Recent Trends in Microwave/Millimeter-Wave Technology and Their Applications in Wireless Communication and Defence Perspectives [14-19 October 2019: Convener: M. Thottappan (IIT-BHU)]

Electromagnetic Boundary Conditions

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"Was there ever a more horrible blasphemy than the statement that all the knowledge of God is confined to this or that book? How dare men call God infinite, and yet try to compress Him within the covers of a little book!"

— Swami Vivekananda (Raja-Yoga)

“What man ‘learns’ is really what he discovers by taking the cover off his own soul, which is a mine of infinite knowledge.”

Who was the inventor of what?

“SUCCESS HAS MANY FATHERS, BUT FAILURE IS AN ORPHAN.”

Transmission and Reception of Radio Waves:

G. Marconi?

A. S. Popov?

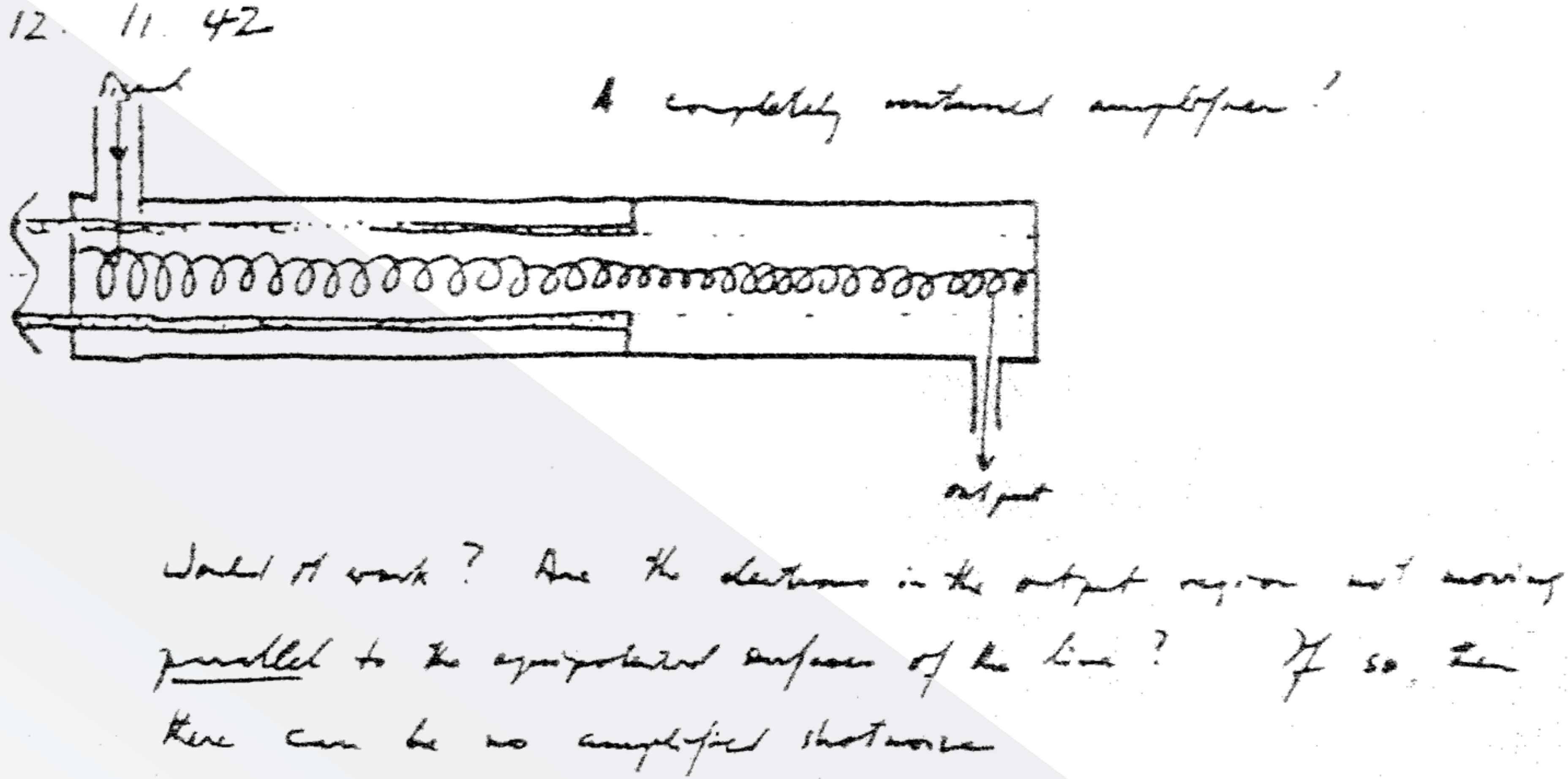
J. C. Bose?

Travelling-Wave Tube:

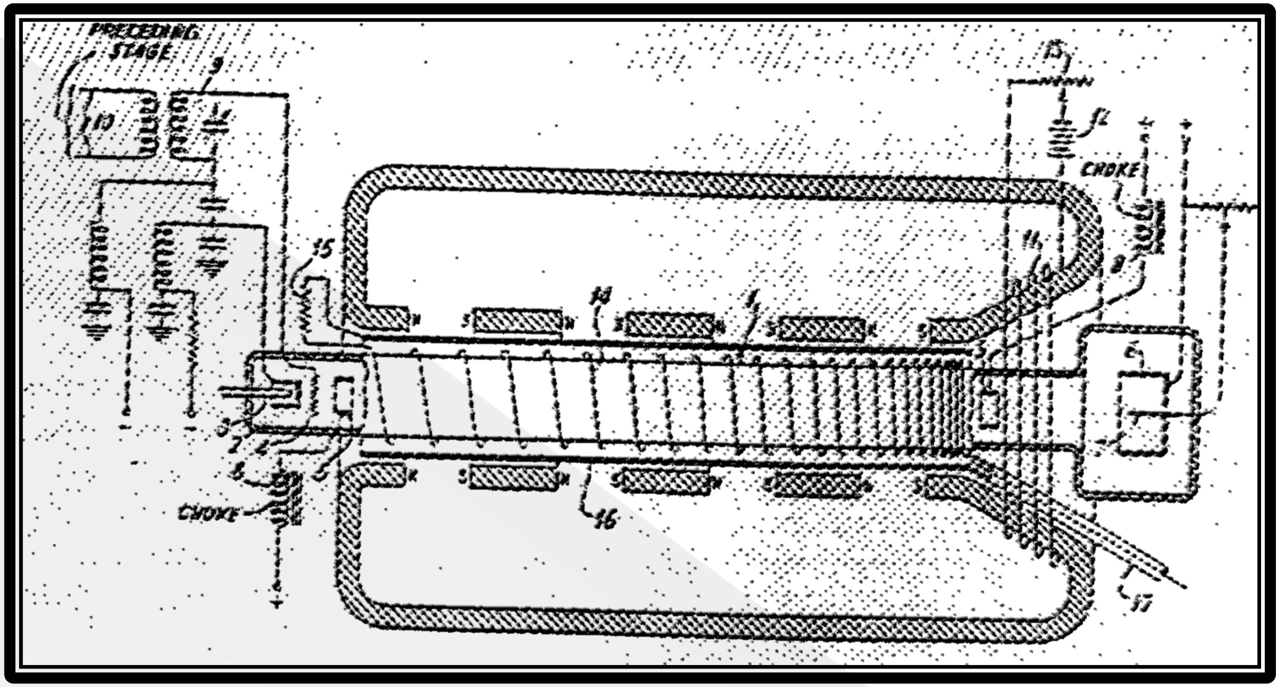
R. Kompfner?

N. E. Lindenblad?

A. Haeff?

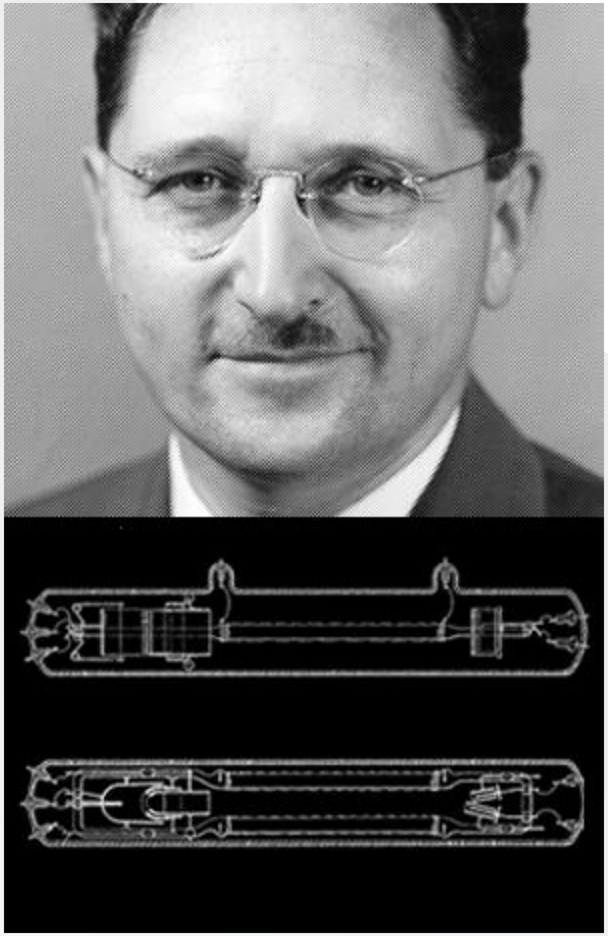


Sketch of the travelling-wave tube from R. Kompfner's note book



N. E. Lindenblad's travelling-wave tube amplification at 390 MHz over a 30 MHz band (U. S. Patent 2,300,052, filed on May 4, 1940 issued on October 27, 1942)

Helix wound around the outside the glass envelope. Signal applied to the grid of the electron gun (also applied to the helix in other experiments). Series of permanent magnets (non-periodic). Pitch tapered for velocity re-synchronization

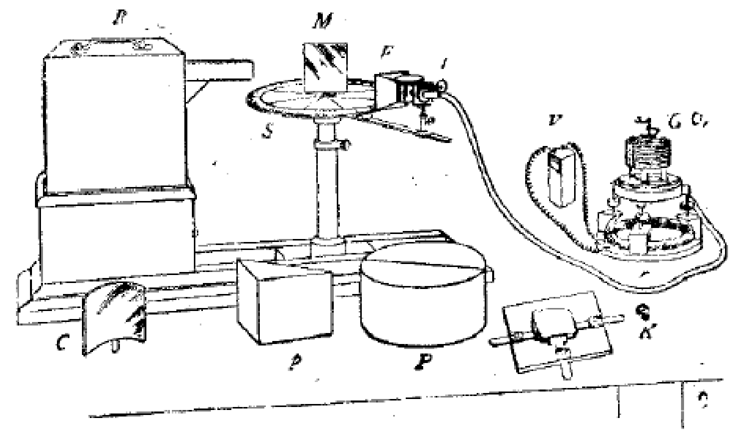


The patent Andrei Haeff filed in 1933 for a primitive type of traveling-wave tube has been largely ignored.

