of length  $l_1$  synchronous with the beam only at lower frequencies and the other nearly dispersion-free helix section of effective length length  $l_2$  synchronous with the beam both at lower and higher frequencies.

> Effective length increased to  $l_1 + l_2$  at lower frequencies Effective length reduced to  $l_2$  at higher frequencies (since the section of length  $l_1$  goes out of synchronism at higher frequencies

Gain is proportional to  $K^{1/3}f l$ 

We have to control (i) the nature and the amounts of dispersion of of the sections by suitably loading the sections and (ii) the lengths of the two sections

## **Select structure sections such as segment loaded helices of controllable dispersion**

**Analysis should be capable of finding the dispersion and interaction impedance characteristics of the structure sections, say, with metal segment loaded envelopes and their control by structure section parameters like segment dimensions and relative section lengths.**

## **Two-section configuration with one of the sections providing a doublehump in the gain-frequency response**

One of the seconds provides a double-hump peaks while the second section provides a single peak between the humps in the gain frequency response

## **Twystron**

The first section is a klystron providing a double-hump gain-frequency response. The second section is a TWT providing a peak between the two humps of the first section in the gain-frequency response.

## **Gyro-twystron**

## **Two-section Gyro-TWT**

Two-hump peaks result from the beam-mode dispersion line intersecting with the waveguide dispersion hyperbola at two points. One may use two dielectric loaded sections, one of which should provide a single peak between the two peaks provided by the other section, in the gain-frequency response.





 $Ln$   $Col$   $1$ REC TRK EXT OVR English (Indi **The Common** 

 $\sqrt{1 + \frac{1}{2}}$ 

40

**A** 



**Source: MTRDC (DRDO)**





**Source: MTRDC (DRDO)**











**Source: MTRDC (DRDO)**

K (ohm)







Angular segment Straight segment T- segment

**Source: MTRDC (DRDO)**



# Embedded rod



**Source: MTRDC (DRDO)**

Metal-coated support rods



Provides negative dispersion with quite high interaction impedance compared to other segment variants

**Source: MTRDC (DRDO)**





## **Methods of broadbanding a single-stage gyro-TWT**

• Controlled dispersion characteristics of the waveguide for wideband coalescence with beam-mode dispersion characteristics

- i) by dielectric lining the metal wall of a circular waveguide
	- $\rightarrow$  Method entails the risk of dielectric charging that results into heating if the dielectric is lossy
- ii) by metal disc loading a circular waveguide
	- $\rightarrow$  Optimisation of the disc parameters brings about the desired shape of the structure dispersion for wide coalescence bandwidth for wideband device performance
- Tapered waveguide cross section and profiled magnetic field
	- $\rightarrow$  Device bandwidth is increased
	- $\rightarrow$  Device gain is decreased due to reduction of the effective interaction length at each of frequency zones of the bandwidth



Electron orbit

## Disc-loaded Circular Waveguide



Beam-absent (cold) analysis

- 
- 
- 
- 
- 

considering



- Clarricoats and Olver Surface impedance model - Amari *et al.* Coupled-integral-equation technique - Esteban and Rebollar Modal expansion technique - Choe and Uhm Field matching technique - Kesari *et al.* Field matching technique

higher order propagating wave space harmonics in disc-free region higher order standing wave modal harmonics in disc-occupied region finite disc thickness

# Field Matching Technique for Dispersion Relation



Infinitesimally Thin Disc-loaded Circular Waveguide



### Disc-loaded Circular Waveguide having Finite Disc Thickness



Dispersion and azimuthal interaction impedance characteristics of the disc-loaded circular waveguide typically for the  $TE_{01}$  mode, taking disc periodicity as the parameter (Broken curves refer to circular waveguide loaded with infinitesimally thin discs)



## Circular Waveguide loaded with Alternate Dielectric and Metal Annular Discs



Dispersion characteristics of a circular waveguide loaded with alternate dielectric and metal annular discs showing wider coalescence bandwidth of  $TE_{02}$  mode than that of  $TE_{01}$ mode excited conventional metal disc-loaded circular waveguide (broken curve)



### Tapering the Structure Cross-section

## bandwidth is increased device gain is decreased due to reduction of effective interaction length



waveguide-wall radius of  $p<sup>th</sup>$  step axial length of  $p^{\text{th}}$  step  $L_p \rightarrow$ disc-hole radius of  $p^{\text{th}}$  disc  $r_{D,p} \rightarrow$ disc thickness of  $p^{\text{th}}$  disc  $T_p \rightarrow$ total number of discs in tube length  $n_D \rightarrow$ 





