CHU'S POWER CONSERVATION CONCEPTS

B.N. Basu
<bnbasu.india@gmail.com>

♣L. J. Chu, "A kinetic power theorem", paper presented at the IRE-PGED Electron Tube Research Conference, Durham, New Hampshire, June, 1951.

♣H. A. Haus and D. Bobroff, "Small-signal power theorem for electron beams", *J. Appl. Phys. 28* (1957), 694-703.

♣ J. W. Kliiver, "Small-signal power conservation theorem for irrotational electron beams", *J. Appl. Phys. 29 (1958), 618-622.*

Space-charge waves

Prerequisite

$J = \rho v$ (Current density equation)

$$J = J_0 + J_1$$

$$\rho = \rho_0 + \rho_1$$

$$v = v_0 + v_1$$

$\frac{\partial J_1}{\partial z} + \frac{\partial \rho_1}{\partial t} = 0$ (Continuity equation)

$\frac{dv_1}{dt} = \frac{e}{m} E_s = \eta E_s$ (Force equation)

$\frac{\partial E_s}{\partial z} = \frac{\rho_1}{\varepsilon_0}$ (Poisson's equation)

$J = \rho v$ (one-dimensional) $\longleftarrow$ $\vec{J} = \rho\vec{v}$ (current density equation)

$\downarrow$ $\longleftarrow$ $J = J_0 + J_1, \rho = \rho_0 + \rho_1, v = v_0 + v_1$

$$J_0 + J_1 = (\rho_0 + \rho_1)(v_0 + v_1) = \rho_0 v_0 + \rho_0 v_1 + \rho_1 v_0 + \rho_1 v_1 \quad \longleftarrow \quad J_0 = \rho_0 v_0$$

$J_1 = \rho_0 v_1 + v_0 \rho_1 + \rho_1 v_1 = \rho_0 v_1 + v_0 \rho_1$ (small-signal approximation)

$$\frac{\partial J_1}{\partial z} = \rho_0 \frac{\partial v_1}{\partial z} + v_0 \frac{\partial \rho_1}{\partial z} \quad \longleftarrow \quad \frac{\partial J_1}{\partial z} + \frac{\partial \rho_1}{\partial t} = 0 \quad \longleftarrow \quad \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

(continuity equation)

$$-\frac{\partial \rho_1}{\partial t} = \rho_0 \frac{\partial v_1}{\partial z} + v_0 \frac{\partial \rho_1}{\partial z}$$

$$\frac{\partial \rho_1}{\partial t} + v_0 \frac{\partial \rho_1}{\partial z} = -\rho_0 \frac{\partial v_1}{\partial z} \quad \longleftarrow \quad D = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}$$

$$D\rho_1 = -\rho_0 \frac{\partial v_1}{\partial z}$$

Force equation

$$m\frac{dv_1}{dt} = eE_s \qquad v = v_0 + v_1$$

$$\frac{dv_1}{dt} = \frac{e}{m}E_s = \eta E_s$$

$$\frac{dv_1}{dt} = \frac{\partial v_1}{\partial t} + \frac{dz}{dt}\frac{\partial v_1}{\partial z} = \frac{\partial v_1}{\partial t} + v\frac{\partial v_1}{\partial z} = \frac{\partial v_1}{\partial t} + (v_0 + v_1)\frac{\partial v_1}{\partial z} = \frac{\partial v_1}{\partial t} + v_0\frac{\partial v_1}{\partial z}$$

$$\frac{\partial v_1}{\partial t} + v_0\frac{\partial v_1}{\partial z} = \eta E_s \longleftarrow D = \frac{\partial}{\partial t} + v_0\frac{\partial}{\partial z}$$

$$Dv_1 = \eta E_s$$

$$D\rho_1 = -\rho_0 \frac{\partial v_1}{\partial z}$$ (obtained earlier) $$[\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} = D]$$

$$Dv_1 = \eta E_s$$ (obtained earlier)

$$D^2\rho_1 = -\rho_0 D \frac{\partial v_1}{\partial z} = -\rho_0 \frac{\partial}{\partial z} Dv_1 = -\rho_0 \frac{\partial}{\partial z} \eta E_s = -\eta\rho_0 \frac{\partial E_s}{\partial z}$$

$$D^2\rho_1 = -\eta\rho_0 \frac{\partial E_s}{\partial z} \longleftarrow \frac{\partial E_s}{\partial z} = \frac{\rho_1}{\varepsilon_0}$$

