Some more analytical concepts in helical slow-wave structures

Some Analytical Concepts in Helical Slow-wave Structures

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Conceptual Development

- Helix in a glass tube
- Model for finite helix thickness
- Field and equivalent circuit analyses
- Modeling discrete helix supports by continuous dielectric tube(s) vis-à-vis azimuthal harmonic effects
- Homogeneous and inhomogeneous loading of the structure due to helix supports
- Anisotropic loading of the structure due to metal envelope with vanes or segments
- Synthesis of helix supports for dispersion controlled structures
- Sheath-helix and tape-helix models (axial harmonic effects)
- Heuristic tape-helix model
- Structure losses due to the finite resistivity of helix material
- Structure losses due to attenuator coating on dielectric helix-support rods
- Asymmetry of helix supports
- Dispersion control for wide device bandwidths
- Multi-dispersion structure for wide device bandwidths

D. T. Swift-Hook's field analysis of a helix closely fitting in a glass envelope, taking into account the effect of the finite helix thickness, using the sheath-helix model

Swift-Hook's study was of direct relevance to the first TWT with a glass envelope developed at CEERI (in the country)

Swift-Hook considered a free-space gap equal to half the helix thickness between the mean helix radius and the beginning of the dielectric region outside the helix to account for the finite thickness of the helix

Sinha, Ghosh, Kartikeyan improved the Swift-Hook's model for the finite helix thickness

[D. T. Swift-Hook, *Proc. IRE* 105b Suppl. (1958), 747-755]

Two internal reports of CEERI, Pilani in quick succession: one on Field Analysis and the other on Equivalent Circuit Analysis of a helix supported by dielectric rods in a metal envelope under the supervision of Dr. SSS Agarwala

These two analyses gave one and the same dispersion relation of the structure

$$\boxed{\frac{k\cot\psi}{\gamma} = \left(\frac{I_0(\gamma a)K_0(\gamma a)}{I_1(\gamma a)K_1(\gamma a)}\right)^{1/2} D}$$

ψ **is the helix pitch angle**

$\gamma = (\beta^2 - k^2)^{1/2}$ **is the radial propagation constant**

β **is the axial phase propagation constant**

$k = \omega(\mu_0\varepsilon_0)^{1/2} = \omega / c$ **is the free-space propagation constant**

D **is a function of structure parameters known as the dielectric loading factor.**

S. F. Paik provided the design formula (dispersion relation) for a helix supported by dielectric wedge bars in a metal envelope

Heuristic approach: Discrete dielectric wedge bars may be modelled by a continuous dielectric tube of an effective relative permittivity obtained by considering the relative volume occupied by the discrete dielectric supports in the structure

$$\varepsilon_{r,eff} = \frac{(\varepsilon_r)(A_s) + (1)(A - A_s)}{(A_s) + (A - A_s)} = 1 + (\varepsilon_r - 1)\frac{A_s}{A}$$

$\varepsilon_{r,eff} =$ **Effective relative permittivity of the equivalent continuous dielectric tube**

$\varepsilon_r =$ **Relative permittivity of the discrete dielectric support rods**

$A_s =$ **Cross-sectional area of the dielectric support rods**

$A =$ **Cross-sectional area of the region between the helix and the envelope (covering the regions both occupied and unoccupied (free-space) by the supports)**

Sinha, considering azimuthal harmonics due to azimuthally periodic dielectric supports, obtained by the rigorous field analysis the same dispersion relation as obtained by the heuristic approach

[A. K. Sinha and others *J. Appl. Phys*. 58 (1985) 3625-3627]

Swift-Hook suggested, in a personal communication, that one could realize the effect of continuous dielectric tube claddings surrounding the helix by loading the helix by discrete dielectric helix-supports of tapered cross-sectional geometry that could provide the required non-homogeneity for the desired structure dispersion for wideband TWTs.

This gives the idea of the synthesis of supports for the desired helix dispersion!

Sinha, and later on Ghosh, with foreign collaborators worked on the synthesis of supports

Modelling of dielectric helix-support rods deviating from the simple wedge cross-sectional geometry such as of circular, rectangular, half-moon-shaped, and T-shaped cross sections causing inhomogeneous helix loading

One can use Sinha's number-of-dielectric-tube model to study non-homogeneous helix loading due to dielectric helix-support rods deviating from the simple wedge cross-sectional geometry

Effective relative permittivity values of the equivalent dielectric tube regions into which the discrete supports can be smoothed out in the model can be found from geometrical considerations

Dielectric helix-supports such as of half-moon-shaped and T-shaped cross sections have the potential for dispersion control required for widening the bandwidth of a TWT

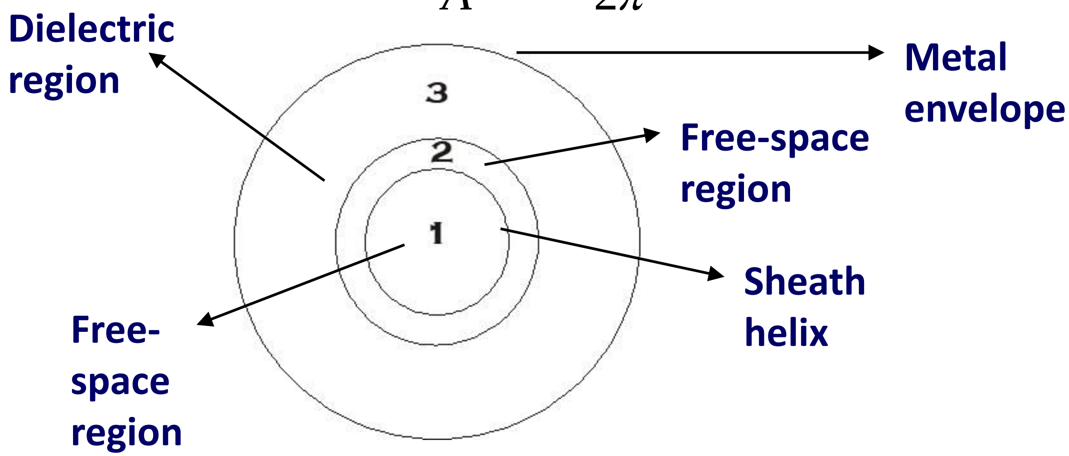
Jain, Raju, Gupta, Kapoor, and, more extensively, Ghosh used the Sinha's number-of-dielectric-tubes model to study inhomogeneously loaded structures for widening the bandwidth of a TWT at high interaction impedance values

Ghosh added a rigour to the model by considering nonuniform radial propagation constant over the equivalent dielectric tube regions of the model

