Magnetic Focusing Structure

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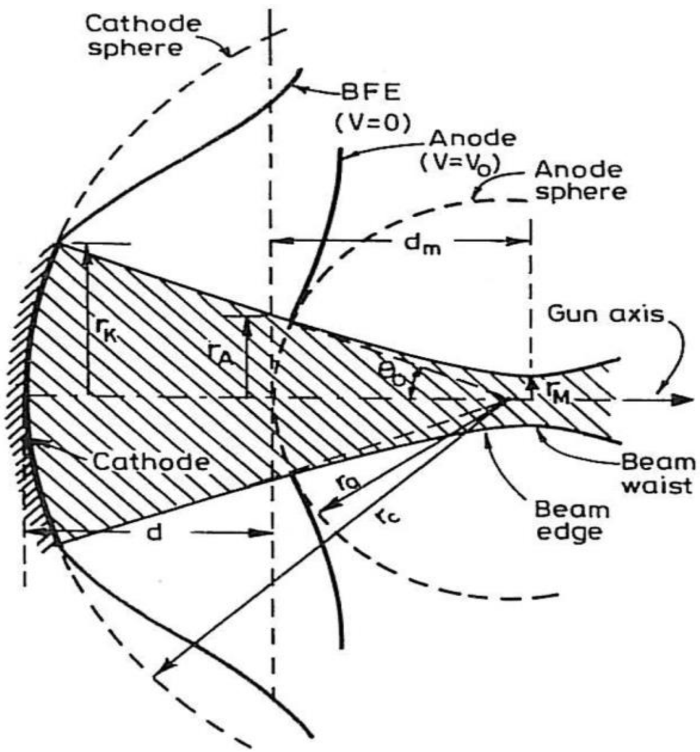
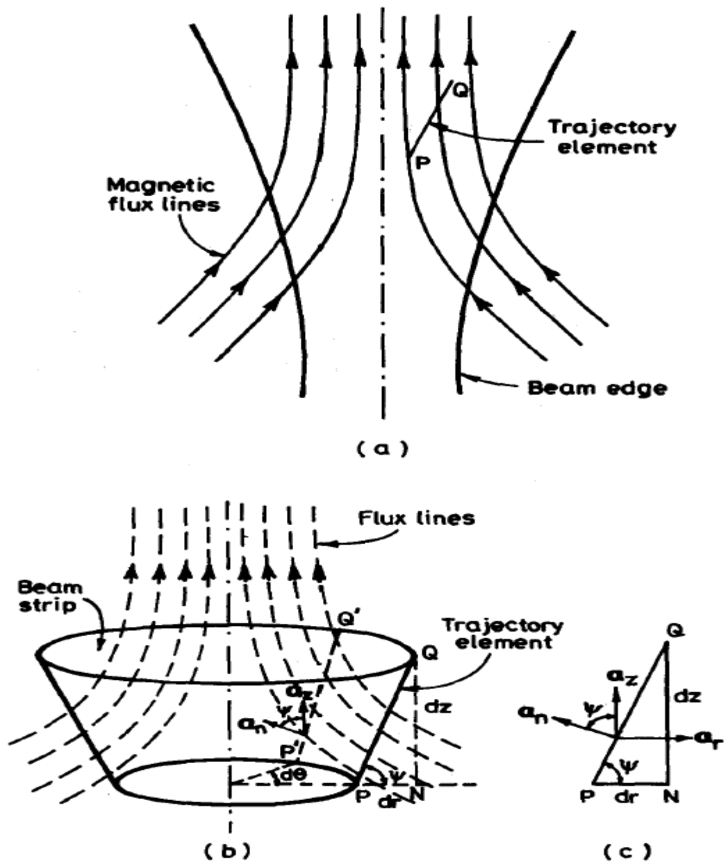
The electrons beyond the beam-waist of the gun if left to them would diverge out due to Coulomb repulsive force between them. The function of the magnetic focussing structure is to provide the Lorentz force to counteract this Coulomb repulsive force thereby constraining the electron beam to move parallel to the beam axis beyond the beam-waist of the gun and transmit the beam into the interaction region of a linear-beam microwave tube (such as the TWT).





Cross-section of the beam through its axis showing the magnetic flux lines cutting through the beam edge which down the axis become parallel to the latter, and also showing an element of electron trajectory PQ inside the beam (a); an element of area dS (= PQQ′P′) generated by rotating the element of electron trajectory PQ through an infinitesimal angle $d\theta$, for the estimate of magnetic flux through such an area element, and the unit vector $\boldsymbol{a}_n$ inwardly normal to the area element and the axial unit vector $\boldsymbol{a}_x$, (b) and the unit vectors $\boldsymbol{a}_n$ and $\boldsymbol{a}_z$ drawn on the element of trajectory (c).