$$\omega_p = \sqrt{\frac{|\eta||\rho_0|}{\varepsilon_0}}$$

$$D^2\rho_1 = -\eta\rho_0 \frac{\rho_1}{\varepsilon_0} = \frac{-\eta\rho_0}{\varepsilon_0}\rho_1 = \frac{-|\eta||\rho_0|}{\varepsilon_0}\rho_1 = -\omega_p{}^2\rho_1$$

$$D^2 = -\omega_p{}^2$$

$$D = \pm j\omega_p$$

RF quantities vary as $\exp j(\omega t - \beta z)$

$$\frac{\partial}{\partial t} = j\omega, \quad \frac{\partial}{\partial z} = -j\beta$$

$$D = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} = j\omega - j\beta v_0 = j(\omega - \beta v_0)$$

$D = \pm j\omega_p$ (recalled)

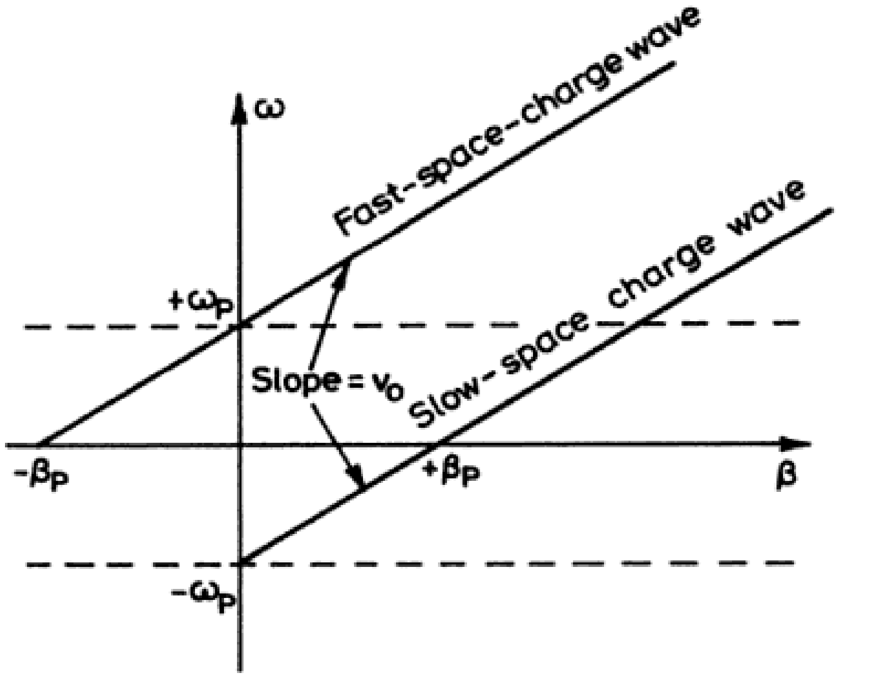
$$\downarrow$$

$$\pm j\omega_p = j(\omega - \beta v_0)$$

$$\downarrow$$

$$\boxed{\omega - \beta v_0 = \pm\omega_p}$$

Dispersion relation of Hahn and Ramo space-charge waves



$$\omega - \beta v_0 = \pm\omega_p \text{ (recalled)} \longrightarrow \beta = \frac{\omega \mp \omega_p}{v_0}$$

$$\frac{\omega}{\beta} = \frac{\omega}{\omega \mp \omega_p} v_0$$

Fast space-charge wave

$$\left(\frac{\omega}{\beta}\right)_f = \frac{\omega}{\omega - \omega_p} v_0 \quad \text{(phase velocity)}$$

$$\beta_f = \frac{\omega - \omega_p}{v_0} \quad \text{(phase propagation constant)}$$

Slow space-charge wave

$$\left(\frac{\omega}{\beta}\right)_s = \frac{\omega}{\omega + \omega_p} v_0 \quad \text{(phase velocity)}$$

$$\beta_s = \frac{\omega + \omega_p}{v_0} \quad \text{(phase propagation constant)}$$

$$J_1 = \rho_0 v_1 + v_0 \rho_1$$ (recalled)

$$\frac{\partial J_1}{\partial z} + \frac{\partial \rho_1}{\partial t} = 0$$ (continuity equation recalled)