Helix with dielectric wedge bar supports is modelled by an equivalent dielectric tube of an effective relative permittivity

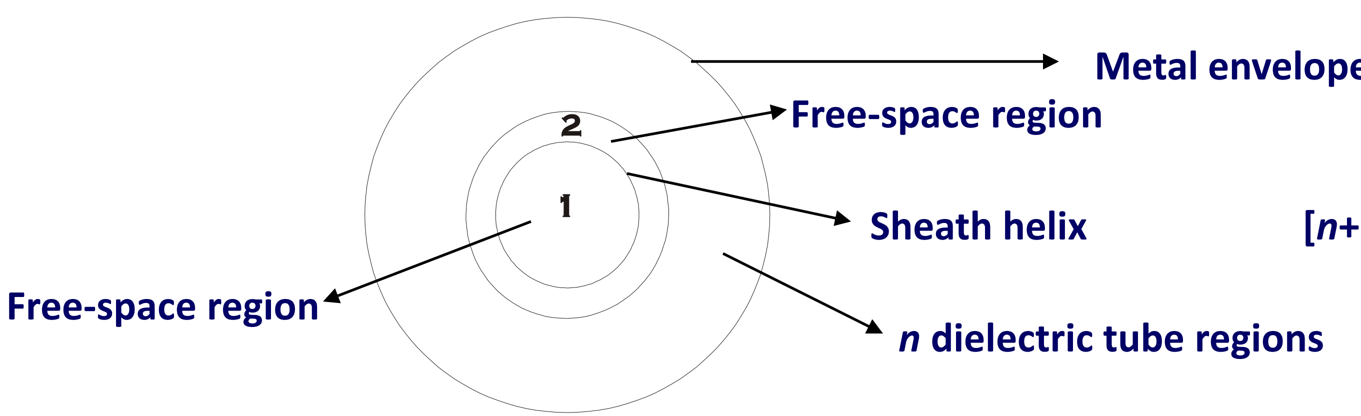
$$\varepsilon_{r,eff} = 1 + (\varepsilon_r - 1)\frac{A_s}{A} = 1 + \frac{\phi N}{2\pi}$$

ϕ = wedge angle N = number of supports



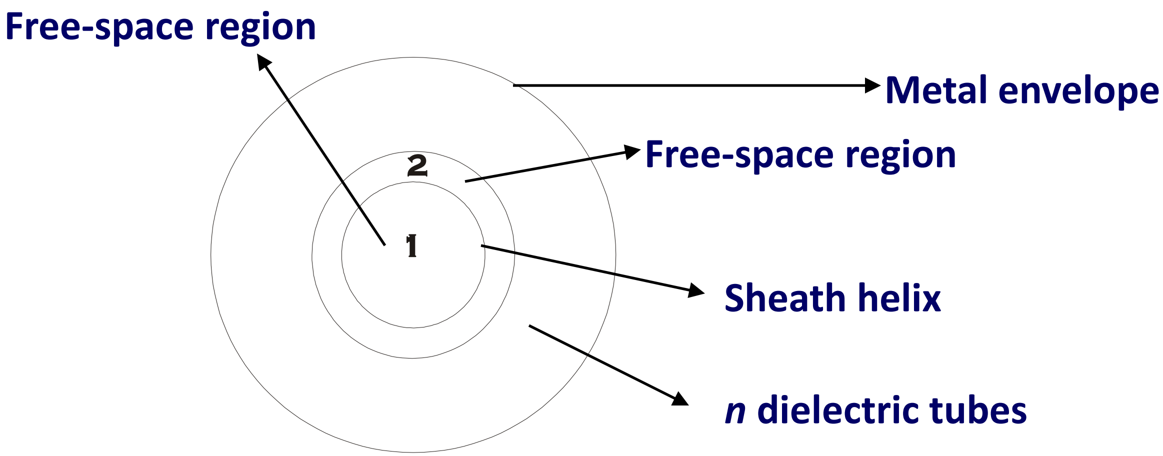
[3 structure regions]

Helix with dielectric with dielectric supports deviating from simple wedge geometry is modelled by Sinha's *n*-dielctric-tube model



[*n*+2 structure regions]

n dielectric tube regions: *n* + 2 structure regions



4(*n*+2) field constants out of which 2 constants are zero in order to prevent the fields to blow up to infinity at the helix axis

4*n* + 6 non-zero field constants

Radial propagation constant considered to be uniform over the structure regions

n + 2 regions

n + 1 region interfaces

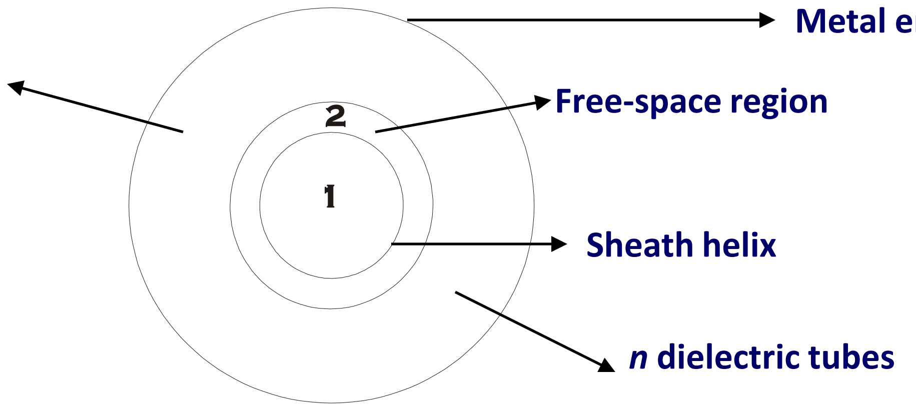
4(*n* + 1) boundary conditions at region interfaces

+ 2 boundary conditions at the metal envelope

= 4*n* + 6 boundary conditions

Field expressions are substituted into 4*n* + 6 boundary conditions to obtain 4*n* + 6 simultaneous equations in 4*n* + 6 field constants (so expressed that the right hand side of each equation is zero). The condition for non-trivial solution that the (4*n* + 6) x (4*n* + 6) determinant formed by the coefficients of field constants occurring in these equations is equal to zero yields the dispersion relation of the structure

Sanjay's rigorous nonuniform-radial-propagation-constant model for non-homogeneously loaded helical structure