Though the magnetic flux density of the focussing structure is predominantly axial yet it has adequate radial component due to which the axially moving electrons of the beam will experience Lorentz force to have an azimuthal velocity component. Consequently, the interaction between the azimuthal component of electron velocity and the axial component of magnetic flux density provided by the structure would give rise to the required radial Lorentz force to counter-balance the space-charge force plus the centrifugal force of the circular electronic motion.

$$\frac{1}{r}\frac{d}{dt}\left(\frac{r^2 d\theta}{dt}\right) = \eta(v \times B)_\theta$$ **(angular acceleration)**

$$\frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \eta(E_r + (v \times B)_r)$$ **(radial acceleration)**

$\eta(= e/m)$: **charge-to-mass ratio of an electron**

E: **electric field**

B: **magnetic flux density**

$$\frac{1}{r}\frac{d}{dt}\left(\frac{r^2 d\theta}{dt}\right) = \eta(v \times B)_\theta$$ **(rewritten)**

$$\downarrow$$

$$d\left(r^2 \frac{d\theta}{dt}\right) = \eta r (B_r v_z - B_z v_r) dt$$

Element $d\phi_B$ of magnetic flux ϕ_B through an element of beam strip generated by making a complete revolution of the element of an electron trajectory in the beam

$$d\phi_B = -2\pi r (B_r dz - B_z dr)$$

$$\downarrow$$

$$d\left(\frac{r^2 d\theta}{dt}\right) = -\frac{\eta}{2\pi} d\phi_B$$

$$\left.\begin{aligned} \frac{d\theta}{dt} &= 0 \\ \phi_B &= \phi_{Bk} \end{aligned}\right\}$$

(condition at the cathode: zero electron velocity, to find the integration constant, the subscript k referring to the cathode)

Integrating

$$\downarrow$$

$$\frac{r^2 d\theta}{dt} = -\frac{\eta\phi_B}{2\pi} + \frac{\eta\phi_{Bk}}{2\pi}$$

Second term in the right hand side is the integration constant.

$\phi_B = \pi r^2 B$: **magnetic flux treating the magnetic flux density as predominantly axial and perpendicular to the circular cross section of the portion of the beam of radius r, that is, of area πr^2**

$\phi_{Bk} = \pi r_k^2 B_k$: **magnetic flux at the cathode, r_k being the beam radius at the cathode and B_k being the magnetic flux density at the cathode**

$$\frac{r^2 d\theta}{dt} = -\frac{\eta \phi_B}{2\pi} + \frac{\eta \phi_{Bk}}{2\pi}$$ **(recalled)**

$$\frac{d\theta}{dt} = -\frac{\eta B}{2}\left(1 - \left(\frac{B_k}{B}\right)\left(\frac{r_k}{r}\right)^2\right)$$

Angular velocity of a beam electron, in general, depends on its radial position.

$$\omega_c = -\eta B = |\eta| B$$

$$\boxed{\frac{d\theta}{dt} = \frac{\omega_c}{2}\left(1 - \left(\frac{B_k}{B}\right)\left(\frac{r_k}{r}\right)^2\right)}$$ **(*Busch's theorem*)**

$$\boxed{\left.\begin{array}{l} B_k = 0 \\ \dfrac{d\theta}{dt} = \dfrac{\omega_c}{2} \end{array}\right\}}$$ **(independent of r)**

In the absence of magnetic flux linking with the cathode (under *Brillouin condition*), all beam electrons rotate with the same velocity $\omega_c/2$ independent of their radii as if the beam is a rotating rigid bar.

Brillouin Focusing

Equation of motion for beam-edge electrons

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \eta(E_r + (v\times B)_r)$$ **(radial acceleration)**

$$E_r = \rho r/(2\varepsilon_0)$$ **(easily obtainable using Gauss's law)**

$$(v\times B)_r = v_\theta B$$

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \eta(\rho r/(2\varepsilon_0) + (v\times B)_r)$$

$$\frac{d^2r}{dt^2} = \eta\left(\frac{\rho r}{2\varepsilon_0} + v_\theta B\right) + r\left(\frac{d\theta}{dt}\right)^2$$ **(beam-edge electrons)**

$$\frac{d^2r}{dt^2} = \eta\left(\frac{\rho r}{2\varepsilon_0} + v_\theta B\right) + r\left(\frac{d\theta}{dt}\right)^2$$

(rewritten) (beam-edge electrons)

$$\left.\begin{aligned} v_\theta &= r\,d\theta/dt \\ d\theta/dt &= \omega_c/2 \\ \omega_c &= -\eta B = |\eta| B \\ J &= \rho v \\ J &= -I_0/(\pi r^2) \end{aligned}\right\}$$

(under Brillouin condition)

$B_k = 0$ (implied)

$$\frac{d^2r}{dt^2} = \left(\frac{|\eta| I_0}{2\pi\varepsilon_0 v}\right)\frac{1}{r} - \left(\frac{|\eta|^2 B^2}{4}\right) r$$

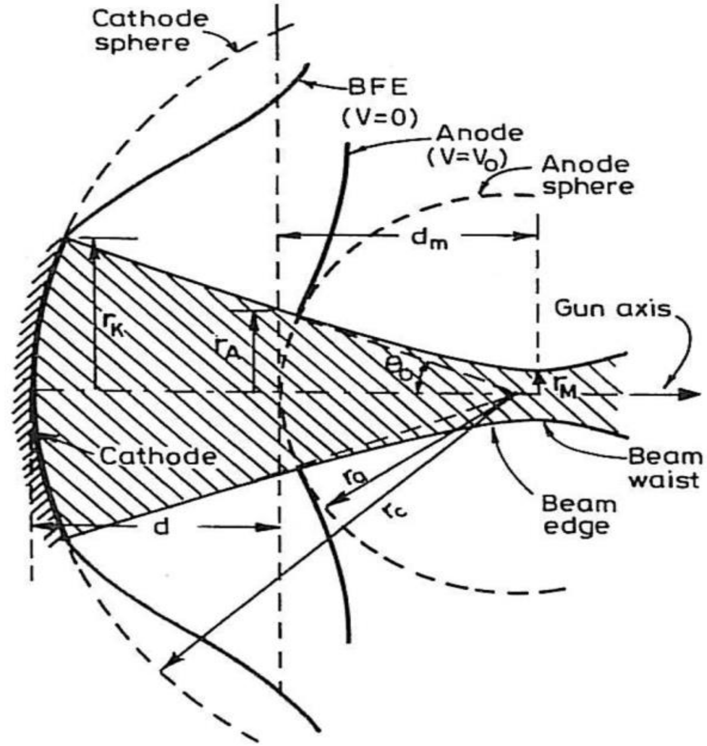
$$\left.\begin{aligned} v &= \frac{dz}{dt} \\ \frac{dr}{dt} &= \frac{dr}{dz}\frac{dz}{dt} = v\frac{dr}{dz} \\ \frac{d^2r}{dt^2} &= v\frac{d^2r}{dz^2}\frac{dz}{dt} = v^2\frac{d^2r}{dz^2} \end{aligned}\right\}$$