Sir J C Bose

Source: Subhradeep Chakraborty (CEERI-Pilani)

IEEE Milestone Plaque for Sir JC Bose



R, radiator ; S, spectrometer-circle ; M, plane mirror ; C, cylindrical mirror ; p, totally reflecting prism ; P, semi-cylinders ; K, crystal-holder ; F, collecting funnel attached to the spiral spring receiver ; t, tangent screw, by which the receiver is rotated ; V, voltaic cell ; r, circular rheostat ; G, galvanometer.

By 1895, Sir. J. C. Bose made the first public demonstration of radio waves in the Kolkata town hall. ……at a distance of 75 feet, he remotely rang an electric bell and ignited a small charge of gunpowder. He called it *Adrisya Alok,* or "Invisible Light". The frequency of operation is nearly 60 GHz. He termed horn antenna as collecting funnel.

J. C. Bose

All Indians should be inspired by the momentous work of J.C. Bose. He is the first ever scientist in the world to demonstrate the wireless transmission of electromagnetic waves. Hertz in 1888 generated and detected electromagnetic waves typically at 30 and 450 MHz; on the contrary, J. C. Bose of India demonstrated these phenomena in the millimetre-wave frequency range. (Edited by Subhradeep)

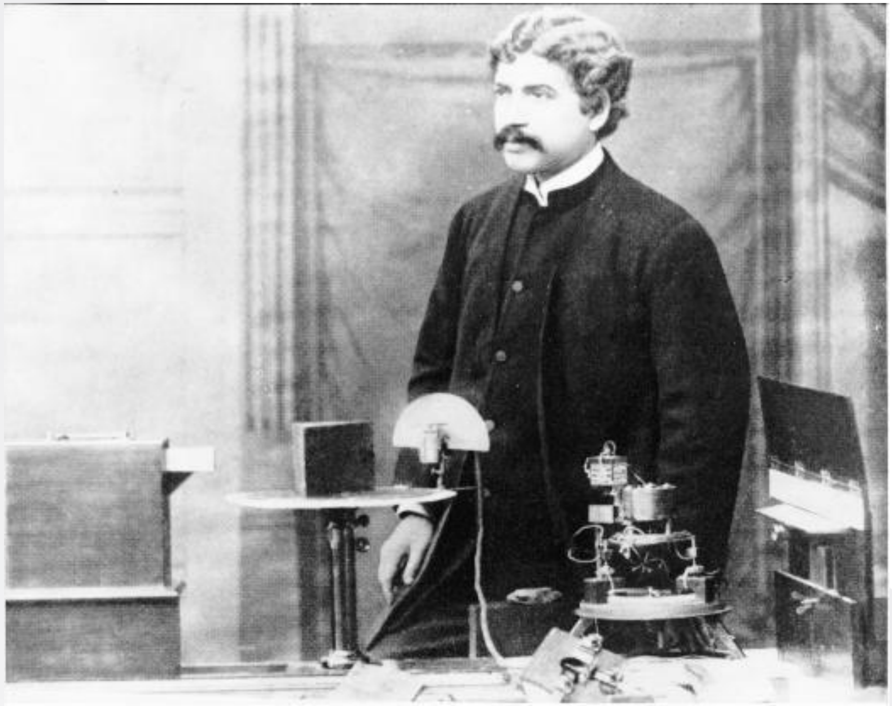
J.C. Bose (1858-1937) at the Royal Institution, London, 1897

জগদীশ চন্দ্র বসু

Jagadish Chandra Basu

Jagadis Chunder Bose

J.C. Bose



J.C. Bose (1858-1937) at the Royal Institution, London, 1897

In 1895 Bose gave his first public demonstration of electromagnetic waves, using them to ring a bell remotely and to explode some gunpowder. In 1896 the Daily Chronicle of England reported: ***"The inventor (J.C. Bose) has transmitted signals to a distance of nearly a mile and herein lies the first and obvious and exceedingly valuable application of this new theoretical marvel."***

"Popov in Russia was doing similar experiments, but had written in December 1895 that he was still entertaining the hope of remote signaling with radio waves."

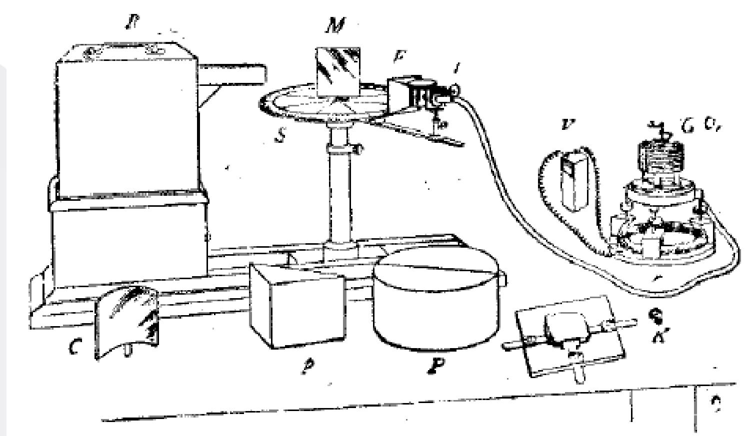
"The first successful wireless signaling experiment by Marconi on Salisbury Plain in England was not until May 1897."

Source: D. T. Emerson, "The work of Jagadis Chunder Bose: 100 years of mm-wave research," *IEEE Trans. Microwave Th. Tech.* December 1997, 45, No. 12 (2267-2273)

J C Bose

IEEE Milestone Plaque for Sir JC Bose

By 1895, Sir. J. C. Bose made the first public demonstration of radio waves in the Kolkata town hall. Details of the apparatus used are vague, but at a distance of 75 feet, he remotely rang an electric bell and ignited a small charge of gunpowder. He called it *Adrisya Alok,* or "Invisible Light". The frequency of operation is nearly 60 GHz. He termed horn antenna as collecting funnel.



R, radiator ; S, spectrometer-circle ; M, plane mirror ; C, cylindrical mirror ; p, totally reflecting prism ; P, semi-cylinders ; K, crystal-holder ; F, collecting funnel attached to the spiral spring receiver ; t, tangent screw, by which the receiver is rotated ; V, voltaic cell ; r, circular rheostat ; G, galvanometer.

Courtesy: Subhradeep (CEERI)

IEEE Milestone Plaque

IEEE MILESTONE IN ELECTRICAL ENGINEERING AND COMPUTING
First Millimeter-Wave Communication Equipment by JC Bose, 1894-1896

Sir Jagadish Chadra Bose, in 1895, first demonstrated at Presidency College, Calcutta, India, transmission and reception of electromagnetic waves at 60 GHz over a distance of 23 meters, through two intercepting walls by remotely ringing a bell and detonating gunpowder. For this communication system, Bose developed entire millimetre-wave components such as: a spark transmitter, coherer, dielectric lens, polarizer, horn antenna and cylindrical diffraction grating.

September 2012

IEEE Monogram

Courtesy: Subhradeep (CEERI)

J.C. Bose published his paper on ‘polarisation of electric rays by double-refracting crystals’ in the Asiatic Society Journal in May1895. He delivered a demonstration lecture at the Town Hall of Calcutta in November 1895 in the presence of the then Governor Sir William Mackenzie.

In this experiment, he sent a signal longer than the infrared and the invisible ray penetrated blocks of wood, human body, two walls and rang a bell and fired a cannon ball 23 meters. Earlier he did similar experiments at Presidency College, Calcutta, as detailed in IEEE Milestone Plaque.

Courtesy: Subhradeep (CEERI)

“Bose’s experiment is believed to be the first ever microwave experiment in artificial materials (on twisted structures) for electromagnetic applications which exhibit the chiral characteristics!”

(Nader Engheta and R. W. Ziolkowski (Ed.): Metamaterials — Physics and engineering exploration)

THIS LECTURE IS DEDICATED TO

Professor N. C. Vaidya — the founder of Centre of Research in Microwave Tubes (CRMT), Electronics Engineering Department, IIT-BHU, Varanasi

Professor R. K. Jha

Professor S. K. Srivastava

I SINCERELY THANK Dr. M. Thottapan for inviting me to present this lecture.

Role of Centre of Research in Microwave Tubes of IIT-BHU has been described in the article: B. N. Basu, "Indian efforts in vacuum electron devices: organisations and persons caught in my glimpse," www.vedas.org.in.

In order to see the article kindly go to: www.vedas.org.in → ABOUT US → Creation of VEDAS.

VEDAS is the acronym for Vacuum Electron Devices and Applications Society.

Vacuum Electron Devices and Applications (VEDA) Society

Organises either a workshop or a symposium usually every alternate year in the country

VEDA 2004 Symposium: MTRDC, Bangalore (30 & 31 October 2004)
VEDA 2005 Workshop: CRMT-BHU, Varanasi (18 & 19 January 2006)
VEDA 2006 Symposium: CEERI, Pilani (CSIR) (11-13 October 2006)
VEDA 2007 Workshop: SAMEER, Mumbai (22 & 23 November 2007)
VEDA 2008 Workshop: MTRDC, Bangalore (DRDO) (8-10 January 2009)
VEDA 2009 Symposium: CRMT-BHU, Varanasi (30 & 31 October 2009)
VEDA 2010 Workshop: CET, Moradabad (18 & 19 November 2010)
VEDA 2011 Workshop: RKGIT, Ghaziabad (18 & 19 November 2011)
IEEE-EDS IVEC-2011: Organized in Bangalore jointly with VEDA Society
VEDA 2012 Symposium: CEERI, Pilani (CSIR) (21-24 September 2012)
VEDA 2013 Workshop: IIT-R, Roorkee (18-20 October 2013)
VEDA 2014 Workshop: DAVV, Indore (20 & 21 March 2015)
VEDA 2015 Conference: MTRDC-DRDO, Bangalore (3-5 December 2015)
VEDA 2016 Conference: IPR-DAE, Gandhinagar (16-18 March 2017)
VEDA 2017 Symposium: IIT-R, Roorkee (17-19 November 2017)
VEDA 2018 Symposium: IIT-G, Guwahati (22-24 November 2018)
VEDA 2019 Symposium: NIT-Patna (21-23 November 2019)

Concern of Professor Pradip K Saha in a message:

"….I have observed sadly that even the teachers teaching microwave engineering do not always have very clear concepts about various aspects of microwaves. And for understanding microwave engineering, the most essential pre-requisite is a thorough knowledge of circuit theory, engineering electromagnetics and computational electromagnetics. With proper background, research in the field of microwaves can be managed more comfortably. The present trend, in fact for quite a few years, is to launch the research career with simulation tools. The more powerful tools you have at your disposal, more the number of papers you would be able to churn out, regrettably, often without understanding the problem or digging deep into it. …….. the pleasure that is obtained from solving a problem analytically or semi-analytically using elegant mathematical techniques is immense. Powerful simulation software is always welcome as additional supporting tools, particularly when a problem is not easily amenable to theoretical formulation. But understanding the basics is paramount to solving a problem; otherwise, the paper produced from the results may be just a paper to add to the list of publications but not treated as a 'contribution'."