Field constants appear in the basic field expressions for the p^{th} structure region

$$E_{z,p} = A_p I_0(\gamma_p r) + B_p K_0(\gamma_p r)$$

$$H_{z,p} = C_p I_0(\gamma_p r) + D_p K_0(\gamma_p r)$$

$$\gamma_p = (\beta^2 - \omega^2 \mu_0 \varepsilon_p)^{1/2} = (\beta^2 - \omega^2 \mu_0 \varepsilon_0 \varepsilon_{r,p})^{1/2} = (\beta^2 - k_0^{\ 2} \varepsilon_{r,p})^{1/2}$$

$r = a =$ mean helix radius

$r = b_0 =$ outer helix radius $=$ inner radius of the first dielectric tube

$r = b_1 =$ outer radius of the first dielectric tube $=$ inner radius of the second dielectric tube; and so on

$r = b_n =$ outer radius of the nth dielectric tube $=$ radius of the metal envelope

With the help of the boundary condition at the sheath helix (at the interface between regions 1 and 2) (*r* = *a*)

$$H_{\theta 1}\cos\psi + H_{z1}\sin\psi = H_{\theta 2}\cos\psi + H_{z2}\sin\psi$$

(No current perpendicular to the helix winding direction)

$$\frac{k_0 \cot\psi}{\gamma} = -\left(\frac{I_0(\gamma_0 a)}{I_1(\gamma_0 a)}\right)\left(\frac{(B_2/A_2)^{-1} + K_0(\gamma_0 a)/I_0(\gamma_0 a)}{(D_2/C_2)^{-1} - K_1(\gamma_0 a)/I_1(\gamma_0 a)}\right)^{1/2}$$

(Dispersion relation)

$$E_{z,n+2} = 0 \ (r = b_n = b) \text{ at the metal envelope}$$

$$\frac{B_{n+2}}{A_{n+2}} = -\frac{I_0(\gamma_{n+2} b)}{K_0(\gamma_{n+2} b)}$$

$$E_{\theta,n+2} = 0 \ (r = b_n = b) \text{ at the metal envelope}$$

$$\frac{D_{n+2}}{C_{n+2}} = \frac{I_1(\gamma_{n+2} b)}{K_1(\gamma_{n+2} b)}$$

For the p^{th} structure region, the normalized wave impedance functions are defined as

$$Z_{E,p} = \frac{j}{(\mu_0/\varepsilon_0)^{1/2}} \frac{E_{z,p}}{H_{\theta,p}} = \left(\frac{\gamma_p}{k_0 \varepsilon_{r,p}}\right)\left(\frac{I_0\{\gamma_p r\} + (B_p/A_p) K_0\{\gamma_p r\}}{I_1\{\gamma_p r\} - (B_p/A_p) K_1\{\gamma_p r\}}\right)$$

$$Z_{H,p} = -\frac{j}{(\mu_0/\varepsilon_0)^{1/2}} \frac{E_{\theta,p}}{H_{z,p}} = -\left(\frac{k_0}{\gamma_p}\right)\left(\frac{I_1\{\gamma_p r\} - (D_p/C_p) K_1\{\gamma_p r\}}{I_0\{\gamma_p r\} + (D_p/C_p) K_0\{\gamma_p r\}}\right)$$

***p* = 1 refers to the free-space region inside the sheath-helix; *p* = 2 refers to the free-space region outside the sheath-helix of half the helix thickness that accounts for the finite helix thickness; *p = n*+2 to refers to the outermost dielectric tube region**

Impedance boundary conditions at the inner surface of the outermost dielectric tube, that is, at the interface $(r = b_{n-1})$:

$$Z_{E,(n+2)} = Z_{E,(n+1)}\,(r = b_{n-1}) \Rightarrow B_{n+1}/A_{n+1} \text{ in terms of } B_{n+2}/A_{n+2}, \text{the latter already obtained}$$

$$Z_{H,(n+2)} = Z_{H,(n+1)}\,(r = b_{n-1}) \Rightarrow D_{n+1}/C_{n+1} \text{ in terms of } D_{n+2}/C_{n+2}, \text{the latter already obtained}$$

Impedance boundary conditions at the next inward interface $(r = b_{n-2})$:

$$Z_{E,(n+1)} = Z_{E,n}\,(r = b_{n-2}) \Rightarrow B_n/A_n \text{ in terms of } B_{n+1}/A_{n+1}, \text{obtained as above}$$

$$Z_{H,(n+1)} = Z_{H,n}\,(r = b_{n-2}) \Rightarrow D_n/C_n \text{ in terms of } D_{n+1}/C_{n+1}, \text{obtained as above}$$

$$Z_{E,(n+1)} = Z_{E,n}\,(r = b_{n-2}) \Rightarrow B_n/A_n \text{ in terms of } B_{n+1}/A_{n+1}$$

$$Z_{H,(n+1)} = Z_{H,n}\,(r = b_{n-2}) \Rightarrow D_n/C_n \text{ in terms of } D_{n+1}/C_{n+1}$$

Considering the impedance boundary conditions at progressively inward interfaces, one obtains the expressions for B_2/A_2 and D_2/C_2, to be substituted into

$$\frac{k_0 \cot\psi}{\gamma} = -\left(\frac{I_0(\gamma_0 a)}{I_1(\gamma_0 a)}\right)\left(\frac{(B_2/A_2)^{-1} + K_0(\gamma_0 a)/I_0(\gamma_0 a)}{(D_2/C_2)^{-1} - K_1(\gamma_0 a)/I_1(\gamma_0 a)}\right)^{1/2}$$

(Dispersion relation re-written)

Factors of practical relevance that can be included in the analysis of helical slow-wave structure:

Finite helix thickness, Non-homogeneity of helix-supports deviating from simple wedge geometry, Nonuniform radial propagation constant over the structure cross section, Effect of axial periodicity of the helix, Finite resistivity of helix material, Resistivity of attenuator coating, Asymmetry of dielectric helix-supports, Anisotropic loading of metal envelope caused by vanes or segments projecting radially inward from the metal envelope, Multi-dispersion structure

Pierce's Sheath-helix model

Cylindrical sheath of infinitesimal thickness that has infinite and zero conductivity in the direction of helical winding and perpendicular to this direction, respectively, replacing the actual helix

At the sheath-helix surface

1. Electric field intensity parallel to the helix winding direction in the region inside the sheath-helix helix is zero.
2. Electric field intensity parallel to the helix winding direction in the region outside the sheath-helix helix is zero
3. Magnetic field intensity parallel to the helix winding direction in the region inside the sheath-helix helix is continuous with that outside
4. Axial electric field intensity (or azimuthal electric field intensity) is continuous with that outside