$$v^2\frac{d^2r}{dz^2} = \left(\frac{|\eta| I_0}{2\pi\varepsilon_0 v}\right)\frac{1}{r} - \left(\frac{|\eta|^2 B^2}{4}\right) r$$

(beam-edge electrons)

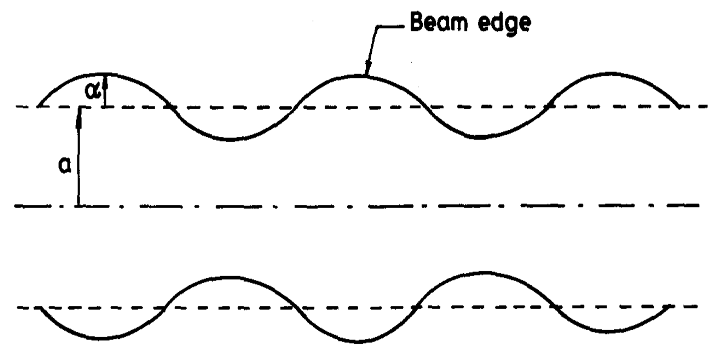
$$v^2 \frac{d^2 r}{dz^2} = \left(\frac{|\eta| I_0}{2\pi\varepsilon_0 v} \right) \frac{1}{r} - \left(\frac{|\eta|^2 B^2}{4} \right) r$$ **(beam-edge electrons)**

$$\left. \begin{aligned} v &= (2\eta V_0)^{1/2} \\ C_1 &= \frac{|\eta| I_0}{2\pi\varepsilon_0 (2|\eta| V_0)^{3/2}} \\ C_2 &= \frac{|\eta|^2 B^2}{4(2|\eta| V_0)} \end{aligned} \right\}$$



$$\frac{d^2 r}{dz^2} = \frac{C_1}{r} - C_2 r$$ **(beam-edge electrons)**

$$\frac{d^2 r}{dz^2} = \frac{C_1}{r} - C_2 r$$ **(beam-edge electrons) (rewritten)**



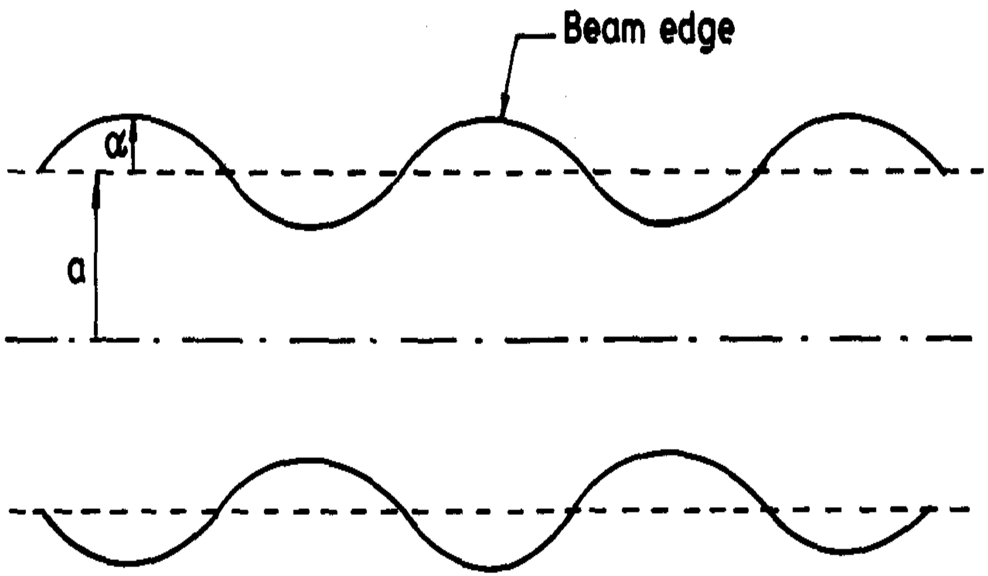
$r = a + \delta;\ \delta << a$ **(beam radius beyond the beam-waist of radius which desired to be held at a constant value $a = r_M$).**

$$\frac{d^2 \delta}{dz^2} = \frac{C_1}{a+\delta} - C_2(a+\delta) = \frac{C_1}{a}\left(1+\frac{\delta}{a}\right)^{-1} - C_2 a - C_2 \delta$$

Expanding binomially and ignoring higher powers of δ/a

$$\frac{d^2 \delta}{dz^2} = \frac{C_1}{a}\left(1-\frac{\delta}{a}\right) - C_2 a - C_2 \delta = \left(\frac{C_1}{a} - C_2 a\right) - \left(\frac{C_1}{a^2} + C_2\right)\delta$$

Beam-scalloping: sinusoidal variation of the beam-edge radius along the structure if the cathode is magnetically shielded $(B_k = 0)$ and the beam radius at the entry plane $(z = 0)$ has the value that is desired to be maintained constant at $r = r_M = a$. Here, α is the maximum value of the radius variation δ.