AS Gilmour wrote an article on my request in
Special Issue on Microwave Tubes and Applications
in
Journal of Electromagnetic Waves and Applications

A.S. Gilmour authored three world famous books:
(1) Microwave Tubes
(2) Principles of Traveling Wave Tubes
(3) Klystrons, Traveling Wave Tubes, Magnetrons, Crossed-Field Amplifiers, and Gyrotrons

Journal of Electromagnetic Waves and Applications, 2017, Vol. 31, No . 17, 1771–1774. [https://doi.org/10.1080/09205071.2017.1375646]
AN OVERVIEW OF MY EFFORTS TO BRIDGE THE GAP IN THE MICROWAVE TUBE AREA BETWEEN WHAT UNIVERSITIES PROVIDE AND WHAT THE INDUSTRY NEEDS

A.S. Gilmour, Jr.
Department of Electrical Engineering, State University of New York, Buffalo,NY, USA

Journal of Electromagnetic Waves and Applications, 2017, Vol. 31, No . 17, 1771–1774 [https://doi.org/10.1080/09205071.2017.1375646]

An overview of my efforts to bridge the gap in the microwave tube area between what universities provide and what the industry needs

A.S. Gilmour, Jr.
Department of Electrical Engineering, State University of New York, Buffalo,NY, USA

ABSTRACT

My efforts to "bridge the gap" span a period of nearly 40 years and consist of over 100 courses on microwave tubes presented to well over 2000 scientists, engineers, and technicians. I developed the first five-day course for the US Navy. After several more courses, I wrote Microwave Tubes. In 1988, I developed the equivalent of a one-semester course on traveling wave tubes for the Navy and presented it several times each at Teledyne, Litton, Varian, and Hughes. In 1994, I wrote Principles of Traveling Wave Tubes, which was translated and published in Russia. I continued expanding and refining the courses and, in 2011, I wrote Klystrons, Traveling Wave Tubes, Magnetrons, Crossed-Field Amplifiers, and Gyrotrons, which was translated and published in China. Most recently, my courses have been attended by scientists and engineers from China, Sweden, Turkey, the United Kingdom, and Germany as well as the United States.

There is only one nature — the division into science and engineering is a human imposition, not a natural one. Indeed, the division is a human failure; it reflects our limited capacity to comprehend the whole.

— Sir William Cecil Dampier

Electromagnetic theory → Science

Circuit theory → Engineering

Maxwell's equations

$$\nabla.\vec{D} = \rho$$

$$\nabla.\vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

"Simple enough to imprint on a T-shirt, and yet rich enough to provide new insights throughout a lifetime of study"

— J. R. Whinnery

"The teaching of Electromagnetics," *IEEE Trans. Education*, Vol. 33, pp. 3-7 (1990)

Whinnery's T-shirt has enough space to accommodate both Maxwell's equations and electromagnetic boundary conditions

$$\nabla . \vec{D} = \rho$$

$$\nabla . \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$(\vec{D}_2 - \vec{D}_1) . \vec{a}_n = \rho_S$$

$$(\vec{B}_2 - \vec{B}_1) . \vec{a}_n = 0$$

$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$$

$$\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0$$

General boundary conditions

$$(\vec{D}_2 - \vec{D}_1).\vec{a}_n = \rho_S$$
$$(\vec{B}_2 - \vec{B}_1).\vec{a}_n = 0$$
$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$$
$$\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0$$

$$\rho_s = Lt\rho dh$$
$$dh \to 0$$

$$\vec{J}_S = Lt\vec{J}dh$$
$$dh \to 0$$

$$\vec{J}_S = Lt\vec{J}dh$$
$$dh \to 0$$

Dielectric (1)-dielectric (2) interface

Both time-dependent and time-independent

$$(\vec{D}_2 - \vec{D}_1).\vec{a}_n = 0$$
$$(\vec{B}_2 - \vec{B}_1).\vec{a}_n = 0$$
$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = 0$$
$$\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0$$

Conductor (1)-dielectric (2) interface

Time-independent

$$\vec{D}_2.\vec{a}_n = \rho_S$$
$$(\vec{B}_2 - \vec{B}_1).\vec{a}_n = 0$$
$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = 0$$
$$\vec{a}_n \times \vec{E}_2 = 0$$

Time-dependent

$$\vec{D}_2.\vec{a}_n = \rho_S$$
$$\vec{B}_2.\vec{a}_n = 0$$
$$\vec{a}_n \times \vec{H}_2 = \vec{J}_s$$
$$\vec{a}_n \times \vec{E}_2 = 0$$

Electromagnetic theory and circuit theory are the two sides of the same coin

Electromagnetic theory		Circuit theory	
$\vec{J} = \sigma\vec{E}$	**Ohm's law**	$V = IR$	
$\vec{S} = \vec{E} \times \vec{H}$ **Poynting vector**	**Joule's law**	I^2R	**power loss**
$\vec{a}_n \times (E_2 - E_1) = 0$ **Electromagnetic boundary condition**	**Law of parallel resistances**	$\dfrac{1}{R_{\text{equivalent}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$	

Electromagnetic theory

Circuit theory

Dispersion relation of a hollow-pipe waveguide

Boundary-value problem

Equivalent circuit transmission line circuit

$$\omega^2 - \beta^2 c^2 - \omega_c^2 = 0$$

(Dispersion relation of a waveguide)

Electromagnetic theory

Circuit theory

Dispersion relation of a helical slow-wave structure of a travelling-wave tube

Boundary-value problem

$$\beta^2 = \omega^2 LC$$

Transmission line equivalent circuit

$$\frac{k \cot \psi}{\gamma} = \left(\frac{I_0(\gamma a) K_0(\gamma a)}{I_1(\gamma a) K_1(\gamma a)} \right)^{1/2}$$

(Helix in free space)

(Dispersion relation of a helix in free-space)

Maxwell's equations

***Simple enough to imprint on a T-shirt and yet rich enough to provide new insights throughout a lifetime of study* — J.R. Whinnery**

"The teaching of electromagnetics", *IEEE Trans. Education* ED-33 (1990) p.327

Several disciplines hang as gems on one priceless necklace which it was Maxwell's privilege and honour to recognize as capricious Nature's enduring ornament

— P. Khastagir

"Apologia," *Seminar on Electromagnetics and their applications*, 22-23 December 1988, Varanasi, India

***James Clerk Maxwell* originally gave as many as <u>twenty equations in twenty variables</u>; it was <u>Oliver Heaviside</u>, who is one of the founders of vector calculus, who reduced these equations to four equations.**

Maxwell's equations

Integral form	Differential form
$\oint_S \vec{D}\cdot d\vec{S} = \oint_S \vec{D}\cdot\vec{a}_n\, dS = \int_\tau \rho\, d\tau$	$\nabla\cdot\vec{D} = \rho$
$\oint_S \vec{B}\cdot d\vec{S} = \oint_S \vec{B}\cdot\vec{a}_n\, dS = 0$	$\nabla\cdot\vec{B} = 0$
$\oint_l \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot\vec{a}_n\, dS$	$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
$\oint_l \vec{H}\cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)\cdot\vec{a}_n\, dS$	$\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$

Four electromagnetic boundary conditions are derived respectively from four Maxwell's equations in 'integral form'.

Relaxation time

Relaxation time is a measure of how fast or slow a medium of uniform conductivity and permittivity approaches electrostatic equilibrium.

Continuity equation → **Relaxation time** ← **Ohm's law**

Poisson's equation ↓

$$\nabla.\vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad \longleftarrow \quad \vec{J} = \sigma \vec{E}$$

↓ **Conductivity is uniform in the medium**

$$\sigma(\nabla.\vec{E}) + \frac{\partial \rho}{\partial t} = 0 \quad \longleftarrow \quad \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

↓

$$\sigma(\frac{\rho}{\varepsilon}) + \frac{\partial \rho}{\partial t} = 0$$

↓

$$\frac{\partial \rho}{\partial t} = -\frac{\sigma}{\varepsilon}\rho$$

$$\frac{\partial \rho}{\partial t} = -\frac{\sigma}{\varepsilon}\rho$$ **(rewritten)**

$$\int \frac{d\rho}{\rho} = -\frac{\sigma}{\varepsilon}\int dt$$

$$\ln \rho = -\frac{\sigma}{\varepsilon}t + \text{constant}$$

We tacitly choose constant in terms of ρ_0

$$\text{constant} = \ln \rho_0$$

$$\ln \rho = -\frac{\sigma}{\varepsilon}t + \ln \rho_0$$

$$\ln \rho - \ln \rho_0 = -\frac{\sigma}{\varepsilon}t$$

$$\ln \frac{\rho}{\rho_0} = -\frac{\sigma}{\varepsilon}t$$

$$\ln\frac{\rho}{\rho_0} = -\frac{\sigma}{\varepsilon}t$$ **(rewritten)**

$$\downarrow$$

$$\frac{\rho}{\rho_0} = \exp\left(-\frac{\sigma}{\varepsilon}t\right)$$

$$\downarrow$$

$$\rho = \rho_0 \exp\left(-\frac{\sigma}{\varepsilon}t\right)$$

$$\downarrow$$

$$\rho = \rho_0 \exp\left(-\frac{t}{\mathrm{T}}\right) \longleftarrow \text{Relaxation time } T = \frac{\varepsilon}{\sigma} = \frac{\varepsilon_r \varepsilon_0}{\sigma}$$

For very large values of relaxation time *T*

$$\left.\begin{aligned} -\frac{t}{\mathrm{T}} &\rightarrow 0 \\ \exp\left(-\frac{t}{\mathrm{T}}\right) &\rightarrow 1 \\ \rho &\rightarrow \rho_0 \end{aligned}\right\}$$

For very small values of relaxation time *T*

$$\left.\begin{aligned} -\frac{t}{\mathrm{T}} &\rightarrow -\infty \\ \exp\left(-\frac{t}{\mathrm{T}}\right) &\rightarrow 0 \\ \rho &\rightarrow 0 \end{aligned}\right\}$$

For very large values of relaxation time T

(dielectric medium)

$$\mathrm{T}=\frac{\varepsilon}{\sigma}=\frac{\varepsilon_r\varepsilon_0}{\sigma}$$

$$-\frac{t}{\mathrm{T}}\to 0$$

$$\exp\left(-\frac{t}{\mathrm{T}}\right)\to 1$$

$$\rho\to\rho_0$$

For very small values of relaxation time T

(good condutor)

$$\mathrm{T}=\frac{\varepsilon}{\sigma}=\frac{\varepsilon_r\varepsilon_0}{\sigma}$$

$$-\frac{t}{\mathrm{T}}\to -\infty$$

$$\exp\left(-\frac{t}{\mathrm{T}}\right)\to 0$$

$$\rho\to 0$$

For a dielectric medium, the value of conductivity σ is very small that renders T a very large value. This makes the volume charge density ρ in the bulk of the dielectric tend to ρ_0 (equilibrium volume charge density). Therefore, within a time of interest *t*, the bulk of a dielectric medium can be charged with the equilibrium volume charge density (ρ_0).