At the dielectric-dielectric interface

Axial and azimuthal electric field intensities as well as axial and azimuthal magnetic field intensities are each continuous

At the metal envelope

Axial and azimuthal electric field intensities are each equal to zero

J. R. Pierce: Traveling-Wave Tubes. D. Van Nostrand (Princeton, 1950)

Sensiper's tape-helix model

Axial harmonic effects due to the axial periodicity of the helix taken into account

Actual helix replaced by a tape of infinitesimal thickness that conducts in all directions

Zero tangential electric field intensity everywhere on the tape surface corresponding to the actual current distribution on the tape surface

Tape surface current density assumed to be predominantly along the helix winding direction, with a defined distribution, presumably caused by an electric field intensity parallel to the winding direction $\mathrm{E}_{//}$, under narrow-tape approximation

Constant amplitude of surface current density over the tape width with its phase varying along the centreline of the tape winding as $\exp-(j\beta_0 p\theta/(2\pi))$, where β_0 is the fundamental axial phase propagation constant, and p is the helix pitch, as per a typically assumed current distribution

Expected satisfaction of the boundary condition $E_{//}=0$ over the entire tape surface, for the narrow tape, provided the latter is satisfied along the centreline of the tape surface

Field expressions found using Floquet's theorem considering the helix periodicity

Relevant field constants and hence $E_{//}$ along the centreline of the tape surface found in terms of the assumed tape surface current density distribution

$E_{//}$ along the centreline of the tape surface set equal to zero $(E_{//}=0)$ yielding the dispersion relation

“If the tape is taken to be very narrow, that is, with δ small compared with *a*, *p*, and λ, it seems quite reasonable to assume that essentially all of the current flows only along the tape.”

…….. “If the point of view is taken that the fields are produced by the currents which flow, with the tape narrow and current flowing primarily in the direction of the tape, the specific distribution of current across the tape will affect only to a small degree the fields in the near neighborhood of the wire and to a much less degree the fields on adjacent and faraway turns. Thus, if some reasonable assumptions are made concerning this current distribution, it is to be expected that only small errors will be made in the field expressions”.

…….. “If an inexact current distribution on the tape is assumed, the tangential electric field can no longer be made zero everywhere on the tape, and this boundary conditions can be only approximately satisfied. This may be done in several ways. One could, for example, require the average value, or better the mean square value, of the tangential electric field on the tape to be a minimum, with the propagation constant, which gives this minimum as the solution. However, another procedure is used here which leads to a somewhat simpler determinantal equation for calculative purposes and which appears to be a quite adequate approximation. In this it is required that $E_{//}$ be zero along the centerline of the tape; in other words, one of the boundary conditions is matched exactly along a line. (As noted before), for a narrow tape the dominant current density is $K_{//}$, and, loosely speaking, it is $E_{//}$ which forces the current to flow along the tape. Thus, if the most important boundary condition is satisfied on a line, one may hope to obtain a reasonable good approximation to the exact case where the condition must be satisfied over a surface”……………. “in the neighborhood of this line which is almost a narrow tape.”

— Samuel Sensiper: Electromagnetic wave propagation on helical conductors. Technical Report No. 194; May 16, 1951. Research Laboratory of Electronics, Institute of Technology, Cambridge, Massachusetts (Sc D Thesis); and *Proc. IRE* 42 (1955) 144-161

Heuristic tape-helix model

Makes the analysis leading to the dispersion relation of a loaded helical slow-wave structure rather simple!

Dispersion relation of an unloaded helix in free-space in the tape-helix model:

$$\sum_{-\infty}^{\infty}(M_m(\gamma_m a)+N_m(\gamma_m a)\left(\frac{\sin(\beta_m\delta/2}{\beta_m\delta/2}\right)=0$$

$$\delta=\text{tape width},\ \gamma_m=(\beta_m{}^2-k_0{}^2)^{1/2},\ k_0=\omega(\mu_0\varepsilon_0)^{1/2}$$

$$M_m(\gamma_m a)=(\frac{m\beta_m}{\gamma_m}\cot\psi-\gamma_m a)^2 I_m(\gamma_m a)K_m(\gamma_m a)$$

$$N_m\{\gamma_m a\}=k_0{}^2a^2\cot^2\psi\, I'_m\{\gamma_m a\}K'_m\{\gamma_m a\}$$

Prime indicates the derivative with respect to argument.

$$\psi=\text{helix pitch angle}$$

$$a=\text{mean helix radius}$$

S. Sensiper: *Proc. IRE* 43 (1955) 149-161

Dispersion relation of a loaded helix in the sheath-helix model:

$$\frac{k_0 \cot\psi}{\gamma_0} = \left(- \frac{I_0(\gamma_0 a) K_0(\gamma_0 a)}{I_0'(\gamma_0 a) K_0'(\gamma_0 a)} \right)^{1/2} D_0(\gamma_0 a)$$

$$\Downarrow$$

$$M_0\{\gamma_0 a\} D_0^{\ 2}\{\gamma_0 a\} + N_0\{\gamma_0 a\} = 0$$

Combinational approach

Combine the dispersion relation of an unloaded helix in free-space in the tape-helix model with the dispersion relation of a loaded helix in the sheath-helix model

A. K. Sinha and others: *IEE Proceedings-H: Microwave, Antenna & Propagation* 139 (1992), 347-350

Dispersion relation of an unloaded helix in free-space in the tape-helix model:

$$\sum_{-\infty}^{\infty}(M_m(\gamma_m a)+N_m(\gamma_m a)\left(\frac{\sin(\beta_m\delta/2}{\beta_m\delta/2}\right)=0$$

Dispersion relation of a loaded helix in the sheath-helix model:

$$M_0(\gamma_0 a)D_0^{\ 2}(\gamma_0 a)+N_0(\gamma_0 a)=0$$

Heuristic combinational approach:

Dispersion relation of a loaded helix in the tape-helix model

$$\sum_{-\infty}^{\infty}(M_m(\gamma_m a)D_m^{\ 2}(\gamma_m a)+N_m(\gamma_m a)\left(\frac{\sin(\beta_m\delta/2}{\beta_m\delta/2}\right)=0$$

Dispersion relation of a loaded helix in the tape-helix model

$$\sum_{-\infty}^{\infty}(M_m(\gamma_m a)D_m{}^2(\gamma_m a)+N_m(\gamma_m a)\left(\frac{\sin(\beta_m\delta/2}{\beta_m\delta/2}\right)=0$$

$D_m(\gamma_m a)$ is obtained by replacing $\gamma_0 a$ by $\gamma_m a$ in the expression for $D_0(\gamma_0 a)$

The above dispersion relation of a loaded helix obtained by the heuristic tape-helix model exactly agrees with the dispersion relation obtained by the analysis of the loaded helix by the rigorous tape-helix model!