Beam-scalloping: sinusoidal variation of the beam-edge radius along the structure if the cathode is magnetically shielded (B_k = 0) and the beam radius at the entry plane (z = 0) is the value that is desired to be maintained constant at $r = r_M = a$. Here, α is the maximum value of the radius variation δ.

$$\frac{d^2\delta}{dz^2} = \frac{C_1}{a}\left(1-\frac{\delta}{a}\right) - C_2 a - C_2\delta = \left(\frac{C_1}{a} - C_2 a\right) - \left(\frac{C_1}{a^2} + C_2\right)\delta$$ **(rewritten)**

$$C_1 = \frac{|\eta| I_0}{2\pi\varepsilon_0 (2|\eta| V_0)^{3/2}}$$

$$C_2 = \frac{|\eta|^2 B^2}{4(2|\eta| V_0)}$$

Choosing to put $\frac{C_1}{a} - C_2 a = 0$

$$\frac{d^2\delta}{dz^2} = -\left(\frac{C_1}{a^2} + C_2\right)\delta = -2C_2\delta$$

$$\boxed{B = B_B = \left(\frac{\sqrt{2} I_0}{\pi\varepsilon_0 |\eta|^{3/2} V_0^{1/2} a^2}\right)^{1/2}}$$

$$m^2 = 2C_2$$

$$\frac{d^2\delta}{dz^2} = -m^2\delta$$

Condition imposed as the required magnetic flux density — one of the Brillouin conditions, the subscript B referring to the Brillouin magnetic field)

$$\delta = A\sin mz + B\cos mz$$ **(with A and B as constants)**

$\delta = A\sin mz + B\cos mz$ **(rewritten)**

$r = a + \delta = a + A\sin mz + B\cos mz$

The radius of the beam edge changes periodically or, in other words, the beam scallops if either or both of A or B have non-zero values.

For no beam scalloping one has to have $A = B = 0$.

$\delta = A\sin mz + B\cos mz$ **(rewritten)**

Impose the condition $\delta = 0$ at $z = 0$ implying that the beam enters the magnetic field region at the beam-waist with beam-edge radius equal to $r = a = r_M$.

$B = 0$

$\delta = A\sin mz + B\cos mz = A\sin mz$

$d\delta / dz = mA\cos mz = mA = 0$

Impose the condition $d\delta/dz = 0$ at $z = 0$ implying that there should be no radial component of beam velocity at the entry of the focussing structure: $r = a = r_M$.

$A = 0$

Impose the conditions $\delta = 0$ and $d\delta/dz = 0$ both at $z = 0$ — the entry of the focussing structure where the beam-wedge radius: $r = a = r_M$.

$\downarrow$

$$A = B = 0$$

$\downarrow$

$$\delta = A\sin mz + B\cos mz = 0$$

$\downarrow$

No beam scalloping

Brillouin conditions for no beam-scalloping

$\Downarrow$

(i) The cathode is magnetically shielded: $B_k = 0$

(ii)

$$\boxed{B = B_B = \left(\frac{\sqrt{2} I_0}{\pi \varepsilon_0 |\eta|^{3/2} V_0^{1/2} a^2} \right)^{1/2}}$$

(iii) $\delta = 0$ **and at** $z = 0$ **implying that the beam enters the magnetic field region at the beam- waist with beam-edge radius equal to** $r = a = r_M$.

(iv) $d\delta/dz = 0$ **at** $z = 0$ **implying that there should be no radial component of beam velocity at the entry of the focussing structure**: $r = a = r_M$.

$$B = B_B = \left(\frac{\sqrt{2} I_0}{\pi \varepsilon_0 |\eta|^{3/2} V_0^{1/2} a^2} \right)^{1/2}$$ **(Brillouin magnetic flux density)**