Tapered Disc-loaded circular waveguide and its analytical model



- i) disc-hole radius, to spread the device gain over a wide range of frequencies without regard to the gain value
- ii) waveguide-wall radius, to compensate for the gain values that would be reduced at the band edges due to tapering of the disc-hole radius







## **Broadbanding techniques**

### Conventional TWT Conventional TWT

- ❑ Homogeneous dielectric loading cannot shape structure dispersion for wide device bandwidths
- ❑ Inhomogeneous dielectric loading by tapered-crosssection helix supports can shape structure dispersion for wide device bandwidths
- ❑ Anisotropic loading by azimuthally periodic metal vanes provided with the envelope can shape structure dispersion for wide device bandwidths

- □ Homogeneous dielectric loading can shape dispersion, and can provide wideband coalescence between the beam-mode and waveguide-mode dispersion characteristics and consequently wide device bandwidths
- ❑ Two-stage homogeneous dielectric loading can widen gain-frequency response
- ❑ Anisotropic loading by azimuthally periodic metal vanes provided with the waveguide wall cannot shape structure dispersion for wide device bandwidths

### **Broadbanding techniques** (*Continued***)**

### Conventional TWT Gyro-TWT

\_\_\_\_\_\_\_\_

 $\overline{\phantom{a}}$ 

❑ Axially periodic disc loading cannot shape structure dispersion and cannot widen device bandwidth but can enhance device gain

- ❑ Axially periodic disc loading can shape structure dispersion and can widen device bandwidth
- ❑ Waveguide cross section tapering can broadband the device at small gains
- ❑ Axially periodic discs plus tapering the waveguide cross section can broadband the device at large gains

### **CARM, SWCA, and Gyro-TWT**

### **Beam-mode dispersion relation**

- $\mathcal Y$  $\gamma$  $\Delta \omega_{D}^{} = \beta \, \Delta \rm v_{z}^{} - \frac{s \, \omega_{c}^{}}{c^{2}} \, \Delta \,$  $\omega = \beta v_z + s\omega_c / \gamma = \omega_D$  $\omega - \beta v_z - s \omega_c / \gamma = 0$ *c*  $D$   $\mu$   $\Delta r$ <sub>*z*</sub> *s v s* is the beam harmonic number Doppler-shifted cyclotron angular frequency
	- $\Delta v_z$  is caused by Weibel instability due to Lorentz force on electrons in transverse motion in transverse RF magnetic field
	- is caused by energy exchange between electrons with transverse motion in transverse RF electric field  $\Delta \gamma$

$$
\Delta \omega_{D} = \beta \, \Delta v_{z} - \frac{s \, \omega_{c}}{\gamma^{2}} \, \Delta \gamma
$$

$$
\gamma m \frac{dv_z}{dt} \vec{a}_z = e \vec{v}_\perp \times \vec{B}_\perp
$$

$$
\gamma m \Delta v_z = e \vec{v}_{\perp} \times \vec{B}_{\perp} \Delta t = e \vec{v}_{\perp} \times (B_r \vec{a}_r + B_\theta \vec{a}_\theta) \Delta t
$$

$$
\Delta v_z = \frac{e\vec{v}_{\perp} \times (B_r\vec{a}_r + B_\theta\vec{a}_\theta)}{mn} \Delta t
$$

Let us express magnetic field quantities in terms of electric field quantities with the help of Maxwell's equations

$$
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \rightarrow \qquad \frac{1}{r} \begin{vmatrix} \vec{a}_r & r\vec{a}_\theta & \vec{a}_z \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial z \\ E_r & rE_\theta & 0 \end{vmatrix} = -\frac{\partial}{\partial t} (B_r \vec{a}_r + B_\theta \vec{a}_\theta + B_z \vec{a}_z)
$$

$$
\frac{1}{r}\begin{vmatrix} \vec{a}_r & r\vec{a}_\theta & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ F_r & rF_\theta & 0 \end{vmatrix} = -\frac{\partial}{\partial t}(B_r\vec{a}_r + B_\theta\vec{a}_\theta + B_z\vec{a}_z) \quad \Delta v_z = \frac{e\vec{v}_\perp \times (B_r\vec{a}_r + B_\theta\vec{a}_\theta)}{m} \Delta t
$$