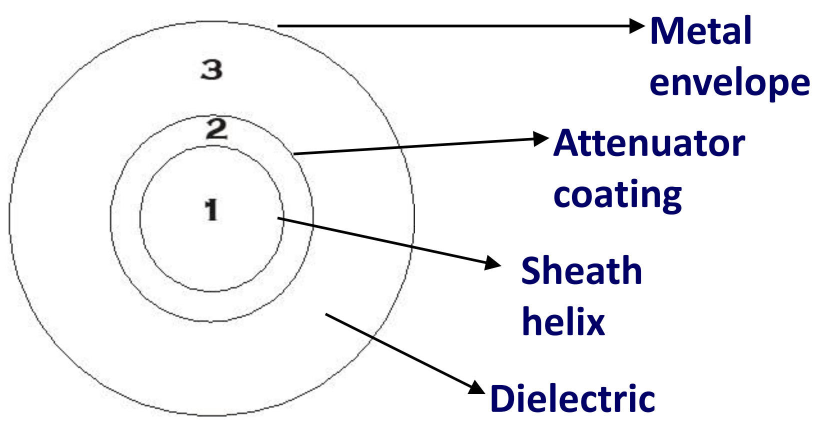
A. K. Sinha and others: *IEE Proceedings-H: Microwave, Antenna & Propagation* 139 (1992), 347-350

Effect of attenuator coating in the analysis of helical structures

Helix is surrounded by a dielectric tube in a metal envelope

A resistive coating is applied on the inner surface of the dielectric tube

Helix turns are short-circuited by the resistive coating



At the sheath-helix interface between regions 1 and 2

$$E_{\theta 1}\cos\psi + E_{z1}\sin\psi = 0$$

$$E_{\theta 2}\cos\psi + E_{z2}\sin\psi = 0$$

$$H_{\theta 1}\cos\psi + H_{z1}\sin\psi = H_{\theta 2}\cos\psi + H_{z2}\sin\psi$$

$$E_{z1} = E_{z2}$$

At the inner surface of the dielectric tube where the resistive coating is present: At the interface between regions 2 and 3

$$H_{\theta 3} - H_{\theta 2} = \sigma_S E_{z2}$$

$$H_{z2} - H_{z3} = \sigma_S E_{\theta 2}$$

[In general, γ is a complex quantity

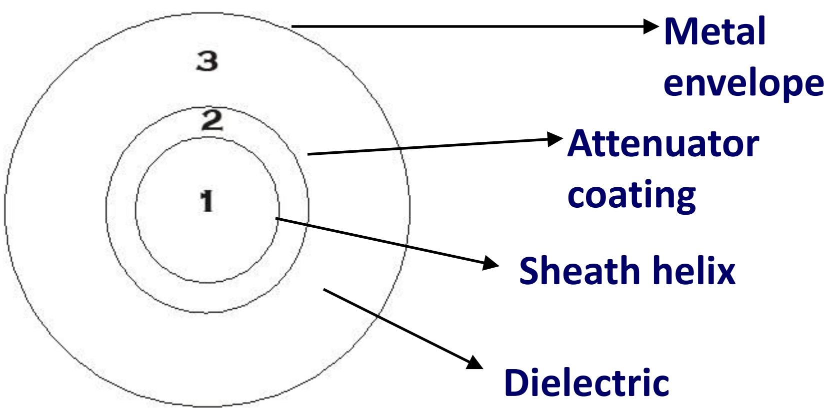
$\sigma_S = 0$ ($R_S = \infty$) corresponds to the absence of the attenuator coating

$\sigma_S = \infty$ ($R_S = 0$) corresponds to a perfectly conducting coating]

At the metal envelope

$$E_{z3} = 0$$

$$E_{\theta 3} = 0$$



RF quantities vary as $\exp(j\omega t - \Gamma z)$

$$\Gamma = \alpha + j\beta z$$

$$\gamma^2 = -\Gamma^2 - k^2$$

$$\gamma = (-\Gamma^2 - k^2)^{1/2}$$

α **is the attenuation constant**

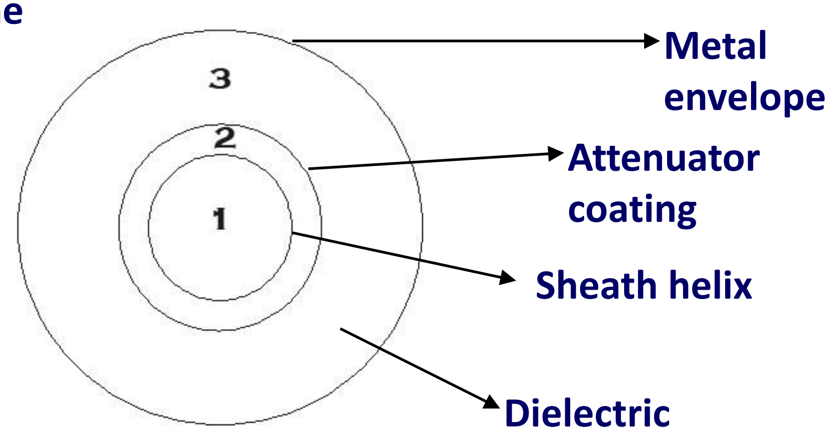
RF quantities depend on the modified Bessel functions with argument γr

The dispersion relation can be obtained with the help of the field expressions and the boundary conditions and solved for the complex argument γr

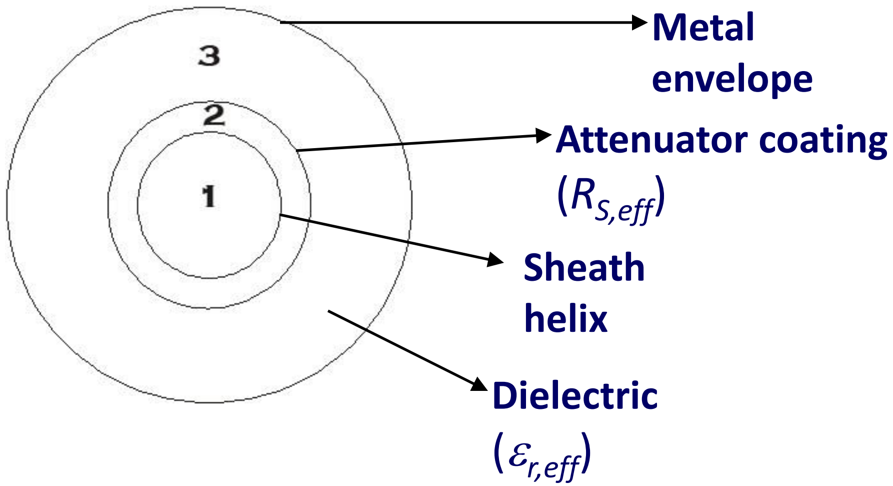
The solution for γ **and hence for** Γ **gives the** β **and** α