$$\downarrow$$

$$B_B \propto \left(\frac{I_0}{V_0^{1/2} a^2} \right)^{1/2} \longrightarrow$$

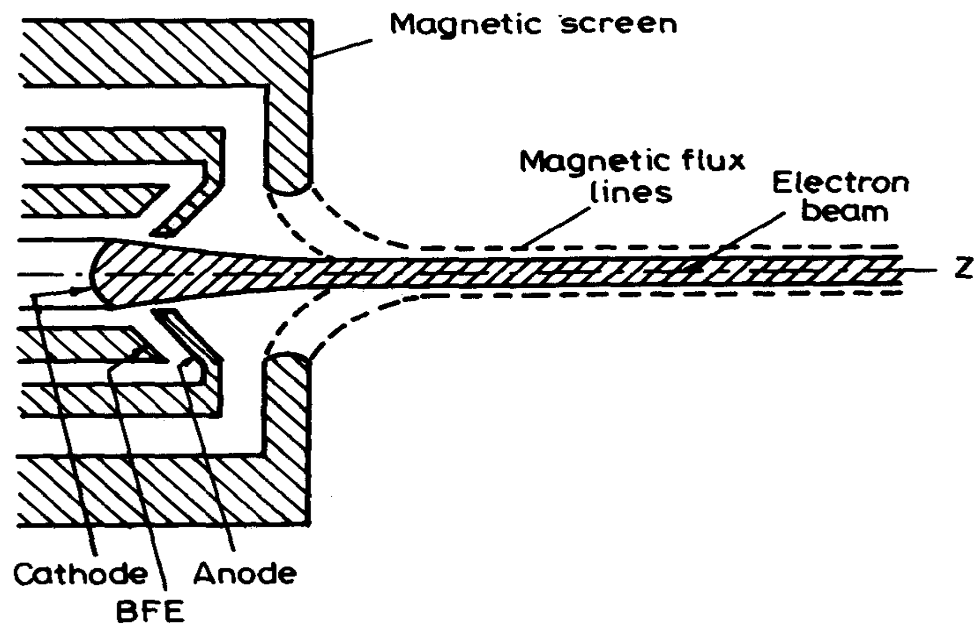
Larger focusing magnetic flux density is required to confine an electron beam of higher beam current / lower beam voltage / higher beam perveance ($perv = I_0 / V_0^{3/2}$)**.**

$$\downarrow$$

$$\left. \begin{aligned} B_B &\propto I_0^{1/2} \\ B_B &\propto \frac{1}{V_0^{1/4}} \\ B_B &\propto \frac{1}{a} \end{aligned} \right\} \longrightarrow$$

The magnetic flux density needs to be doubled if the beam current is increased 4 times / the beam voltage is decreased 16 times / the beam radius is halved.

In Brillouin focusing, the cathode is shielded from magnetic field by employing a screen in the form of a pole piece made of a magnetic material, with a hole through which the electron beam can pass in order to realize Brillouin conditions.



Schematic representation of Brillouin focusing showing a magnetically shielded cathode

Some limitations of Brillouin focusing:

1. In order to realize the Brillouin conditions, the cathode has to be magnetically shielded; the precise Brillouin value of magnetic flux density related to the beam voltage, beam current and beam radius has to be attained abruptly at the entry of the focusing structure; and the beam waist-radius has to be precisely set to the beam radius desired to be maintained constant.

2. Uniform cathode emission has been assumed and thermal velocity effects have been ignored. As a result, the predicted Brillouin magnetic field would be lower than the actual in practice.

3. Brillouin conditions are prone to being offset by the unpredictable space-charge forces due to the formation of positive ions in the tube.

4. Brillouin conditions imply DC conditions. Thus, the beam current has been assumed to be constant. However, it changes due to local RF bunching and more so in the large-signal regime. Consequently, this is likely to change the beam radius and can cause beam scalloping.

Confined-Flow Focusing

Confined-flow focusing

- **Makes the focusing conditions to be satisfied less stringent than Brillouin focusing conditions.**

- **Allows magnetic flux lines to thread into the cathode unlike in Brillouin focusing.**

- **Provides lesser sensitivity to beam current density modulation than Brillouin focusing.**

- **Requires, however, larger magnetic field for beam confinement than Brillouin focusing.**

In confined-flow focusing, the magnetic flux lines cutting through the beam-waist are considered to extend to the cathode and follow the beam trajectories.

$\downarrow$

$$\left.\begin{aligned} B_W \pi a^2 &= B_k \pi r_k{}^2 \\ B_k &= \frac{B_W}{(r_k/a)^2} \\ B_W &= B_k (r_k/a)^2 \end{aligned}\right\}$$

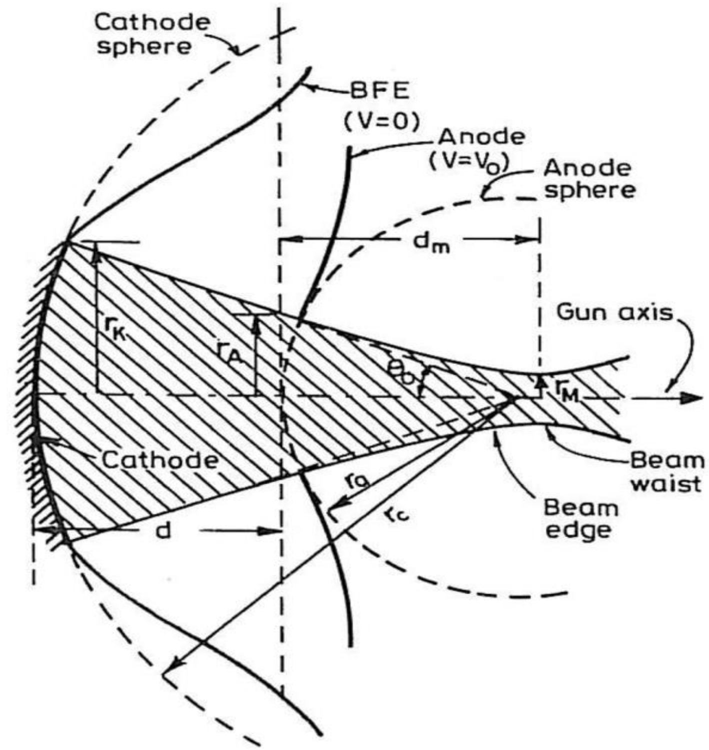
r_k : **Cathode-disc radius**

$a(=r_M)$: **Beam radius to confine = Beam-waist radius**

B_k : **Magnetic flux density at the cathode**

B_W : **Magnetic flux density at the beam-waist**

$$\frac{d^2 r}{dz^2} = \frac{C_1}{r} - C_2 r$$ **(beam-edge electrons)**



$$\frac{d^2r}{dt^2}=\eta\left(\frac{\rho r}{2\varepsilon_0}+v_\theta B\right)+r\left(\frac{d\theta}{dt}\right)^2 (B\neq B_B) \longleftarrow \frac{d\theta}{dt}=-\frac{\eta B}{2}\left(1-\left(\frac{B_k}{B}\right)\left(\frac{r_k}{r}\right)^2\right)(B\neq B_B)$$