On the other hand, for a medium of good conductivity, the value of conductivity σ is very large that renders T a very small value. This makes the volume charge density ρ in the bulk of the dielectric tend to 0. Thus, the bulk of a medium of a good conductor cannot be charged; any charge injected into such a medium of good conductivity will not stay long within the bulk of the conductor only to reappear at the outer surface of the conducting medium in compliance with the requirement of the conservation of charge.

Conductivity, permittivity, and relaxation time of typical medium materials

Medium material	σ (mho/m)	ε_r	$T = \varepsilon_r\varepsilon_0/\sigma$
Copper	5.8×10^{7}	1	1.5×10^{-19} s
Sea-water	4	81	2×10^{-10} s
Corn oil	5×10^{-4}	3.1	0.55 s
Mica	10^{-15}	5.8	~1/2 a day
Quartz (fused)	10^{-17}	5	~50 days

The concept of the relaxation time is very useful in understanding the electromagnetic boundary conditions at the interface between two dielectrics as well as those at the interface between a conductor and a dielectric.

Surface charge density is defined as the product of the volume charge density and the infinitesimal thickness over which the charge is spread at the interface, in the limit of the infinitesimal thickness tending to zero. (This definition emerges in course of the deduction of general electromagnetic boundary conditions).

Definition of surface charge density ρ_s

$$\rho_s = \underset{dh \to 0}{Lt} \rho \, dh \quad (\mathrm{C/m^2})$$

<u>Surface current density</u> is defined as the product of the current density and the infinitesimal thickness at the interface over which the current density is significant, in the limit of the infinitesimal thickness tending to zero. (This definition emerges in course of the deduction of general electromagnetic boundary conditions).

Definition of surface current density $\vec{J}_S$

$$\vec{J}_S = \underset{dh \to 0}{Lt} \vec{J} dh \quad \text{(A/m}^2\text{)}$$

Derivation of electromagnetic boundary conditions

Field quantities, for both steady (time-independent) and time-varying (time-dependent) situations, will, in general, get modified when the medium is perturbed by the presence of another medium because of the abrupt change in the medium properties at the interface (common boundary) between the media. However, the field quantities would pass through a common set of electromagnetic boundary conditions at the interface or common boundary between the media.

$$\oint_S \vec{D}\cdot\vec{a}_n dS = \int_\tau \rho\, d\tau$$ **(recalled)**

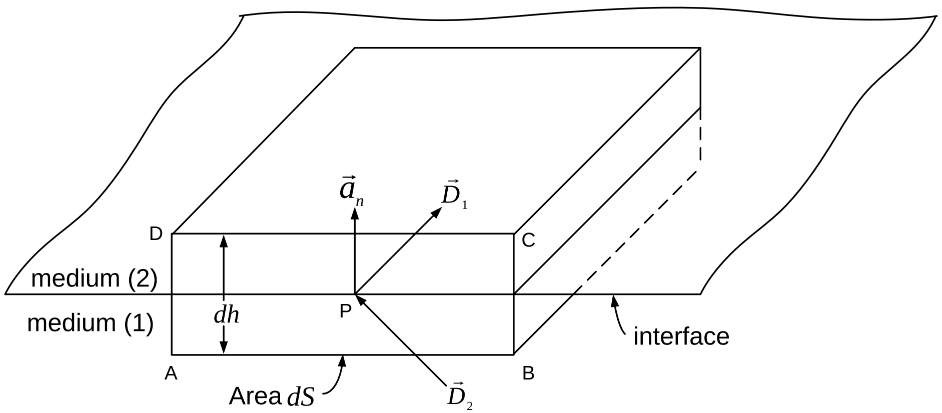
↓

Applied to volume element $d\tau$

↓

$$\vec{D}_2.\vec{a}_n dS + \vec{D}_1.(-\vec{a}_n)dS \simeq \rho d\tau$$

(To be elaborated later)



$d\tau$ — **a parallelepiped volume element $d\tau$ in the form of a pill box of infinitesimal thickness *dh* and of infinitesimal area dS of each of its bottom and top faces, enclosing the point P where the boundary condition is sought**

$\vec{a}_n$ = unit vector directed from region 1 to 2

$\vec{D}_1$ = electric displacement in region 1 at the point P on the interface

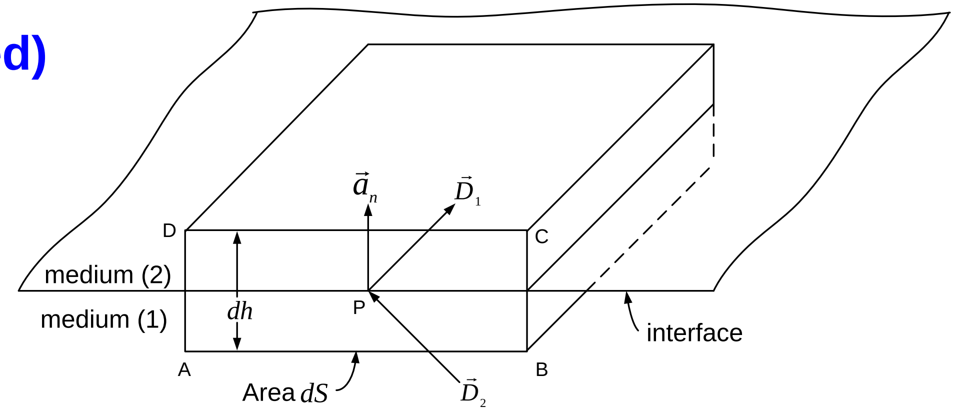
$\vec{D}_2$ = electric displacement in region 2 at the point P on the interface

dS = element of area on each of top and bottom faces of volume element

dh = infinitesimal thickness of volume element

$d\tau = dS\, dh$ = volume element

$\vec{D}_2.\vec{a}_n dS + \vec{D}_1.(-\vec{a}_n)dS \cong \rho d\tau$ **(recalled)**



(taking the electric displacements to be constant over the area elements)

(<u>ignoring the contribution of area elements on the side faces</u> of volume element considering such area elements to be insignificant taking negligible infinitesimal thickness *dh* of the volume element and hence prompting us to use the <u>approximate sign of equality</u>)

Contribution to the left hand side by the area element *dS* on top face $= \vec{D}_2.\vec{a}_n dS$

Contribution to the left hand side by the area element *dS* on bottom face $= (\vec{D}_2).(-\vec{a}_n)dS = -\vec{D}_2.\vec{a}_n dS$

(Outward unit vector at the bottom face being downward being opposite to that at the top face)

Let us remove the approximation from the sign of equality by taking in the limit *dh* tending to zero.

$$\vec{D}_2.\vec{a}_n dS + \vec{D}_1.(-\vec{a}_n)dS = \underset{dh \to 0}{Lt} \rho\, dS dh$$

$$(\vec{D}_2 - \vec{D}_1).\vec{a}_n = \underset{dh \to 0}{Lt} \rho\, dh$$

$$(\vec{D}_2 - \vec{D}_1).\vec{a}_n = \mathop{Lt}_{dh \to 0} \rho\, dh$$ **(rewritten)**

The surface charge density ρ_s is defined as:

$$\rho_s = \mathop{Lt}_{dh \to 0} \rho\, dh$$ (C/m^2)

(since ρ is in C/m^3 **and *dh* is in** m**)**

$$\boxed{(\vec{D}_2 - \vec{D}_1).\vec{a}_n = \rho_s}$$

(First of four general electromagnetic boundary conditions)

We already gave the method and deduced the said boundary condition.

$$\oint_S \vec{D} \cdot \vec{a}_n dS = \int_\tau \rho\, d\tau$$ **(Maxwell's equation in integral form)** → $$(\vec{D}_2 - \vec{D}_1).\vec{a}_n = \rho_s$$

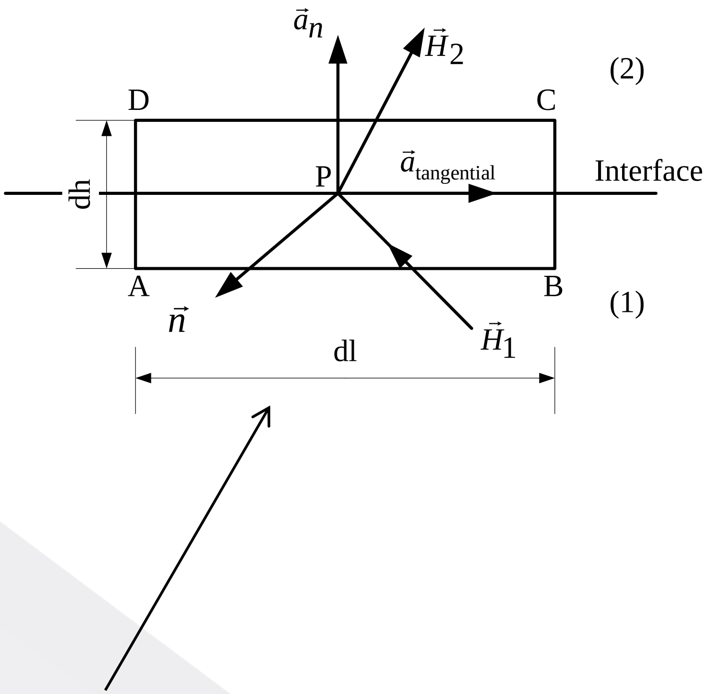
Similarly, following the same method we obtain

$$\oint_S \vec{B} \cdot d\vec{S} = \oint_S \vec{B} \cdot \vec{a}_n\, dS = 0$$ **(another Maxwell's equation in integral form)** → $$\boxed{(\vec{B}_2 - \vec{B}_1).\vec{a}_n = 0}$$

(Second of four general electromagnetic boundary conditions)

Free magnetic charge being absent or magnetic flux lines being continuous

Let us next consider a rectangle element of infinitesimal length *dl*, infinitesimal thickness *dh* and area element *ds* = (*dh*)(*dl*) enclosing the point P on the interface between the medium 1 and medium 2 (where the boundary condition is sought) such that the bottom and top lengths of the rectangle lay in medium 1 and medium 2 respectively.