There exists an optimum value of the surface resistance $R_S = 1/\sigma_S$ **for maximum** α

P. K Jain and others: *IEEE Trans. Electron Devices* 35 (1988), 549-558



Could it be possible to interpret the surface resistance of coating on individual dielectric helix-supports R_S for the effective surface resistivity on the inner surface of the equivalent dielectric tube $R_{S,eff}$?



$$\varepsilon_{r,eff} = 1 + (\varepsilon_r - 1)\frac{A_s}{A}$$

$$R_{S,eff} = \left(\frac{A}{A_s}\right)^{1/2} R_S$$

A_s = **Area of all the support rods**

A = **Area of the region outside the helix within the envelope (including the support rods and the free-space regions)**

$$\varepsilon_{r,eff} A = (\varepsilon_r) A_s + (1)(A - A_s)$$

$$\varepsilon_{r,eff} = 1 + (\varepsilon_r - 1)\frac{A_s}{A}$$

Complex permittivity:

$$\vec{J} = (\sigma + j\omega\varepsilon)\vec{E} = j\omega(\varepsilon + \frac{\sigma}{j\omega})\vec{E}$$

$$= j\omega(\varepsilon' - j\varepsilon'')\vec{E} = j\omega\varepsilon^* \vec{E}$$

$$\varepsilon' = \varepsilon \qquad \varepsilon'' = \frac{\sigma}{\omega}$$

$\varepsilon^* = \varepsilon' - j\varepsilon'' =$ **Complex permittivity**

$\varepsilon_r^* = \varepsilon_r' - j\varepsilon_r'' =$ **Relative complex permittivity**

$$\varepsilon_r' = \frac{\varepsilon'}{\varepsilon_0} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r \qquad \varepsilon_r'' = \frac{\sigma}{\omega\varepsilon_0}$$

$$\sigma = \omega\varepsilon_0\varepsilon_r''$$

$$\varepsilon_{r,eff} = 1 + (\varepsilon_r - 1)\frac{A_s}{A}$$

$$\varepsilon^*_{r,eff} = 1 + (\varepsilon^*_r - 1)\frac{A_s}{A} = 1 + (\varepsilon'_r - j\varepsilon''_r - 1)\frac{A_s}{A}$$

$$= 1 + \frac{A_s}{A}(\varepsilon'_r - 1) - j\frac{A_s}{A}\varepsilon''_r$$

$$\varepsilon^*_{r,eff} = 1 + \frac{A_s}{A}(\varepsilon'_r - 1) - j\frac{A_s}{A}\varepsilon''_r$$

$$\varepsilon^*_{r,eff} = \varepsilon'_{r,eff} - j\varepsilon''_{r,eff}$$

$$\varepsilon'_{r,eff} = 1 + \frac{A_s}{A}(\varepsilon'_r - 1)$$

$$\sigma = \omega\varepsilon_0\varepsilon''_r$$

$$\downarrow$$

$$\varepsilon''_{r,eff} = \frac{A_s}{A}\varepsilon''_r \longrightarrow \sigma_{eff} = \omega\varepsilon_0\varepsilon''_{r,eff}$$

$$\downarrow$$

$$\sigma_{eff} = \omega\varepsilon_0\frac{A_s}{A}\varepsilon''_r$$

$$\sigma_{eff} = \omega\varepsilon_0 \frac{A_s}{A} \varepsilon_r''$$

$$\downarrow$$

$$R_{S,eff} = \left(\frac{\pi f \mu_0}{\sigma_{eff}}\right)^{1/2} \longleftarrow R_s = \left(\frac{\pi f \mu_0}{\sigma}\right)^{1/2}$$

$$\downarrow$$

$$\eta_0 = \left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2}$$

$$R_{S,eff} = \left(\frac{\pi f \mu_0}{\omega\varepsilon_0 \frac{A_s}{A}\varepsilon_r''}\right)^{1/2} = \eta_0\left(\frac{A}{A_s}\frac{1}{2\varepsilon_r''}\right)^{1/2} \leftarrow \eta_0 = \left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2}$$

$$\downarrow$$

$$\varepsilon_r'' = \frac{\sigma}{\omega\varepsilon_0}$$

$$R_{S,eff} = \eta_0\left(\frac{A}{A_s}\frac{1}{2\varepsilon_r''}\right)^{1/2}$$

$$\downarrow$$

$$\tan\delta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon_0\varepsilon_r}$$

$$R_{S,eff} = \eta_0\left(\frac{A}{A_s}\frac{\omega\varepsilon_0}{2\sigma}\right)^{1/2} \longrightarrow R_{S,eff} = \eta_0\left(\frac{A}{A_s}\frac{1}{2\varepsilon_r \tan\delta}\right)^{1/2}$$

$$= \left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2}\left(\frac{A}{A_s}\frac{\omega\varepsilon_0}{2\sigma}\right)^{1/2} = \left(\frac{A}{A_s}\right)^{1/2}\left(\frac{\pi f \mu_0}{\sigma}\right)^{1/2} \leftarrow R_s = \left(\frac{\pi f \mu_0}{\sigma}\right)^{1/2}$$

$$\downarrow$$

$$R_{S,eff} = \left(\frac{A}{A_s}\right)^{1/2} R_S$$

Asymmetry of the dielectric helix-supports

Dr. Amarjit Singh once in a social gathering at Pilani suggested me that we should take into account in our analysis the effect of the asymmetry of the dielectric helix-supports, a problem of relevance to the design of wideband TWTs. In turn, I passed on the problem to Sinha, who succeeded in developing the analysis for such asymmetry and demonstrated the presence of a band gap in the dispersion diagram.