RF quantities vary as $\exp j(\omega t - \beta z)$

$$-j\beta J_1 + j\omega\rho_1 = 0$$

$$\rho_1 = \frac{\beta J_1}{\omega}$$

$$\beta_f = \frac{\omega - \omega_p}{v_0}$$

$$\beta_s = \frac{\omega + \omega_p}{v_0}$$

$$v_{1f} = \frac{\omega - v_0\beta_f}{\omega\rho_0} J_{1f}$$

$$v_{1s} = \frac{\omega - v_0\beta_s}{\omega\rho_0} J_{1s}$$

$$v_{1f} = \frac{\omega_p}{\omega\rho_0} J_{1f}$$

$$v_{1s} = \frac{-\omega_p}{\omega\rho_0} J_{1f}$$

Chu's power conservation concepts

$$\boldsymbol{J_1} = \rho_0 \boldsymbol{v_1} + \boldsymbol{v_0}\rho_1$$

$$j\omega \boldsymbol{J_1} = j\omega\rho_0 \boldsymbol{v_1} + j\omega \boldsymbol{v_0}\rho_1$$

$$j\omega\rho_1 \boldsymbol{v_0} = j\omega \boldsymbol{J_1} - j\omega\rho_0 \boldsymbol{v_1}$$

$$\nabla.\boldsymbol{J_1} = -j\omega\rho_1$$

Multiplying by $\boldsymbol{v_0}$

$$(\nabla.\boldsymbol{J_1})\boldsymbol{v_0} + j\omega\rho_1 \boldsymbol{v_0} = 0$$

$$(\nabla.\boldsymbol{J_1})\boldsymbol{v_0} + j\omega \boldsymbol{J_1} - j\omega\rho_0 \boldsymbol{v_1} = 0$$

$$(\nabla.\boldsymbol{J_1}^*)\boldsymbol{v_0} - j\omega \boldsymbol{J_1}^* + j\omega\rho_0 \boldsymbol{v_1}^* = 0$$

$$\boldsymbol{v_0}(\nabla.\boldsymbol{J_1}^*) = j\omega \boldsymbol{J_1}^* - j\omega\rho_0 \boldsymbol{v_1}^*$$

$$\boldsymbol{J_1} = J_1 \mathbf{a}_z$$

$$\boldsymbol{v_0} = v_0 \mathbf{a}_z$$

$$\boldsymbol{v_1} = v_1 \mathbf{a}_z$$

$$\boldsymbol{J_1} = \rho_0 \boldsymbol{v_1} + v_0 \rho_1$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v_0}\cdot\nabla\right)\boldsymbol{v_1} = \eta \boldsymbol{E}$$

$$j\omega \boldsymbol{v_1} + (\boldsymbol{v_0}\cdot\nabla)\boldsymbol{v_1} = \eta \boldsymbol{E}$$

$$\nabla.\boldsymbol{J_1} = -\frac{\partial\rho_1}{\partial t} = -j\omega\rho_1$$

$$j\omega \boldsymbol{v_1} + (\boldsymbol{v_0} \cdot \nabla)\boldsymbol{v_1} = \eta \boldsymbol{E}$$

$$\boldsymbol{E} = \frac{j\omega \boldsymbol{v_1} + (\boldsymbol{v_0} \cdot \nabla)\boldsymbol{v_1}}{\eta} = \frac{j\omega \boldsymbol{v_1} + (\boldsymbol{v_0} \cdot \nabla)\boldsymbol{v_1}}{-|\eta|}$$

$$\boldsymbol{J_1}^* \cdot \boldsymbol{E} = \boldsymbol{J_1}^* \cdot [\frac{j\omega \boldsymbol{v_1} + (\boldsymbol{v_0} \cdot \nabla)\boldsymbol{v_1}}{-|\eta|}]$$

$$\boldsymbol{J_1}^* \cdot \boldsymbol{E} = -\frac{j\omega}{|\eta|}\boldsymbol{J_1}^* \cdot \boldsymbol{v_1} - \frac{\boldsymbol{J_1}^* \cdot (\boldsymbol{v_0} \cdot \nabla)\boldsymbol{v_1}}{|\eta|}$$

$$\boldsymbol{J_1}^* \cdot \boldsymbol{E} = -\frac{\boldsymbol{v_1}}{|\eta|}.\boldsymbol{v_0}(\nabla.\boldsymbol{J_1}^*) - \frac{\boldsymbol{v_1}}{|\eta|}.j\omega\rho_0 \boldsymbol{v_1}^* - \frac{\boldsymbol{J_1}^* \cdot (\boldsymbol{v_0} \cdot \nabla)\boldsymbol{v_1}}{|\eta|}$$

$$\boldsymbol{v_0}(\nabla.\boldsymbol{J_1}^*) = j\omega \boldsymbol{J_1}^* - j\omega\rho_0 \boldsymbol{v_1}^*$$