(recalled)

$$\longleftarrow \left.\begin{aligned} v_\theta &= r d\theta/dt \\ \omega_c &= -\eta B = |\eta| B \\ J &= \rho v \\ J &= -I_0/(\pi r^2) \end{aligned}\right\}(B_k\neq 0)$$

$$\frac{d^2r}{dt^2}=\left(\frac{|\eta| I_0}{2\pi\varepsilon_0 v}\right)\frac{1}{r}-\frac{|\eta|^2}{4}\left(B^2-B_k{}^2\frac{r_k^4}{r^4}\right)r$$

$$(B_k\neq 0)$$

$$\frac{d^2r}{dt^2}=\left(\frac{|\eta| I_0}{2\pi\varepsilon_0 v}\right)\frac{1}{r}-\left(\frac{|\eta|^2 B^2}{4}\right)r \quad (B_k=0)$$

$$B=B_B \longrightarrow$$

$$\frac{d^2r}{dt^2}=\left(\frac{|\eta| I_0}{2\pi\varepsilon_0 v}\right)\frac{1}{r}-\left(\frac{|\eta|^2 B_B^2}{4}\right)r$$

$$(B_k=0)$$

$$\frac{d^2r}{dt^2} = \left(\frac{|\eta| I_0}{2\pi\varepsilon_0 v}\right)\frac{1}{r} - \frac{|\eta|^2}{4}\left(B^2 - B_k{}^2\frac{r_k^4}{r^4}\right)r$$

$(B_k \neq 0)$ **(rewritten)**

$$\frac{d^2r}{dt^2} = \left(\frac{|\eta| I_0}{2\pi\varepsilon_0 v}\right)\frac{1}{r} - \left(\frac{|\eta|^2 B_B^2}{4}\right)r$$

$(B_k = 0)$ **(rewritten)**

Comparing

$$B^2 = B_B^2 + B_k{}^2\frac{r_k^4}{a^4}$$

$$B_W = B_k (r_k / a)^2$$ **(recalled)**

$$\left.\begin{aligned} &p = B / B_B \\ &B = pB_B;\ B_W = (1 - 1/p^2)^{1/2} B = (p^2 - 1)^{1/2} B_B \\ &B_k = (1 - 1/p^2)^{1/2} (a / r_k)^2 B = (p^2 - 1)^{1/2} (a / r_k)^2 B_B \end{aligned}\right\}$$

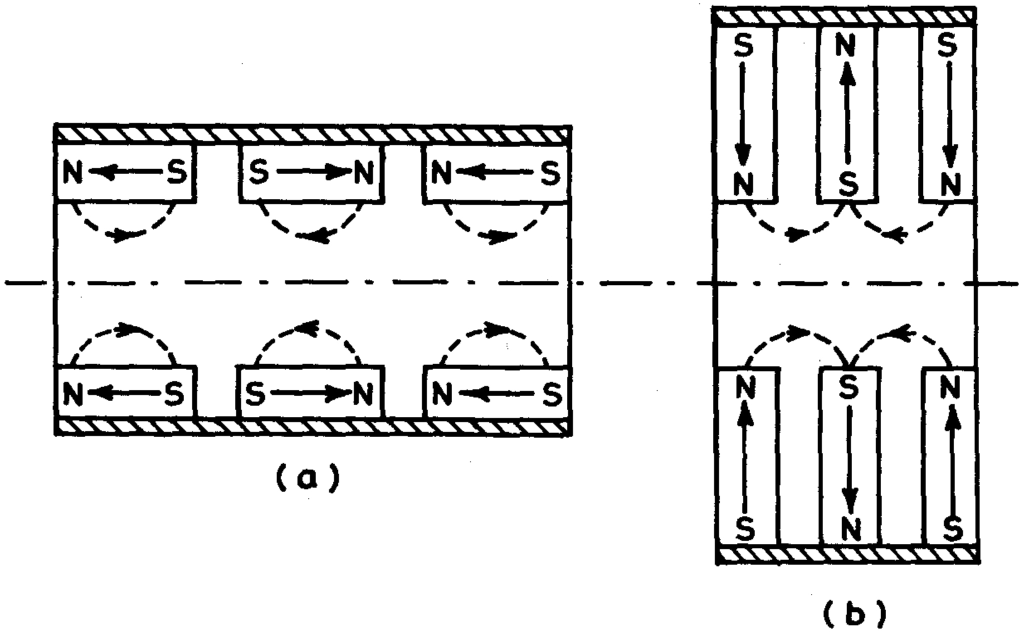
$$\left.\begin{aligned} &p = B/B_B \\ &B = pB_B;\; B_W = (1-1/p^2)^{1/2}B = (p^2-1)^{1/2}B_B \\ &B_k = (1-1/p^2)^{1/2}(a/r_k)^2 B = (p^2-1)^{1/2}(a/r_k)^2 B_B \end{aligned}\right\} \textbf{(rewritten)}$$

$$\left.\begin{aligned} &p = B/B_B = 1.5 \\ &(\pi r_k^{\,2}/\pi a^2 (= (r_k/a)^2) = 50 \end{aligned}\right\} \textbf{(typical)}$$

$$\left.\begin{aligned} &B_W \approx 1.1B_B \\ &B_k \approx 0.02B_B \end{aligned}\right\}$$

The confined-flow focussing requires (i) a magnetic flux density at the beam-waist of the convergent gun, that is, at the entrance of the magnetic focussing structure greater — though not much greater — than the corresponding Brillouin value and (ii) a very small magnetic flux density relative to the Brillouin value threading into the cathode.