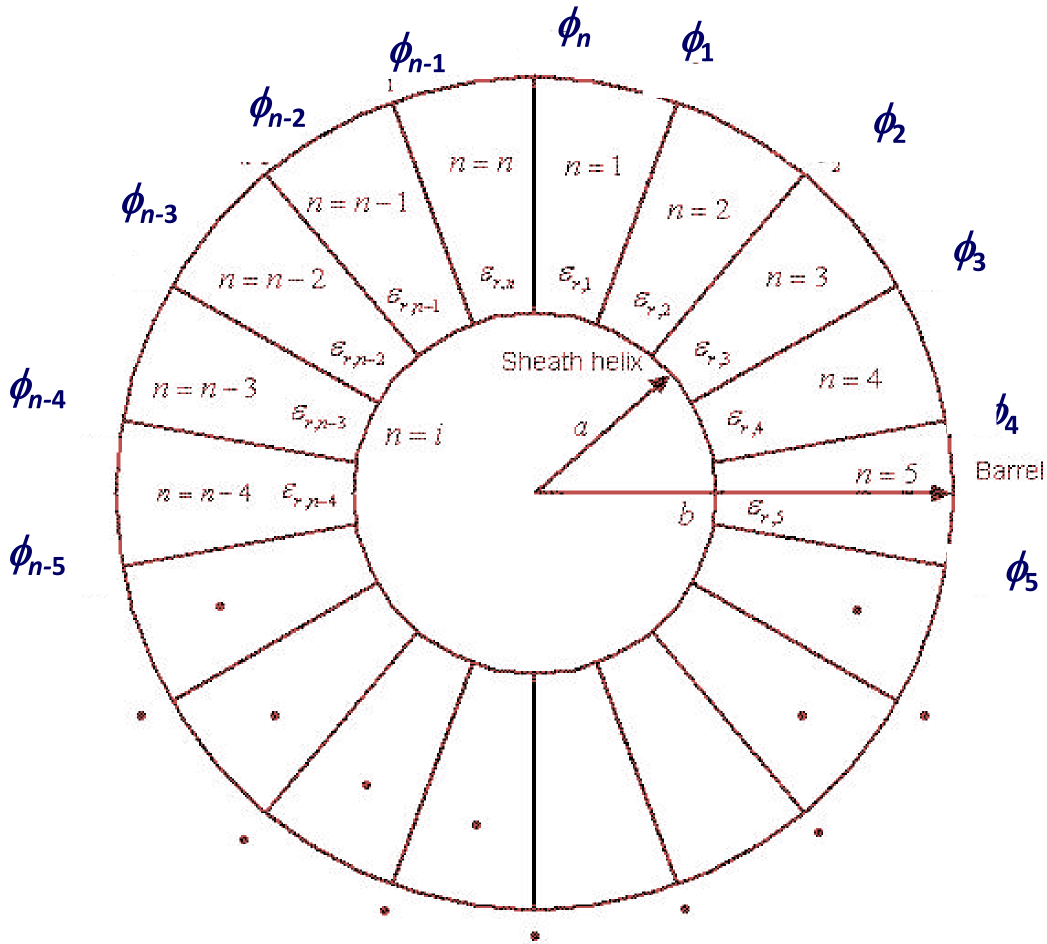
Asymmetry:

Dielectric helix-supports are not symmetrically arranged around the helix

Dimensions of the dielectric helix-supports may be different from support to support

Dielectric constants of the dielectric helix-supports are not the same for all the supports

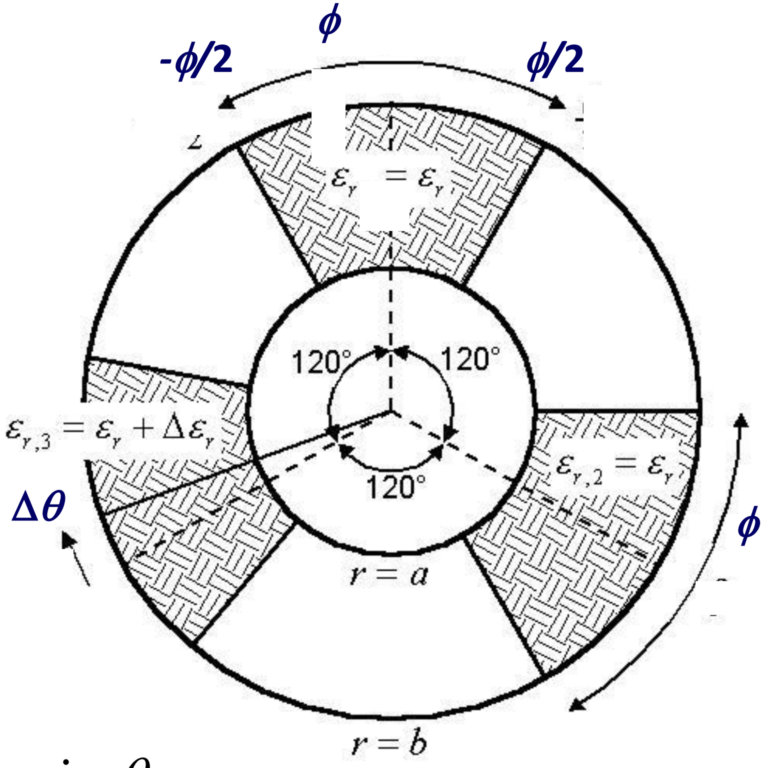
Consequence: Stop band in the ω-β dispersion characteristics



Region outside the helix divided into '*n*' regions

N = **number of supports over** 2π

Θ = **angular periodicity**



$$\text{RF angular dependence : } \exp jp_m\theta$$

$$p_m = m(2\pi/\Theta)$$

$$\Theta = 2\pi/N$$

$$p_m = mN$$

$$\text{RF angular dependence : } \exp jmN\theta$$

Asymmetric support structure

One of the support rods is offset

$$\nabla\phi \neq 0; \nabla\varepsilon_r = 0\,(\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3})\,(\text{angular offset})$$

$$\nabla\varepsilon_r \neq 0\,(\varepsilon_{r1} = \varepsilon_{r2} \neq \varepsilon_{r3}); \nabla\phi = 0\,(\text{permittivity offset})$$

$$N = 1(\text{one period extending over } 2\pi)$$

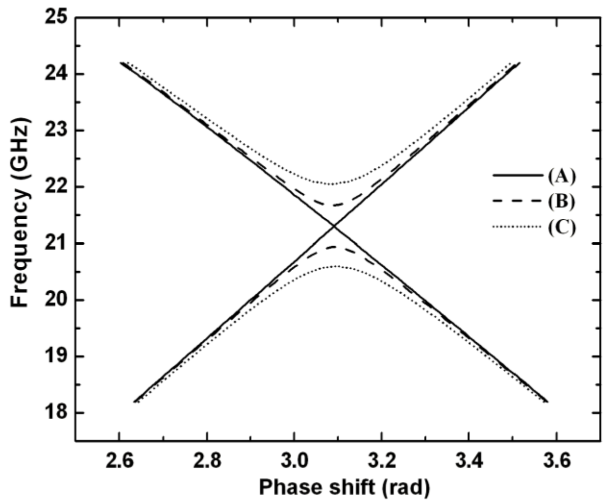
$$n = 6\ (= 3\,(\text{dielectric region}) + 3\,(\text{free-space region})$$
$$\text{over one period } 2\pi)$$