Taking dot product with $\frac{\boldsymbol{v_1}}{|\eta|}$

$$\frac{\boldsymbol{v_1}}{|\eta|}.\mathrm{v}_0(\nabla.\boldsymbol{J_1}^*) = \frac{\boldsymbol{v_1}}{|\eta|}.j\omega \boldsymbol{J_1}^* - \frac{\boldsymbol{v_1}}{|\eta|}.j\omega\rho_0 \boldsymbol{v_1}^*$$

$$\downarrow$$

$$-\frac{j\omega}{|\eta|}\boldsymbol{J_1}^*.\boldsymbol{v_1} = -\frac{\boldsymbol{v_1}}{|\eta|}.\boldsymbol{v_0}(\nabla.\boldsymbol{J_1}^*) - \frac{\boldsymbol{v_1}}{|\eta|}.j\omega\rho_0 \boldsymbol{v_1}^*$$

$$\boldsymbol{J}_1^* \cdot \boldsymbol{E} = -\frac{\boldsymbol{v}_1}{|\eta|}.\boldsymbol{v}_0(\nabla.\boldsymbol{J}_1^*) - \frac{\boldsymbol{v}_1}{|\eta|}.j\omega\rho_0\boldsymbol{v}_1^* - \frac{\boldsymbol{J}_1^* \cdot (\boldsymbol{v}_0 \cdot \nabla)\boldsymbol{v}_1}{|\eta|}$$

$$\nabla \cdot (\psi \boldsymbol{A}) = \boldsymbol{A}.\nabla\psi + \psi\nabla \cdot \boldsymbol{A}$$

$$\boldsymbol{A} = \boldsymbol{J}_1^* \qquad \psi = \boldsymbol{v}_1.\boldsymbol{v}_0$$

$$\boldsymbol{J}_1^* \cdot \boldsymbol{E} = -\frac{\nabla \cdot (\boldsymbol{v}_1.\boldsymbol{v}_0\boldsymbol{J}_1^*)}{|\eta|} + \frac{\boldsymbol{J}_1^*.\nabla\boldsymbol{v}_1.\boldsymbol{v}_0}{|\eta|}$$

$$-\frac{\boldsymbol{v}_1}{|\eta|}.j\omega\rho_0\boldsymbol{v}_1^* - \frac{\boldsymbol{J}_1^* \cdot (\boldsymbol{v}_0 \cdot \nabla)\boldsymbol{v}_1}{|\eta|}$$

$$\nabla \cdot (\boldsymbol{v}_1.\boldsymbol{v}_0\boldsymbol{J}_1^*) = \boldsymbol{J}_1^*.\nabla\boldsymbol{v}_1.\boldsymbol{v}_0 + \boldsymbol{v}_1.\boldsymbol{v}_0\nabla \cdot \boldsymbol{J}_1^*$$

$$\downarrow$$

$$\boldsymbol{v}_1.\boldsymbol{v}_0\nabla \cdot \boldsymbol{J}_1^* = \nabla \cdot (\boldsymbol{v}_1.\boldsymbol{v}_0\boldsymbol{J}_1^*) - \boldsymbol{J}_1^*.\nabla\boldsymbol{v}_1.\boldsymbol{v}_0$$

$$\boldsymbol{J}_1^* \cdot \boldsymbol{E} = -j\omega\rho_0\frac{\boldsymbol{v}_1}{|\eta|}.\boldsymbol{v}_1^* - \frac{\nabla \cdot (\boldsymbol{v}_1.\boldsymbol{v}_0\boldsymbol{J}_1^*)}{|\eta|}$$

$$+\frac{\boldsymbol{J}_1^*.\{\nabla\boldsymbol{v}_1.\boldsymbol{v}_0 - (\boldsymbol{v}_0 \cdot \nabla)\boldsymbol{v}_1\}}{|\eta|}$$