$\vec{a}_n =$ **unit vector directed from medium 1 to medium 2**

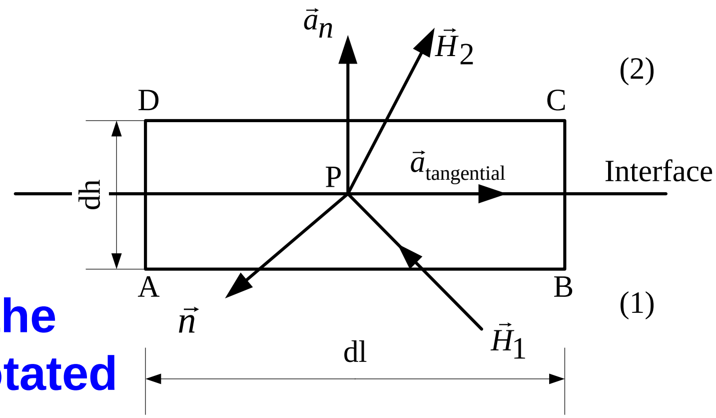
$\vec{n} =$ **unit vector normal to the area element *dS***

$\vec{a}_{\text{tangential}} =$ **unit vector tangential to the interface**

$\vec{a}_n =$ **unit vector directed from medium 1 to medium 2**

$\vec{n} =$ **unit vector normal to the area element** ***dS***

$\vec{a}_{\text{tangential}} =$ **unit vector tangential to the interface**



We take $\vec{n}$

such that it takes its direction as the direction of the linear motion that a screw would have if it were rotated following along the sequence of the closed line integral from A **to** B; B **to** C; C **to** D; **and then from** D **back to** A **in the left hand side of Maxwell's equation:**

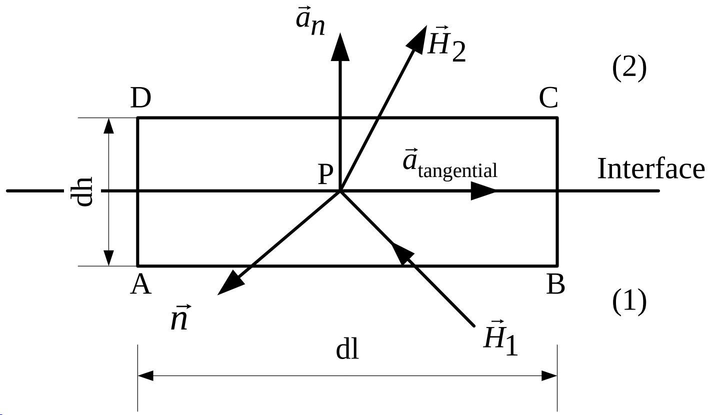
$$\vec{J}_{\text{total}} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\oint_l \vec{H} \cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot \vec{n}\, dS \longrightarrow \oint_l \vec{H}.d\vec{l} = \int_S \vec{J}_{\text{total}}.\vec{n}\, dS$$

(Maxwell's equation so chosen to be expressed)

(So defined for the sake of convenience)

$$\oint_l \vec{H}.d\vec{l} = \int_S \vec{J}_{\text{total}}.\vec{n}\,dS$$ **(recalled)**



Applied to the left hand side following the sequence from A to B; B to C; C to D; and then from D back to A

$$\vec{H}_1.\vec{a}_{\text{tangential}}\,dl + \vec{H}_2.(-\vec{a}_{\text{tangential}})\,dl \cong \vec{J}_{\text{total}}.\vec{n}\,dS$$

Subscripts 1 and 2 refer to regions 1 and 2 respectively.

(<u>Ignoring the contribution of length elements on the side faces</u> of area element considering such length elements to be insignificant taking negligible infinitesimal thickness *dh* of the area element and hence prompting us to use the <u>approximate sign of equality</u>)

$$(\vec{H}_1 - \vec{H}_2).\vec{a}_{\text{tangential}}\,dl \cong \vec{J}_{\text{total}}.\vec{n}\,dS$$

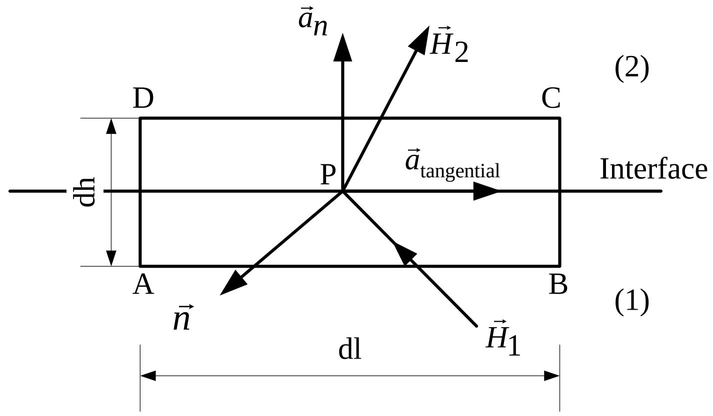
$$dS = (dh)(dl)$$

$$(\vec{H}_1 - \vec{H}_2).\vec{a}_{\text{tangential}} \cong \vec{J}_{\text{total}}.\vec{n}\,dh$$

The unit vectors satisfy the relation of cross product: $\vec{a}_n \times \vec{n} = \vec{a}_{\text{tangential}}$

$$(\vec{H}_1 - \vec{H}_2).\vec{a}_n \times \vec{n} \cong \vec{J}_{\text{total}}.\vec{n}\,dh$$

$$(\vec{H}_1 - \vec{H}_2).\vec{a}_n \times \vec{n} \cong \vec{J}_{\text{total}}.\vec{n}\,dh$$ **(rewritten)**



$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

$$\vec{n}.(\vec{H}_1 - \vec{H}_2)\times\vec{a}_n \cong \vec{J}_{\text{total}}.\vec{n}\,dh$$

$$\left.\begin{aligned}\vec{A} &= \vec{H}_1 - \vec{H}_2\\ \vec{B} &= \vec{a}_n\\ \vec{C} &= \vec{n}\end{aligned}\right\}$$

$$\vec{n}.(\vec{H}_1 - \vec{H}_2)\times\vec{a}_n \cong \vec{n}.\vec{J}_{\text{total}}\,dh$$

$$\vec{n}.[(\vec{H}_1 - \vec{H}_2)\times\vec{a}_n - \vec{J}_{\text{total}}dh] \cong 0 \longleftarrow \vec{G} = (\vec{H}_1 - \vec{H}_2)\times\vec{a}_n - \vec{J}_{\text{total}}dh$$

θ is the angle between $\vec{n}$ and $\vec{G}$.

$$\vec{n}.\vec{G} \cong 0$$

The orientation of the area element ABCD **is arbitrary. That makes the value of** θ **and hence that of** $\cos\theta$ **arbitrary.**

$$\vec{n}\cdot\vec{G} = (1)(G)\cos\theta = G\cos\theta \cong 0 \longrightarrow G \cong 0$$ **since the value of** $\cos\theta$ **is arbitrary.**

$G \cong 0$ **(rewritten)** ⟵ $\vec{G} = (\vec{H}_1 - \vec{H}_2) \times \vec{a}_n - \vec{J}_{\text{total}} dh$

↓

$$(\vec{H}_1 - \vec{H}_2) \times \vec{a}_n - \vec{J}_{\text{total}} dh \cong 0$$

↓

$$(\vec{H}_1 - \vec{H}_2) \times \vec{a}_n \cong \vec{J}_{\text{total}} dh$$ ⟵ $\vec{J}_{\text{total}} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

↓

$$(\vec{H}_1 - \vec{H}_2) \times \vec{a}_n \cong \vec{J} dh + \frac{\partial \vec{D}}{\partial t} dh$$

↓ ⟵ **We can remove the approximation from the sign of equality by taking *dh* tending to zero in the limit.**

$$(\vec{H}_1 - \vec{H}_2) \times \vec{a}_n = \underset{dh \to 0}{Lt} \vec{J} dh + \underset{dh \to 0}{Lt} \frac{\partial \vec{D}}{\partial t} dh$$

$$(\vec{H}_1 - \vec{H}_2)\times\vec{a}_n = \underset{dh\to 0}{Lt}\,\vec{J}dh + \underset{dh\to 0}{Lt}\,\frac{\partial\vec{D}}{\partial t}dh$$ **(rewritten)**

The first term takes a finite value in the limit $dh \to 0.$

$\vec{J} \to \infty$ as $dh \to 0$

The second term becomes nil in the limit $dh \to 0.$

$\vec{D}$ is finite and hence

$\partial\vec{D}/\partial t$ is finite

$$(\vec{H}_1 - \vec{H}_2)\times\vec{a}_n = \vec{J}_S + 0 \longleftarrow \vec{J}_S = \underset{dh\to 0}{Lt}\,\vec{J}dh$$

$$(\vec{H}_1 - \vec{H}_2)\times\vec{a}_n = \vec{J}_S \longleftarrow \vec{A}\times\vec{B} = -\vec{B}\times\vec{A} = (\vec{B})\times(-\vec{A}) \longleftarrow \left.\begin{matrix}\vec{A} = \vec{H}_1 - \vec{H}_2\\ \vec{B} = \vec{a}_n\end{matrix}\right\}$$

$$\boxed{\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s}$$

(Third of four general electromagnetic boundary conditions)

$$\oint_l \vec{H}\cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)\cdot \vec{a}_n\, dS$$

↓

$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = \underset{dh\to 0}{Lt}\ \vec{J}dh = \vec{J}_s$$

(Obtained earlier)

↓

$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$$

(Obtained earlier)

Similarly,

$$\oint_l \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot \vec{a}_n\, dS$$

↓

$$\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = \underset{dh\to 0}{Lt}\left(-\frac{\partial \vec{B}}{\partial t}dh\right) = 0$$

$dh \to 0$ as $\vec{B}$ is finite and hence $\partial \vec{B}/\partial t$ is finite.

$$\boxed{\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0}$$

(Fourth of four general electromagnetic boundary conditions)

Maxwell's equation in integral form	→	**Electromagnetic boundary condition**
$$\oint_S \vec{D}\cdot\vec{a}_n dS = \int_\tau \rho\, d\tau$$	→	$$(\vec{D}_2-\vec{D}_1).\vec{a}_n = \rho_s$$
$$\oint_S \vec{B}\cdot d\vec{S} = \int_S \vec{B}\cdot\vec{a}_n\, dS = 0$$	→	$$(\vec{B}_2-\vec{B}_1).\vec{a}_n = 0$$
$$\oint_l \vec{H}\cdot d\vec{l} = \int_S \left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)\cdot\vec{a}_n\, dS$$	→	$$\vec{a}_n\times(\vec{H}_2-\vec{H}_1) = \vec{J}_S$$
$$\oint_l \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot\vec{a}_n\, dS$$	→	$$\vec{a}_n\times(\vec{E}_2-\vec{E}_1) = 0$$

Electromagnetic boundary condition	Meaning	At a point on the interface between two media
$(\vec{D}_2 - \vec{D}_1).\vec{a}_n = \rho_s$	→	Normal component of electric displacement is discontinuous, the amount of discontinuity being equal to the surface charge density
$(\vec{B}_2 - \vec{B}_1).\vec{a}_n = 0$	→	Normal component of magnetic flux density is continuous
$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_S$	→	Tangential component of magnetic field is discontinuous, the amount of discontinuity being equal to the surface current density
$\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0$	→	Tangential component of electric field is continuous

Electromagnetic boundary conditions at the dielectric-dielectric interface

Electromagnetic boundary conditions at the dielectric-conductor interface

We can interpret general electromagnetic boundary conditions for dielectric-dielectric interface and conductor-dielectric/free-space interface.