Periodic Permanent Magnet (PPM) Focusing

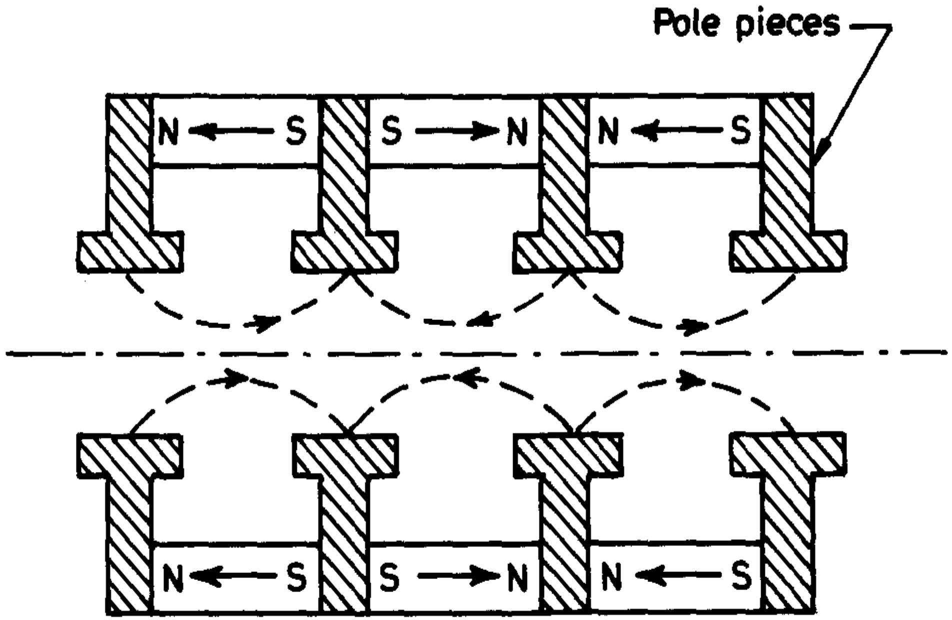


Ring-magnet cells (shown typically, three, in number) in a PPM for the axial (a) and radial magnetization (b) showing magnetic flux lines (dotted) inside the stricture.

A solenoid focussing structure becomes rather heavy; moreover, it also needs an external power supply to make it further heavier.

The permanent magnet (PM) focussing structure is suitable for a microwave tube requiring a small interaction length such as the klystron. However, if the length of a PM is increased by a factor of N, say, then all other dimensions of the structure have also to be increased in the same proportion to ensure that the magnetic flux density in the interaction region does not fall, requiring an increase in the inner and outer radii, for instance, of a tubular PM each by the same factor N. This will amount to increasing the volume and hence the weight of the PM to N^3 times — attributable to the increase in magnetic field and magnetic energy stored outside the magnet which does not help in focussing the electron beam.

In a periodic permanent magnet (PPM) focussing structure, instead of a single PM of its length increased by a factor of N, an array of N identical magnet cells are used. The length of such a structure can be increased N times without requiring to increase its transverse dimensions as is necessary in a PM. Thus one gets the advantages of a PPM over its PM counterpart in terms of weight by a factor of $N^3/N = N^2$. In such a PPM structure, the magnetic flux lines external to the magnet due to consecutive cells are directed oppositely causing a reduction in magnetic field and consequently a reduction in the loss of magnetic energy outside the magnet.



A typical arrangement of magnetic pole pieces between magnets to bring magnetic flux density close to the axis of an axially magnetized PPM.

The advent of light-weight magnetic materials such as samarium-cobalt ($SmCo_5$ and Sm_2Co_{17}) and ALNICO-5 has made it possible to suitable design light-weight PPM.

Let us approximate the magnetic flux density in a PPM structure, supposedly varying periodically along the axial distance z while remaining uniform over its cross section, as:

$$B = B_0 \cos\frac{2\pi z}{L} = B_0\left(\frac{1+\cos(4\pi z/L)}{2}\right)^{1/2}$$

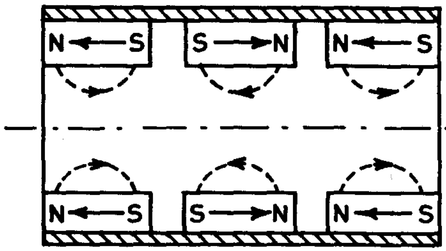
B_0 : **peak magnetic flux density**

L : **axial periodicity of variation in** B

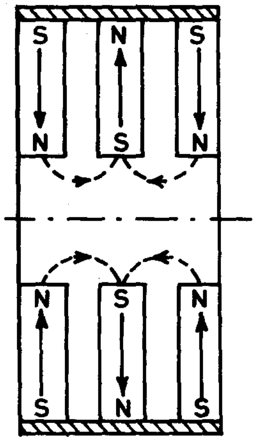
$L/2$: **distance between two consecutive oppositely directed maxima**

$$\frac{d^2r}{dz^2} = \frac{C_1}{r} - C_2 r \longleftarrow \left.\begin{aligned} C_1 &= \frac{|\eta| I_0}{2\pi\varepsilon_0 (2|\eta| V_0)^{3/2}} \\ C_2 &= \frac{|\eta|^2 B^2}{4(2|\eta| V_0)} \end{aligned}\right\}$$