RF quantities vary only with z

$$\nabla\boldsymbol{v}_1.\boldsymbol{v}_0 - (\boldsymbol{v}_0 \cdot \nabla)\boldsymbol{v}_1 = \frac{\partial v_1}{\partial z}v_0 - v_0\frac{\partial v_1}{\partial z} = 0$$

$$\boldsymbol{J}_1^* \cdot \boldsymbol{E} = -j\omega\rho_0\frac{\boldsymbol{v}_1}{|\eta|}.\boldsymbol{v}_1^* - \frac{\nabla \cdot (\boldsymbol{v}_1.\boldsymbol{v}_0\boldsymbol{J}_1^*)}{|\eta|}$$

$$\rho_0 = -|\rho_0|$$

$$\boldsymbol{J}_1^* \cdot \boldsymbol{E} = j\omega|\rho_0|\frac{\boldsymbol{v}_1}{|\eta|}.\boldsymbol{v}_1^* - \frac{\nabla \cdot (\boldsymbol{v}_1.\boldsymbol{v}_0\boldsymbol{J}_1^*)}{|\eta|}$$

$$\nabla\times\boldsymbol{E} = -j\omega\mu_0\boldsymbol{H}$$
$$\nabla\times\boldsymbol{H} = j\omega\varepsilon_0\boldsymbol{E}$$
$$\nabla\times\boldsymbol{H}^* = \boldsymbol{J_1}^* - j\omega\varepsilon_0\boldsymbol{E}^*$$

$$\nabla.(\boldsymbol{E}\times\boldsymbol{H}^*) = \boldsymbol{H}^*.\nabla\times\boldsymbol{E} - \boldsymbol{E}.\nabla\times\boldsymbol{H}^*$$

$$\nabla.(\boldsymbol{E}\times\boldsymbol{H}^*) = \boldsymbol{H}^*.(-j\omega\mu_0\boldsymbol{H}) - \boldsymbol{E}.(\boldsymbol{J_1}^* - j\omega\varepsilon_0\boldsymbol{E}^*)$$

$$\boldsymbol{E}.\boldsymbol{J_1}^* = -\nabla.(\boldsymbol{E}\times\boldsymbol{H}^*) - j\omega\mu_0\boldsymbol{H}.\boldsymbol{H}^* + j\omega\varepsilon_0\boldsymbol{E}.\boldsymbol{E}^*$$

$$\boldsymbol{J_1}^*\cdot\boldsymbol{E} = j\omega|\rho_0|\frac{\boldsymbol{v_1}}{|\eta|}.\boldsymbol{v_1}^* - \frac{\nabla\cdot(\boldsymbol{v_1}.\boldsymbol{v_0}\boldsymbol{J_1}^*)}{|\eta|}$$

On comparison

$$-\nabla.(\boldsymbol{E}\times\boldsymbol{H}^*) - j\omega\mu_0\boldsymbol{H}.\boldsymbol{H}^* + j\omega\varepsilon_0\boldsymbol{E}.\boldsymbol{E}^*$$
$$= j\omega|\rho_0|\frac{\boldsymbol{v_1}}{|\eta|}.\boldsymbol{v_1}^* - \frac{\nabla\cdot(\boldsymbol{v_1}.\mathrm{v}_0\boldsymbol{J_1}^*)}{|\eta|}$$

$$-\nabla.(\boldsymbol{E}\times\boldsymbol{H}^*) - j\omega\mu_0\boldsymbol{H}.\boldsymbol{H}^* + j\omega\varepsilon_0\boldsymbol{E}.\boldsymbol{E}^*$$

$$= j\omega|\rho_0|\frac{\boldsymbol{v_1}}{|\eta|}.\boldsymbol{v_1}^* - \frac{\nabla\cdot(\boldsymbol{v_1}.\boldsymbol{v_0}\boldsymbol{J_1}^*)}{|\eta|}$$

$$\nabla.(\boldsymbol{E}\times\boldsymbol{H}^*) = -j\omega\mu_0\boldsymbol{H}.\boldsymbol{H}^* + j\omega\varepsilon_0\boldsymbol{E}.\boldsymbol{E}^* - \frac{j\omega|\rho_0|}{|\eta|}\boldsymbol{v_1}.\boldsymbol{v_1}^* + \frac{\nabla\cdot(\boldsymbol{v_1}.\boldsymbol{v_0}\mathrm{J}_1^*)}{|\eta|}$$