For this purpose, it is worth reviewing some of the basic behaviours of conductor and dielectric media with regard to relaxation time, existence of a free charge in the bulk of the media, surface resistance, surface current density, and electric field and magnetic in the media.

Dielectric	Conductor
Relaxation time of is very large	**Relaxation time is very small**
A charge can stay longer inside the bulk of a dielectric without appearing at its surface and a finite volume charge density can be established inside the bulk resulting in a zero surface charge density at the surface of a dielectric for both <u>time-independent</u> and <u>time-dependent</u> situations.	**A charge inside the bulk of a good conductor decays very fast to appear with a large volume charge density concentrated over a thin layer on the surface of the conductor, resulting in a finite surface charge density on the conductor surface for both <u>time-independent</u> and <u>time-dependent</u> situations.**

$$\rho_s = \underset{dh \to 0}{Lt} \rho \, dh \text{ (C/m}^2\text{)}$$

$$\vec{J}_S = \underset{dh \to 0}{Lt} \vec{J} dh \text{ (A/m}^2\text{)}$$

<u>Continued</u>

In continuation

Dielectric	Conductor
The electric field and the electric displacement can be established inside a dielectric for both time-independent and time-dependent situations.	**The bulk of a conductor cannot be electrically charged, resulting in no electric field or electric displacement inside the conductor for both time-independent and time-dependent situations.**
A finite magnetic field or magnetic flux density can be established inside a dielectric independently of electric field, for both time-independent and time-dependent situations.	**A finite magnetic field or magnetic flux density can be established inside a conductor independent of electric field for time-independent situations. However, for time-dependent situations, the magnetic field or magnetic flux density is nil inside a conductor since it is coupled to the electric field which is nil inside the conductor for such situations.**

Continued

In continuation

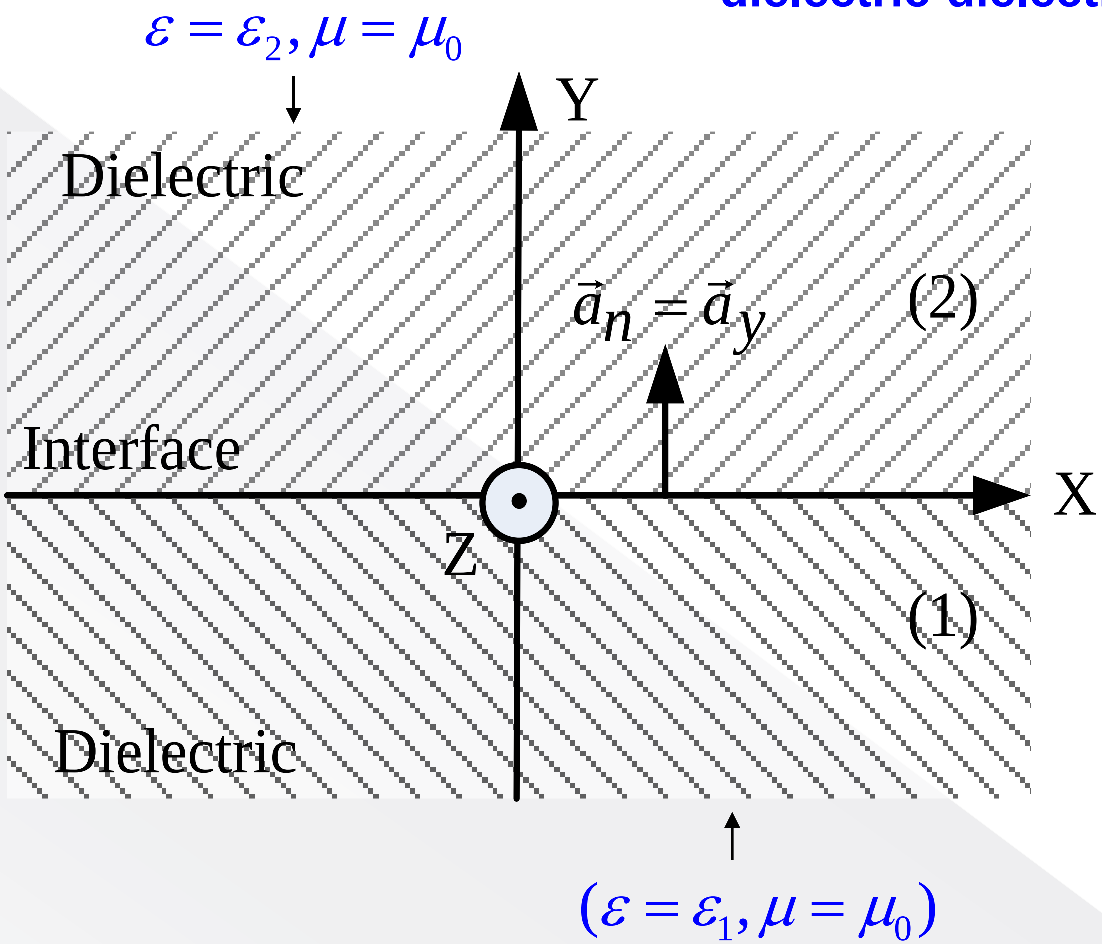
Dielectric	Conductor
No current flows through a dielectric and therefore the surface current density becomes zero at the dielectric surface for both <u>time-independent</u> and <u>time-dependent</u> situations.	**A finite current can be made to flow through the bulk of a conductor for <u>time-independent</u> situations, resulting in zero surface current density. However, for <u>time-dependent</u> situations a large current density can be concentrated over a thin layer on the conductor surface resulting in a finite surface current density.**

$$\rho_s = \underset{dh \to 0}{Lt}\, \rho\, dh \quad \text{(C/m}^2\text{)}$$

$$\vec{J}_S = \underset{dh \to 0}{Lt}\, \vec{J} dh \quad \text{(A/m}^2\text{)}$$

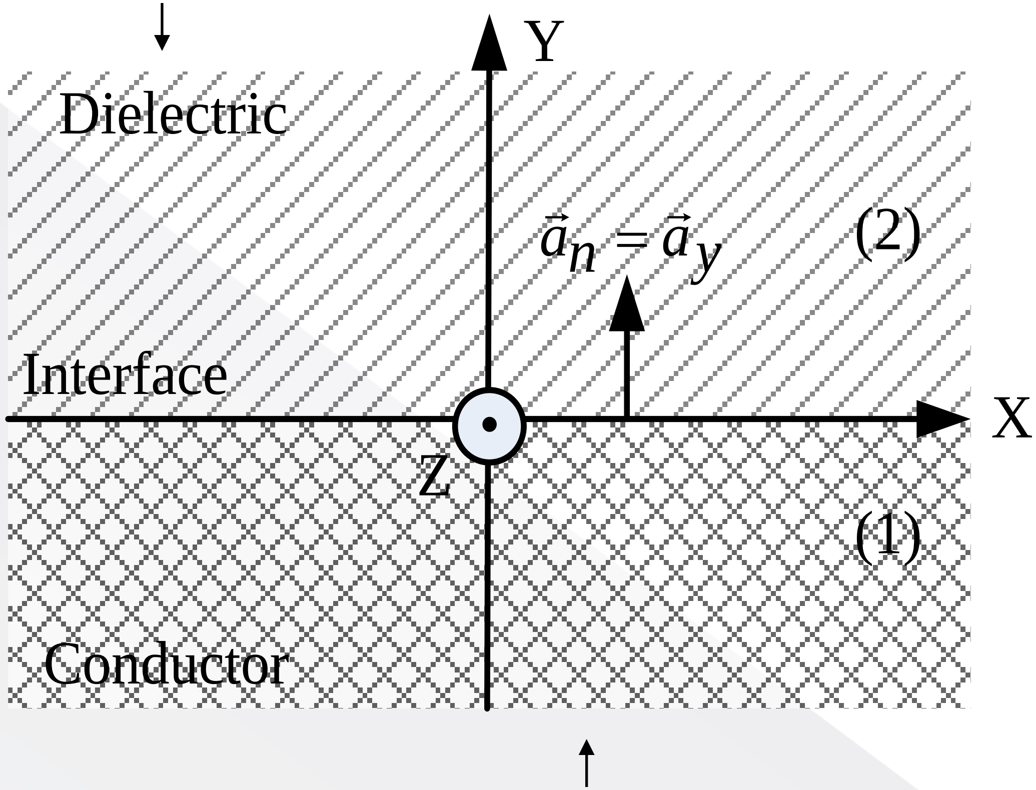
The above behaviours of the dielectric and conductor help in the interpretation of general electromagnetic boundary conditions for dielectric-dielectric interface and conductor-dielectric/free-space to be taken up next in our study, which is of practical relevance.

dielectric-dielectric interface



conductor-dielectric interface

$\varepsilon = \varepsilon_2, \mu = \mu_0$



$\varepsilon = \varepsilon_0, \mu = \mu_0$

Electromagnetic boundary conditions at dielectric-dielectric interface

$$\rho \neq 0 \quad \rightarrow \quad \rho_s = \underset{dh \to 0}{Lt} \int \rho \, dh = 0$$

$$\vec{E}_{1,2} \neq 0, \vec{D}_{1,2} \neq 0$$

$$\vec{H}_{1,2} \neq 0, \vec{B}_{1,2} \neq 0$$

Subscripts 1 and 2 refer to quantities in region1 and 2 respectively

$$(\vec{D}_2 - \vec{D}_1).\vec{a}_n = \rho_S$$

$$(\vec{B}_2 - \vec{B}_1).\vec{a}_n = 0$$

$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$$

$$\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0$$

(general boundary conditions)