*r*- component of magnetic field: 
$$
-\frac{\partial E_{\theta}}{\partial z} = -\frac{\partial B_{r}}{\partial t} \implies -j\beta E_{\theta} = j\omega B_{r} \implies B_{r} = -\frac{\beta E_{\theta}}{\omega}
$$

$$
\theta\text{-component:}\quad \frac{1}{r}\frac{\partial E_r}{\partial z} = -\frac{\partial B_\theta}{\partial t} \implies -j\beta E_r = -j\omega B_\theta \implies B_\theta = \frac{\beta E_r}{\omega}
$$

$$
\Delta v_z = \frac{e \vec{v}_{\perp} \times (B_r \vec{a}_r + B_\theta \vec{a}_\theta)}{ym} \Delta t = \frac{e \beta \vec{v}_{\perp} \times (-E_\theta \vec{a}_r + E_r \vec{a}_\theta)}{ym\omega} \Delta t = \frac{e \beta \vec{v}_{\perp} \times \vec{a}_z \times \vec{E}_{\perp}}{ym\omega} \Delta t
$$
  

$$
\vec{a}_z \times \vec{E}_{\perp} = \vec{a}_z \times (E_r \vec{a}_r + E_\theta \vec{a}_\theta) = E_r \vec{a}_\theta - E_\theta \vec{a}_r = -E_\theta \vec{a}_r + E_r \vec{a}_\theta
$$

$$
\Delta v_z = \frac{e\beta \vec{v}_{\perp} \times \vec{a}_z \times \vec{E}_{\perp}}{m\omega} \Delta t
$$

$$
\Delta v_z = \frac{e\beta \vec{v}_{\perp} \times \vec{a}_z \times \vec{E}_{\perp}}{ \gamma m \omega} \Delta t = \frac{e\beta [(\vec{v}_{\perp} \cdot \vec{E}_{\perp})\vec{a}_z - (\vec{v}_{\perp} \cdot \vec{a}_z)\vec{E}_{\perp}]}{ \gamma m \omega} \Delta t = \frac{e\beta [(\vec{v}_{\perp} \cdot \vec{E}_{\perp})\vec{a}_z]}{ \gamma m \omega} \Delta t
$$
  

$$
\vec{v}_{\perp} \cdot \vec{a}_z = 0
$$
  

$$
\Delta v_z = \frac{e\beta [(\vec{v}_{\perp} \cdot \vec{E}_{\perp})\vec{a}_z]}{ \Delta t} \Delta t
$$
  

$$
\Delta \omega_D = \beta \Delta v_z - \frac{s \omega_c}{v^2} \Delta \gamma
$$

 $\gamma$ 

 $\Delta \gamma = ?$ 

 $\mu$ 

Rate of change of kinetic energy with time  $\Delta \gamma = ?$ 

$$
\frac{d}{dt}(\eta mc^2) = e(\vec{v}_{\perp} \cdot \vec{E}_{\perp})
$$
\n
$$
\Delta v_z = \frac{e\beta[(\vec{v}_{\perp} \cdot \vec{E}_{\perp})\vec{a}_z]}{m\omega} \Delta t
$$
\n(Weibel instability)  
\n
$$
mc^2(\Delta \gamma) = e(\vec{v}_{\perp} \cdot \vec{E}_{\perp})\Delta t
$$
\n(Weibel instability)  
\n
$$
\Delta \gamma = \frac{e}{mc^2}(\vec{v}_{\perp} \cdot \vec{E}_{\perp})\Delta t
$$
\n
$$
\Delta \omega_D = \beta \Delta v_z - \frac{s \omega_c}{\gamma^2} \Delta \gamma
$$
\n
$$
\omega_D = \beta \frac{e\beta}{m\omega}(\vec{v}_{\perp} \cdot \vec{E}_{\perp})\Delta t - \frac{s\omega_c}{\gamma^2} \frac{e}{mc^2}(\vec{v}_{\perp} \cdot \vec{E}_{\perp})\Delta t = [\frac{e\beta^2}{m\omega} - \frac{s\omega_c}{\gamma^2} \frac{e}{mc^2}] \Delta t (\vec{v}_{\perp} \cdot \vec{E}_{\perp})
$$
\n(Weibel instability)  
\n(CRM instability)