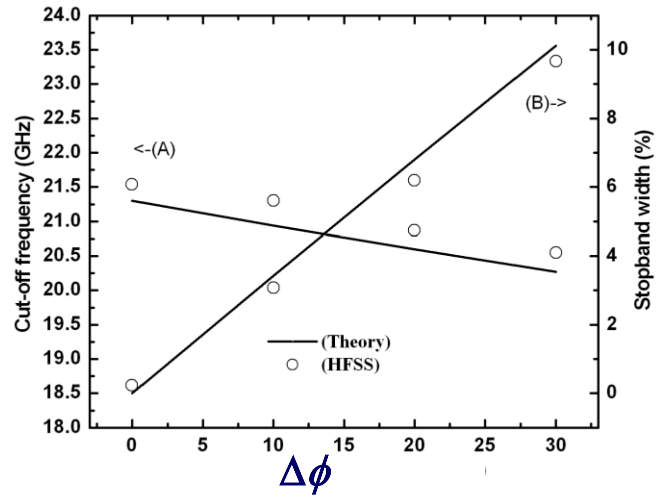
Symmetric support structure

$$\nabla\varepsilon_r = 0; \nabla\phi = 0$$

$$N = 3(\text{three periods extending over } 2\pi, \text{typically for three supports})$$

$$n = 2\,(= 1\,(\text{dielectric region}) + 1\,(\text{free-space region}))$$
$$\text{over one period } 2\pi / N = 2\pi / 3)$$





A: $\Delta\phi = 0^0$

B: $\Delta\phi = 10^0$

C: $\Delta\phi = 20^0$

Phase shift = Axial phase propagation constant times the helix pitch

A: Cutoff frequency

B: Stop-band width

(Helix radius a =0.7 mm, Envelope radius b =1.5 mm, Support rod dielectric constant $\varepsilon_r = 6.5$, Wedge angle ϕ =60^0)

With the increase of offset angle $\Delta\phi$, the cutoff frequency decreases and the stop-band width increases

Agreement with HFSS: cutoff frequency within 1.5% and stop-band width within 0.1%

Computer time: ~ 1s for a data point against ~ 25 min for HFSS

Wideband Multi-Octave TWTs

Zero-to-slightly-negative-dispersion structure for wideband performance

Anisotropically loaded helix

Metal vane/ segment loaded envelope for negative dispersion

Inhomogeneously loaded helix:

Helix with tapered geometry dielectric supports such as half-moon-shaped and T-shaped supports

Negative dispersion ensures constancy of Pierce's velocity synchronization parameter b with frequency

Constancy of b with frequency with negative dispersion

$$b = \frac{v_0 - v_p}{v_p C} = \frac{v_0 - v_p}{v_p (K I_0 / 4V_0)^{1/3}} = \frac{\dfrac{v_0 - v_p}{v_p}}{K^{1/3}} \frac{1}{(I_0 / 4V_0)^{1/3}}$$

Negative dispersion: v_p **increases with frequency**

$v_0 - v_p$ **decreases with frequency**

$\dfrac{v_0 - v_p}{v_p}$ **decreases with frequency**

$\rightarrow$ **Numerator of the expression for** b **decreases with frequency**

K **decreases with frequency and hence the denominator of the expression for** b **decreases with frequency**

$\rightarrow$ **Denominator of the expression for *b* decreases with frequency**

$\Rightarrow$ b **remains <u>constant</u> with frequency**

Multi-section, Multi-Dispersion Structures for Ultra-Wideband TWTs

Examples of wideband multi-section devices

Twystron

The first section is a klystron providing a double-hump gain-frequency response. The second section is a TWT providing a peak between the two humps of the first section in the gain-frequency response.

Two-section Gyro-TWT

A gyro-TWT is inherently a two-hump device corresponding to the beam-mode dispersion line intersecting the waveguide dispersion hyperbola at two points. Grazing intersection, ideally, provides a single intersecting point.

One may use two dielectric loaded sections, one of which should provide a single peak between the two peaks provided by the second section, in the gain-frequency response.

Conventional TWTs with multi-dispersion, multi-section structures

Small-signal gain equation $G \sim BCN$

$$N\lambda_e = l$$

$$N\frac{v_0}{f} = l \qquad C = (KI_0 / 4V_0)^{1/3}$$

$$N = \frac{f\,l}{v_0}$$

$$G \sim B(KI_0 / 4V_0)^{1/3}\frac{f\,l}{v_0}$$

G **is proportional to** $K^{1/3} f\, l$

G is proportional to $K^{1/3} f\, l$

<u>Gain-frequency response</u>:

Lower gain at lower frequencies as G is proportional to f

Lower gain at higher frequencies as *G* is proportional to $K^{1/3}$, the latter decreasing with frequency

Conventional structure: If you had increased the length *l*, then the gain *G* would be compensated at lower frequencies *f*. However, then the gain *G* would become very high at higher frequencies *f*.

Therefore, let us arrive at the design of a helical slow-wave structure the effective length of which is large at lower frequencies but at the same time the effective length becomes relatively smaller at higher frequencies. (The design should ensure that the gain is not enhanced at any frequency to a high value causing oscillation in the device).

The answer lies in a multi-dispersion, multi-section conventional helix TWTs!

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One positive-dispersion helix section of length l_1 synchronous with the beam only at lower frequencies and the other nearly dispersion-free helix section of effective length length l_2 synchronous with the beam both at lower and higher frequencies.

Effective length increased to $l_1 + l_2$ at lower frequencies
Effective length reduced to l_2 at higher frequencies (since the section of length goes out of synchronism at higher frequencies

G **is proportional to** $K^{1/3} f\, l$

We have to control (i) the nature and the amounts of dispersion of of the sections by suitably loading the sections and (ii) the lengths of the two sections

Select structure sections such as segment loaded helices of controllable dispersion

Analysis should be capable of finding the dispersion and interaction impedance characteristics of the structure sections, say, with metal segment loaded envelopes and their control by structure section parameters like segment dimensions and relative section lengths.

Thank you!