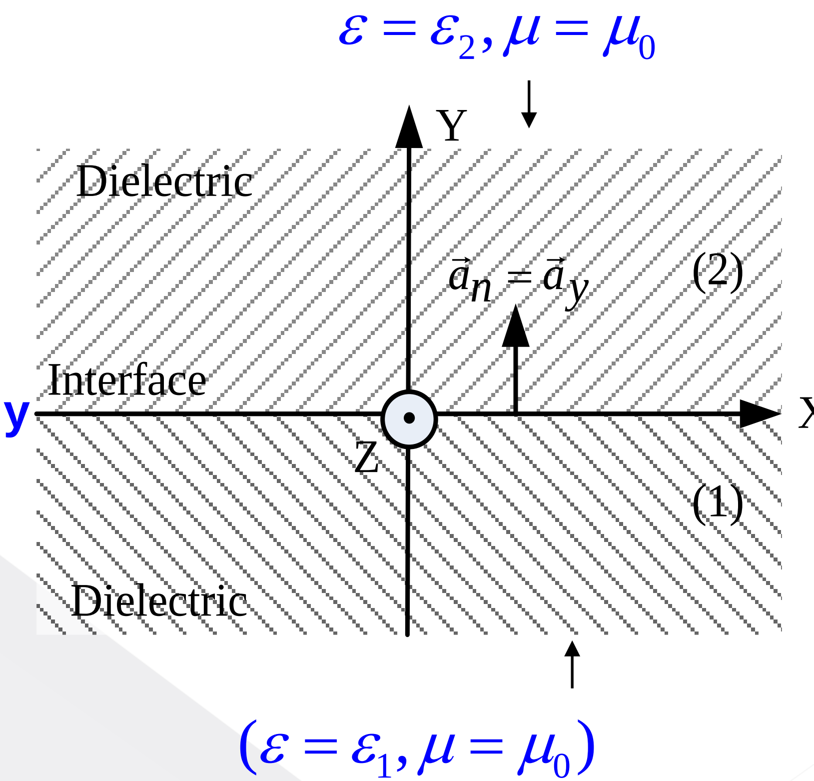
$$(\vec{D}_2 - \vec{D}_1).\vec{a}_n = 0$$

$$(\vec{B}_2 - \vec{B}_1).\vec{a}_n = 0$$

$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = 0$$

$$\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0$$

(boundary conditions at dielectric-dielectric interface for both time-independent and time-dependent situations)



Electromagnetic boundary conditions at conductor-dielectric interface

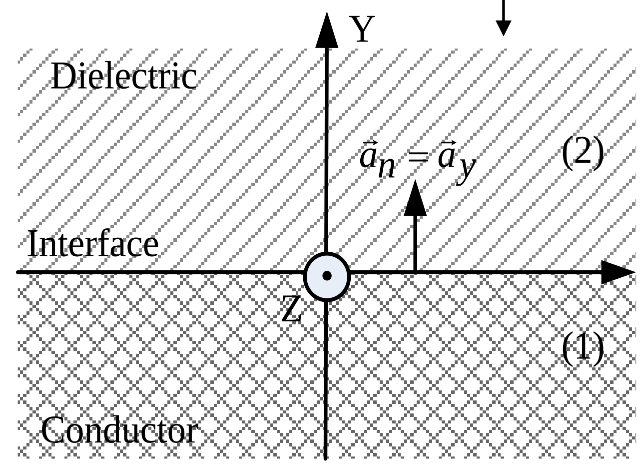
$$\vec{J}_S = \underset{dh \to 0}{Lt} \vec{J} dh$$

$$\vec{E}_1 = 0, \vec{D}_1 = 0$$

$$\vec{H}_1 \neq 0,\ \vec{B}_1 \neq 0$$ ← **for time-independent situations**

$$\vec{H}_1 = \vec{B}_1 = 0$$ ← **for time-dependent situations**

$\varepsilon = \varepsilon_2, \mu = \mu_0$



$\varepsilon = \varepsilon_0, \mu = \mu_0$

$$(\vec{D}_2 - \vec{D}_1).\vec{a}_n = \rho_S$$

$$(\vec{B}_2 - \vec{B}_1).\vec{a}_n = 0$$

$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$$

$$\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0$$

(general electromagnetic boundary conditions)

$$\vec{D}_2.\vec{a}_n = \rho_S$$

$$(\vec{B}_2 - \vec{B}_1).\vec{a}_n = 0$$

$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = 0$$

$$\vec{a}_n \times \vec{E}_2 = 0$$

(for time-independent situations)

$$\vec{D}_2.\vec{a}_n = \rho_S$$

$$\vec{B}_2.\vec{a}_n = 0$$

$$\vec{a}_n \times \vec{H}_2 = \vec{J}_s$$

$$\vec{a}_n \times \vec{E}_2 = 0$$

(for time-dependent situations)

Let us illustrate the application of the boundary conditions at a dielectric-dielectric interface by taking up the problem of finding the electric displacements in region 1 (*x*>0) containing a dielectric of relative permittivity ε_{r1} = 3 and region 2 (*x*<0) containing another dielectric of relative ε_{r2} = 5, the two regions forming an interface at *x* = 0 if the electric field in region 2 is given as:

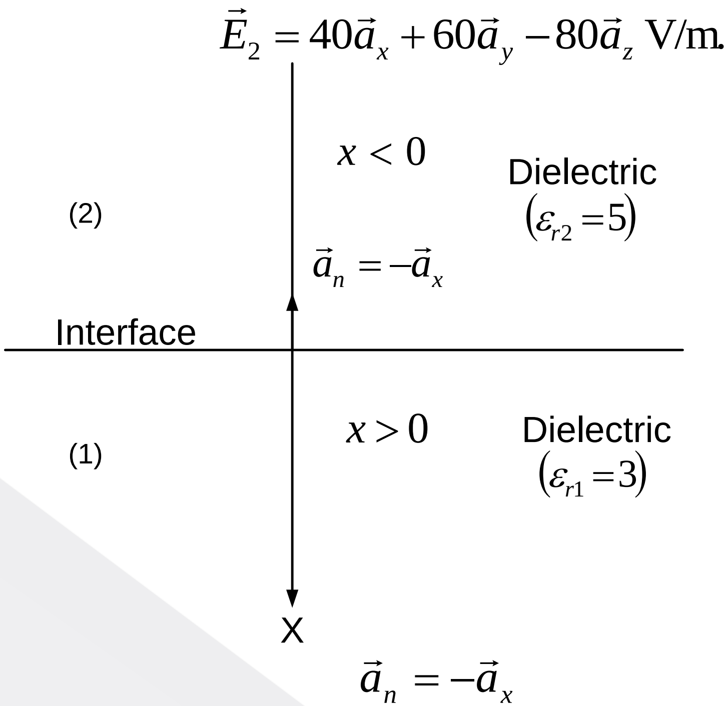
$$\vec{E}_2 = 40\vec{a}_x + 60\vec{a}_y - 80\vec{a}_z \text{ V/m.}$$

$\downarrow$ **(given)**

$$\vec{D}_2 = \varepsilon_2 \vec{E}_2 = \varepsilon_0 \varepsilon_{r2} \vec{E}_2 = 5\varepsilon_0 \vec{E}_2$$
$$= 5\varepsilon_0 \times (40\vec{a}_x + 60\vec{a}_y - 48\vec{a}_z) \text{ V/m}$$

$\downarrow$

$$\vec{D}_2 = (200\vec{a}_x + 300\vec{a}_y - 400\vec{a}_z)\varepsilon_0 \text{ C/m}^2$$



Let us recall the boundary condition

$(\vec{D}_2 - \vec{D}_1).\vec{a}_n = 0$ **(recalled)**

$\downarrow$

$$[(D_{2x} - D_{1x})\vec{a}_x + (D_{2y} - D_{1y})\vec{a}_y + (D_{2z} - D_{1z})\vec{a}_z].[-\vec{a}_x] = 0$$

$\downarrow$

$$-(D_{2x} - D_{1x}) = 0$$

$\vec{D}_2 = (200\vec{a}_x + 300\vec{a}_y - 400\vec{a}_z)\varepsilon_0 \text{ C/m}^2$ **(recalled)**

$\downarrow$

$$D_{1x} = D_{2x} = 200\varepsilon_0 \text{ C/m}^2$$

$$\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0$$ **(boundary condition recalled)**

$$\downarrow$$

$$-\vec{a}_x \times [(E_{2x} - E_{1x})\vec{a}_x + (E_{2y} - E_{1y})\vec{a}_y + (E_{2z} - E_{1z})\vec{a}_z)] = 0$$

$$\downarrow$$

$$-(E_{2y} - E_{1y})\vec{a}_z + (E_{2z} - E_{1z})\vec{a}_y = 0$$

$$\downarrow$$

$$E_{1y} = E_{2y} \text{ and } E_{1z} = E_{2z} \longleftarrow$$

$$\vec{E}_2 = 40\vec{a}_x + 60\vec{a}_y - 80\vec{a}_z \text{ V/m.}$$ **(given)**

$$\downarrow$$

$$E_{2y} = 60\,\text{V/m and } E_{2z} = -80\,\text{V/m}$$

$$\downarrow$$

$$E_{1y} = 60\,\text{V/m and } E_{1z} = -80\,\text{V/m}$$

$$\downarrow$$

$$D_{1y} = \varepsilon_1 E_{1y} = \varepsilon_0 \varepsilon_{r1} E_{1y} = (\varepsilon_0)(3)E_{1y} = (\varepsilon_0)(3)(60) = 180\varepsilon_0\ \text{C/m}^2$$

$$D_{1z} = \varepsilon_1 E_{1z} = \varepsilon_0 \varepsilon_{r1} E_{1z} = (\varepsilon_0)(3)E_{1z} = (\varepsilon_0)(3)(-80) = -240\varepsilon_0\ \text{C/m}^2 \longleftarrow E_{1z} = -80\,\text{V/m}$$ **(recalled)**

$$\downarrow$$

$$\vec{D}_1 = D_{1x}\vec{a}_x + D_{1y}\vec{a}_y - D_{1z}\vec{a}_z)\varepsilon_0 \longleftarrow$$

$$D_{1x} = D_{2x} = 200\varepsilon_0\ \text{C/m}^2$$ **(recalled)**

$$D_{1y} = 180\varepsilon_0\ \text{C/m}^2$$ **(recalled)**

$$\downarrow$$

$$\vec{D}_1 = (200\vec{a}_x + 180\vec{a}_y - 240\vec{a}_z)\varepsilon_0\ \text{C/m}^2$$

$$\vec{D}_2 = (200\vec{a}_x + 300\vec{a}_y - 400\vec{a}_z)\varepsilon_0\ \text{C/m}^2$$ **(recalled)**

'

Take up another similar problem as an exercise to illustrate the application of boundary conditions at a dielectric-dielectric interface in which to find the electric field in region 2 (*y*>0) containing a dielectric of relative permittivity ε_{r2} separated at the interface (*y* = 0) from region 1 (*y*<0) containing another dielectric of relative permittivity ε_{r1} if the electric field in region 1 is given as:

The approach to getting the solution to the problem has already been elaborated in the preceding illustration. Some of the steps are provided as a hint as follows.