$$
\Delta \omega_{D} = \beta \frac{e\beta}{\gamma m \omega} (\vec{v}_{\perp} \cdot \vec{E}_{\perp}) \Delta t - \frac{s\omega_{c}}{\gamma^{2}} \frac{e}{mc^{2}} (\vec{v}_{\perp} \cdot \vec{E}_{\perp}) \Delta t = \left[ \frac{e\beta^{2}}{\gamma m \omega} - \frac{s\omega_{c}}{\gamma^{2}} \frac{e}{mc^{2}} \right] \Delta t (\vec{v}_{\perp} \cdot \vec{E}_{\perp})
$$
\n(Weibel instability) (CRM instability)

$$
\Delta \omega_{D} = \beta \frac{e\beta}{\gamma m \omega} (\vec{v}_{\perp} \cdot \vec{E}_{\perp}) \Delta t - \frac{s\omega_{c}}{\gamma^{2}} \frac{e}{mc^{2}} (\vec{v}_{\perp} \cdot \vec{E}_{\perp}) \Delta t = \left[ \frac{e\beta^{2}}{\gamma m \omega} - \frac{s\omega_{c}}{\gamma^{2}} \frac{e}{mc^{2}} \right] \Delta t (\vec{v}_{\perp} \cdot \vec{E}_{\perp})
$$
\n(Weibel instability) (CRM instability)

$$
\Delta \omega_D = \left[\frac{e\beta^2}{\gamma m \omega} - \frac{s\omega_c}{\gamma^2} \frac{e}{mc^2}\right] \Delta t (\vec{v}_{\perp} \cdot \vec{E}_{\perp})
$$

$$
\frac{s\omega_c}{\gamma} \leq \omega \qquad \qquad \frac{s\omega_c}{\gamma} = F\omega \qquad F \leq 1
$$

$$
\Delta \omega_{D} = \left[\frac{e\beta^{2}}{m\omega} - F\omega \frac{e}{mc^{2}}\right] \Delta t (\vec{v}_{\perp} \cdot \vec{E}_{\perp})
$$

$$
\Delta \omega_D = \left[\frac{e\beta^2}{\gamma m \omega} - F\omega \frac{e}{\gamma mc^2}\right] \Delta t (\vec{v}_{\perp} \cdot \vec{E}_{\perp})
$$

$$
\frac{s\omega_c}{\gamma} = F\omega
$$

(Weibel instability) (CRM instability)

## **If Weibel instability dominates over CRM instability**



$$
\frac{\beta^2}{\omega^2} > \frac{1}{c^2} \quad \longrightarrow \quad \frac{\omega^2}{\beta^2} < c^2
$$
\n
$$
\frac{\omega}{\beta} < c
$$

 $v_p < c$ 

**Destabilization of slow waves: Slowwave cyclotron amplifier (SWCA**)

**If CRM instability dominates over Weibel instability**

$$
F\omega \frac{e}{\gamma mc^2} > \frac{e\beta^2}{\gamma m\omega} \quad (F \approx 1)
$$



 $v_p > c$ 

**Destabilization of fast waves: Gyro-TWT**

**If both Weibel instability and CRM instability are present in equal proportions (auto-resonance)**

$$
\frac{e\beta^2}{\gamma m \omega} = F\omega \frac{e}{\gamma mc^2} \quad (\Delta \omega_D = 0) \qquad \frac{s\omega_c}{\gamma} \le \omega \qquad \frac{s\omega_c}{\gamma} = F\omega \qquad F \le 1
$$
  

$$
\frac{\beta^2}{\omega^2} = F\frac{1}{c^2} \qquad F \le 1
$$
  

$$
\frac{\omega}{\beta} = c\frac{1}{\sqrt{F}}
$$
  

$$
v_p = c\frac{1}{\sqrt{F}} \qquad F \le 1
$$

 $v_p \geq c$  $\,>\,$ 

## **Destabilization of slightly fast waves: Cyclotron auto-resonance maser**

## **Some CARM features**

Weibel and CRM instabilities are simultaneously present in equal proportions balancing the effects due to each other leading to auto-resonance.

Once the electron beam is phase-bunched, a large amount of energy could be extracted without loosing synchronism (auto-resonance).