(recalled)



(a)



(b)

$$B = B_0 \cos\frac{2\pi z}{L} = B_0\left(\frac{1+\cos(4\pi z/L)}{2}\right)^{1/2}$$ **(rewritten)**

$$\downarrow$$

$$\frac{d^2 r}{dz^2} = \frac{C_1}{r} - C_2 r \quad \longleftarrow \quad \left.\begin{aligned} C_1 &= \frac{|\eta| I_0}{2\pi\varepsilon_0 (2|\eta| V_0)^{3/2}} \\ C_2 &= \frac{|\eta|^2 B^2}{4(2|\eta| V_0)} \end{aligned}\right\}$$ **(rewritten)**

(rewritten)

$$\longleftarrow \quad r = a + \delta;\ \delta \ll a$$

$$\longleftarrow \quad \left.\begin{aligned} Z &= 2\pi z / L \\ \sigma &= r/a \\ \alpha &= \frac{|\eta| L^2 B_0^2}{64\pi^2 V_0} \\ \beta &= \frac{|\eta| L^2 I_0}{8\pi^3 a^2 \varepsilon_0 (2|\eta| V_0)^{3/2}} = \frac{|\eta| L^2 (Perv)}{8\pi^3 a^2 \varepsilon_0 (2|\eta|)^{3/2}} \end{aligned}\right\}$$

$$\downarrow$$

$$\frac{d^2\sigma}{dZ^2} + \alpha(1+\cos 2Z)\sigma - \frac{\beta}{\sigma} = 0$$

$$\frac{d^2\sigma}{dZ^2}+\alpha(1+\cos 2Z)\sigma-\frac{\beta}{\sigma}=0$$ **(rewritten)**

$$\beta=\frac{|\eta|L^2 I_0}{8\pi^3 a^2\varepsilon_0(2|\eta|V_0)^{3/2}}=\frac{|\eta|L^2(Perv)}{8\pi^3 a^2\varepsilon_0(2|\eta|)^{3/2}}=0$$

For smaller values of beam current / beam perveance

$$\frac{d^2\sigma}{dZ^2}+\alpha(1+\cos 2Z)\sigma=0$$

$$\alpha=\frac{|\eta|L^2 B_0^2}{64\pi^2 V_0}$$

Has alternating pass and stop bands of solution on the scale of α that is proportional B_0^2 :

$$0\le\alpha\le 0.66 \text{ (first pass band)}$$

$$0.66\le\alpha\le 1.72 \text{ (first stop band)}$$

$$1.72\le\alpha\le 3.76 \text{ (second pass band)}$$

$$3.76\le\alpha\le 6.1 \text{ (second stop band)}$$

and so on.

$$0 \le \alpha \le 0.66 \text{ (first pass band)}$$

$$0.66 \le \alpha \le 1.72 \text{ (first stop band)}$$

$$1.72 \le \alpha \le 3.76 \text{ (second pass band)}$$

$$3.76 \le \alpha \le 6.1 \text{ (second stop band)}$$

and so on.

The stop bands are wider than the pass bands and the widths of both the pass and sop bands increase with the order of the band.

$$\frac{d^2\sigma}{dZ^2} + \alpha(1+\cos 2Z)\sigma = 0$$

$$\left.\begin{aligned} \sigma &= r/a \\ Z &= 2\pi z/L \end{aligned}\right\}$$

(recalled)

In a pass band, the solution for the normalized beam radius σ is stable and periodic with the normalized axial distance Z, the beam radius ripples depending on the value of α relative to that of β.

$$\left.\begin{aligned} \alpha &= \frac{|\eta| L^2 B_0^2}{64\pi^2 V_0} \\ \beta &= \frac{|\eta| L^2 I_0}{8\pi^3 a^2 \varepsilon_0 (2|\eta| V_0)^{3/2}} \end{aligned}\right\}$$

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This finding arrived at for vanishingly small beam currents (β=0) continues to be valid even for higher beam currents and the detailed computational study shows that the minimum beam ripples result if the condition $\alpha \approx \beta$ is satisfied.

$$\alpha = \beta$$

$$B_{\mathrm{rms}} = \frac{B_0}{\sqrt{2}} = \left(\frac{2\sqrt{2 I_0}}{\pi \varepsilon_0 |\eta|^{3/2} V_0^{1/2} a^2} \right)^{1/2} = \left(\frac{\sqrt{2} I_0}{\pi \varepsilon_0 |\eta|^{3/2} V_0^{1/2} a^2} \right)^{1/2} = B_B$$

(root mean square value of the PPM magnetic flux density)

$$L < \left(\frac{0.66 \times 64}{|\eta|} \right)^{1/2} \frac{\pi V_0^{1/2}}{B_0} \longleftarrow \left.\begin{aligned} &\alpha < 0.66 \text{ (first pass band)} \\ &\alpha = \frac{|\eta| L^2 B_0^2}{64\pi^2 V_0} \end{aligned}\right\}$$

(axial periodicity of PPM to be chosen for the first pass band)