$$\vec{E}_1 = l\vec{a}_x + m\vec{a}_y - n\vec{a}_z.$$

$$\vec{D}_1 = \varepsilon_1\vec{E}_1 = \varepsilon_0\varepsilon_{r1}(l\vec{a}_x + m\vec{a}_y + n\vec{a}_z) \longleftarrow \vec{E}_1 = l\vec{a}_x + m\vec{a}_y - n\vec{a}_z$$

(electric field in region 1) (given)

$$\downarrow$$

$$\left.\begin{aligned} D_{1x} &= \varepsilon_0\varepsilon_{r1} l \\ D_{1y} &= \varepsilon_0\varepsilon_{r1} m \\ D_{1z} &= \varepsilon_0\varepsilon_{r1} n \end{aligned}\right\}$$

$$(\vec{D}_2 - \vec{D}_1).\vec{a}_n = 0$$ **(boundary condition)** $\longleftarrow \vec{a}_n = \vec{a}_y$

$$\downarrow$$

$$[D_{2x}\vec{a}_x + D_{2y}\vec{a}_y + D_{2z}\vec{a}_z - (D_{1x}\vec{a}_x + D_{1y}\vec{a}_y + D_{1z}\vec{a}_z)]\cdot\vec{a}_y = 0$$

$$\nwarrow \left.\begin{aligned} \vec{a}_x\cdot\vec{a}_y &= 0 \\ \vec{a}_y\cdot\vec{a}_y &= 1 \\ \vec{a}_z\cdot\vec{a}_y &= 0 \end{aligned}\right\}$$

$$\downarrow$$

$$D_{2y} - D_{1y} = 0$$

$$\vec{E}_1 = l\vec{a}_x + m\vec{a}_y - n\vec{a}_z$$ **(given)**

$$\downarrow$$

$$E_{1y} = m \longrightarrow D_{2y} = D_{1y} = \varepsilon_1 E_{1y} = \varepsilon_0\varepsilon_{r1}E_{1y} = \varepsilon_0\varepsilon_{r1} m$$

,

$$\vec{a}_n = \vec{a}_y, \quad \vec{E}_1 = l\vec{a}_x + m\vec{a}_y - n\vec{a}_z \;\longrightarrow\; \vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0$$

$$\downarrow$$

$$\vec{a}_y \times \vec{a}_x = -\vec{a}_z, \quad \vec{a}_y \times \vec{a}_y = 0, \quad \vec{a}_y \times \vec{a}_z = \vec{a}_x \;\longrightarrow\; \vec{a}_y \times [(E_{2x}\vec{a}_x + E_{2y}\vec{a}_y + E_{2z}\vec{a}_z) - (l\vec{a}_x + m\vec{a}_y + n\vec{a}_z)] = 0$$

$$\downarrow$$

$$-(E_{2x} - l)\vec{a}_z + (E_{2z} - n)\vec{a}_x = 0$$

$$\downarrow$$

$$\left.\begin{aligned} E_{2x} - l &= 0 \\ E_{2z} - n &= 0 \end{aligned}\right\}$$

$$\downarrow$$

$$\left.\begin{aligned} E_{2x} &= l \\ E_{2z} &= n \end{aligned}\right\}$$

$$D_{2y} = \varepsilon_0 \varepsilon_{r1} m$$ **(recalled)**

$$\downarrow$$

$$E_{2y} = \frac{D_{2y}}{\varepsilon_2} = \frac{D_{2y}}{\varepsilon_0 \varepsilon_{r2}} = \frac{\varepsilon_{r1} m}{\varepsilon_{r2}}$$

$$\vec{E}_1 = l\vec{a}_x + m\vec{a}_y - n\vec{a}_z$$

$$\vec{E}_2 = E_{2x}\vec{a}_x + E_{2y}\vec{a}_y + E_{2z}\vec{a}_z = l\,\vec{a}_x + m\frac{\varepsilon_{r1}}{\varepsilon_{r1}}\vec{a}_y + n\,\vec{a}_z$$

Circuit law of parallel resistances

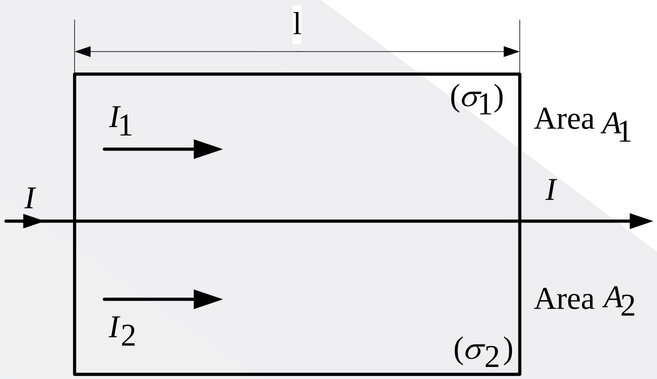
Let us apreciate the circuit law of parallel resistances with the help of the boundary condition that the tangential component of electric field is continuous at the interface between two media.

For this purpose, the said boundary condition is applied at the interface between two rectangular conducting slabs in contact of the same length *l*, conductivities σ_1 and σ_2 and cross-sectional areas A_1 and A_2 respectively.

Current densities J_1 and J_2 are related to electric fields E_1 and E_2 in the slabs through Ohm's law while the current *I* fed into the slabs in contact is divided in currents I_1 and I_2 through the slabs.

$$J_1 = \sigma_1 E_1 \qquad J_2 = \sigma_2 E_2$$

$$E_1 = \frac{J_1}{\sigma_1} \qquad E_2 = \frac{J_2}{\sigma_2}$$



E_1 and E_2, which are tangential at the interface, are continuous at the interface:

$E_1 = E_2$ **(boundary condition)**

$$\downarrow$$

$$\frac{J_1}{\sigma_1} = \frac{J_2}{\sigma_2}$$

$$\downarrow$$

$$\frac{I_1}{A_1\sigma_1} = \frac{I_2}{A_2\sigma_2}$$

$$\downarrow \quad \leftarrow \text{ Multiplying by } l$$

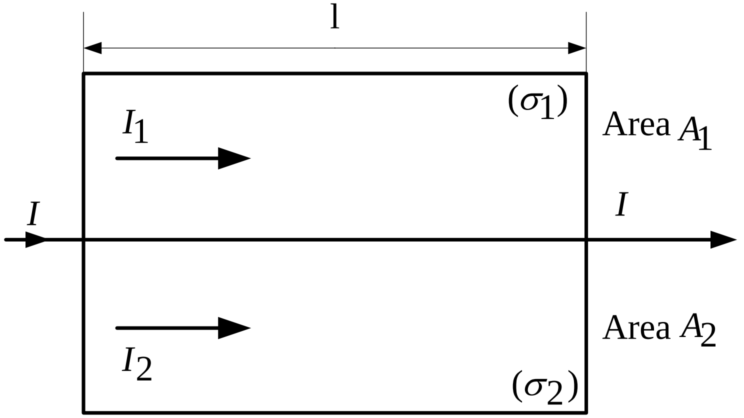
$$I_1 \frac{l}{A_1\sigma_1} = I_2 \frac{l}{A_2\sigma_2}$$

$$\downarrow \quad \leftarrow \quad R_1 = \frac{1}{\sigma_1}\frac{l}{A_1}$$

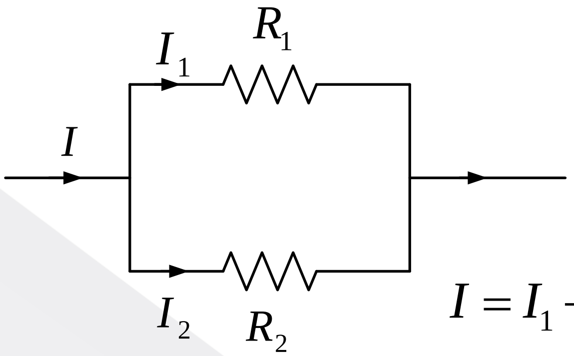
$$R_2 = \frac{1}{\sigma_2}\frac{l}{A_2}$$

$$I_1R_1 = I_2R_2$$

$I = I_1 + I_2 \longrightarrow$



$I = I_1 + I_2 \longrightarrow$



$I = I_1 + I_2$

$$I_1 R_1 = I_2 R_2$$

↓

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

↓

$$\frac{I_1}{I_1 + I_2} = \frac{R_2}{R_2 + R_1} = \frac{R_2}{R_1 + R_2}$$

↓

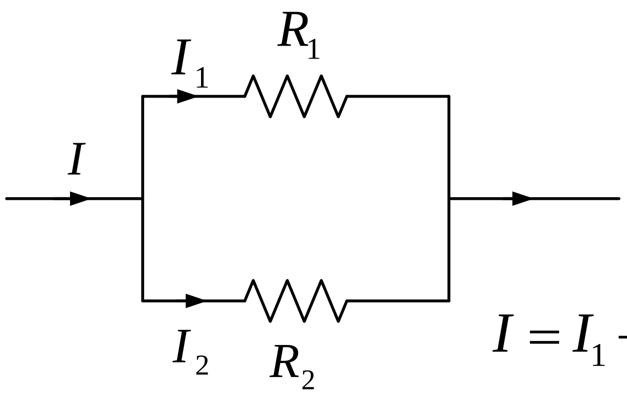
$$\frac{I_1}{I} = \frac{R_2}{R_1 + R_2}$$

Multiplying by R_1

↓

$$I_1 = \frac{R_2}{R_1 + R_2} I$$

$$I = I_1 + I_2$$ →



$$I = I_1 + I_2$$

$$I = I_1 + I_2$$

$$I_1 R_1 = I \frac{R_1 R_2}{R_1 + R_2} = I R_{\text{equivalent}}$$

↓

$$R_{\text{equivalent}} = \frac{R_1 R_2}{R_1 + R_2}$$

↓

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

(Law of parallel resistances)

For further reading, see

B. N. Basu, "Electromagnetic Theory and Applications in Beam-Wave Electronics," World Scientific Publishing Co. Inc., Singapore, New Jersey, London, Hong Kong (1996)
See Section 4.7 of the above book, page 140 [4.7 Electromagnetic Boundary Conditions 140]

B. N. Basu, "Engineering Electromagnetics Essentials," Universities Press, Hyderabad (2015) [See Chapter 7 of the book for Electromagnetic Boundary Conditions]

B. N. Basu, “Engineering Electromagnetics Essentials,” Universities Press, Hyderabad (2015) [See Chapter 7 of the book]

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PPT (converted to pdf) of Chapter 7 of the book: B. N. Basu, "Engineering Electromagnetics Essentials," Universities Press, Hyderabad (2015) is available for the participants of the STC. The convener of the program may be contacted to procure it.

Courtesy: Mr. Uttam Goswami


Thank You!