Mildly fast waveguide mode is destabilized  $v_p > c$  $\,>$ 

Requires lesser magnetic field due to large  $\beta v_z$  Doppler shift (beam-mode dispersion relation)  $s\omega_c$  /  $\gamma = \omega - \beta v_z$ 



Operated far from cutoff at a large value of the axial phase propagation constant  $\beta$  and hence sensitive to beam velocity spread causing inhomogeneous broadening of the cyclotron resonance band

Wideband coalescence between the dispersion plots giving wide bandwidths

Operates at a lower beam pitch factor, higher beam voltage, with higher power capability, and at higher frequencies

Higher efficiency, since the axial kinetic energy is tapped

Increased stability due to higher axial electron velocities

Reduced gain caused by axial bunching offsetting azimuthal bunching

Wider device bandwidth due to wideband coalescence between the the beam-mode and waveguide-mode dispersion characteristics

### **Typical CARM Performance**

10 MW, 35 GHz, 45 dB gain and 3% efficiency (1.5 MV, 0.25 kA) 10 MW at 125 GHz, 2% efficiency (0.5 MV, 1.0 kA)

### **Some SWCA features**



Weibel instability dominates over CRM instability

Slow waveguide mode is destabilized  $v_p < c$ 

Requires lesser magnetic field due to large  $\beta v_z$  Doppler shift (beam-mode dispersion relation)  $s\omega_c$  /  $\gamma = \omega - \beta v_z$ 

Wideband coalescence between the dispersion plots giving wide device bandwidths
Operates at a lower beam voltage,

with relatively lower power capability, and at lower frequencies

Wider device bandwidth due to wideband coalescence between the the beam-mode and waveguide-mode dispersion characteristics

## **Typical SWCA performance**

Ka band, 50 kW, 60 kV, 5 A, 7kG, 24-28 dB saturated gain, 2 dB/cm power gain, 15% efficiency at 2% axial beam velocity spread (JJ Choi *et al.*, IEEE-PS, Aug 1994, 465-475)

Stabilized by a sever against gyro-BWO instability

35 GHz, 1 dB/cm over 25-65 GHz, 71.5 kV, 9.2 A, 5.9 kG, 10% efficiency, bandwidth: 89.6% (10 dB), 67.1% (20 dB), 53.7% (30 dB), 1.6 mm average beam radius, 3.2 mm dielectric inner radius, 4.6 mm W/G radius, low-loss alumina lining, a metal mesh or coating on the dielectric to prevent static charge buildup



### **The bandwidth of a 'slow-wave' TWT can be widened by**

- providing azimuthally periodic metal vanes/segments with the metal envelope of the helical slow-wave structure and optimizing the structure parameters, thereby obtaining the desired dispersion characteristics of the structure for wide device bandwidths
- appropriately shaping the cross-sectional geometry of dielectric helix supports, thereby obtaining the desired dispersion characteristics of the structure for wide device bandwidths
- using multi-dispersion, multi-section helical structures
- using a relatively simple conventional fast-wave guiding structure though heavily dielectric loading it to operate it in the slow-wave regime to realize an SWCA for wideband coalescence between the beam-mode and the waveguide-mode dispersion plots for wide device bandwidths operating at a lower beam voltage, with relatively lower power capability and at lower frequencies

#### **The bandwidth of a 'fast-wave' TWT can be widened by**

- dielectric loading the cylindrical waveguide of a gyro-TWT, by either dielectric lining the waveguide or by providing a coaxial rod insert and optimizing the dielectric parameters for wideband coalescence between the beam-mode and the waveguide-mode dispersion plots resulting in wide device bandwidths
- tapering the waveguide cross section thereby causing the different length elements of the waveguide operate at different elements of frequency ranges, over a wide band of frequencies, thereby providing wide device bandwidths, however, at the cost of the device gain
- using axially periodic disc loading and optimizing the disc parameters for the desired wideband coalescence between the beam-mode and the waveguidemode dispersion plots resulting in wide device bandwidths
- using multi-dispersion, multi-section waveguides for wide device bandwidths
- using tapered-cross-section axially periodic disc loading and optimizing the disc parameters for the desired wideband coalescence between the beam-mode and the waveguide-mode dispersion plots resulting in wide device bandwidths at relatively large gains
- realizing CARM operation in mildly fast-wave waveguide regime providing the desired wideband coalescence between the beam-mode and the waveguidemode dispersion plots for wide device bandwidths at relatively higher beam voltages and at higher frequencies obtaining higher power levels and higher efficiencies though at reduced gains

## **State-of-the-art gyro-TWTs**



**(Source: State-of-the-art of high power gyro-devices and free electron masers updates 2007, FZKA 7392)** 



## Specifications of gyro-TWTs developed in some typical laboratories (Compiled by Vishal Kesari <vishal\_kesari@rediffmail.com>)



[Continued]



## Specifications of gyro-TWTs developed in some typical laboratories (Compiled by Vishal Kesari <vishal\_kesari@rediffmail.com>)



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# **Thank You**