$$\oint_\tau \nabla.(\boldsymbol{E}\times\boldsymbol{H}^*)d\tau = \oint_\tau [-j\omega\mu_0\boldsymbol{H}.\boldsymbol{H}^* + j\omega\varepsilon_0\boldsymbol{E}.\boldsymbol{E}^* - \frac{j\omega|\rho_0|}{|\eta|}\boldsymbol{v_1}.\boldsymbol{v_1}^* + \frac{\nabla\cdot(\boldsymbol{v_1}.\boldsymbol{v_0}\mathrm{J}_1^*)}{|\eta|}]d\tau$$

$$\oint_S (\boldsymbol{E}\times\boldsymbol{H}^*).\mathbf{a_n}dS = \oint_\tau [-j\omega\mu_0\boldsymbol{H}.\boldsymbol{H}^* + j\omega\varepsilon_0\boldsymbol{E}.\boldsymbol{E}^* - \frac{j\omega|\rho_0|}{|\eta|}\boldsymbol{v_1}.\boldsymbol{v_1}^* + \frac{\nabla\cdot(\boldsymbol{v_1}.\boldsymbol{v_0}\mathrm{J}_1^*)}{|\eta|}]d\tau$$

$$\oint_S (\boldsymbol{E} \times \boldsymbol{H}^*).\mathbf{a_n} dS = \oint_\tau [-j\omega\mu_0 \boldsymbol{H}.\boldsymbol{H}^* + j\omega\varepsilon_0 \boldsymbol{E}.\boldsymbol{E}^* - \frac{j\omega|\rho_0|}{|\eta|} \boldsymbol{v_1}.\boldsymbol{v_1}^* + \frac{\nabla \cdot (\boldsymbol{v_1}.\boldsymbol{v_0}\boldsymbol{J_1}^*)}{|\eta|}] d\tau$$

$$\frac{1}{2}\mathrm{Re}\oint_S (\boldsymbol{E} \times \boldsymbol{H}^*).\mathbf{a_n} dS = \frac{1}{2}\mathrm{Re}\oint_\tau [-j\omega\mu_0 \boldsymbol{H}.\boldsymbol{H}^* + j\omega\varepsilon_0 \boldsymbol{E}.\boldsymbol{E}^* - \frac{j\omega|\rho_0|}{|\eta|} \boldsymbol{v_1}.\boldsymbol{v_1}^* + \frac{\nabla \cdot (\boldsymbol{v_1}.\boldsymbol{v_0}\boldsymbol{J_1}^*)}{|\eta|}] d\tau$$

$$\frac{1}{2}\mathrm{Re}\oint_S (\boldsymbol{E} \times \boldsymbol{H}^*).\mathbf{a_n} dS = \frac{1}{2}\oint_\tau \frac{\nabla \cdot (\boldsymbol{v_1}.\boldsymbol{v_0}\boldsymbol{J_1}^*)}{|\eta|}] d\tau = \frac{1}{2}\mathrm{Re}\oint_S \frac{(\boldsymbol{v_1}.\boldsymbol{v_0}\boldsymbol{J_1}^*).\mathbf{a_n} dS}{|\eta|}$$

$$\frac{1}{2}\mathrm{Re}\oint_S (\boldsymbol{E} \times \boldsymbol{H}^*).\mathbf{a_n} dS = -\frac{1}{2}\mathrm{Re}\oint_S \frac{(-\boldsymbol{v_1}.\boldsymbol{v_0}\boldsymbol{J_1}^*).\mathbf{a_n} dS}{|\eta|}$$

$$\frac{1}{2}\mathrm{Re}\oint_S(\boldsymbol{E}\times\boldsymbol{H}^*).\mathbf{a_n}dS = -\frac{1}{2}\mathrm{Re}\oint_S\frac{(-\boldsymbol{v_1}.\boldsymbol{v_0}\boldsymbol{J_1}^*).\mathbf{a_n}dS}{|\eta|}$$

$$\frac{1}{2}\mathrm{Re}\oint_S(\boldsymbol{E}\times\boldsymbol{H}^*).\mathbf{a_n}dS = -\frac{1}{2}\mathrm{Re}\oint_S \mathrm{V_k}\boldsymbol{J_1}^*.\mathbf{a_n}dS$$

$$P_{\mathrm{em}} = -\frac{1}{2}\mathrm{Re}\oint_S \mathrm{V_k}\boldsymbol{J_1}^*.\mathbf{a_n}dS$$

$$P_{\mathrm{em}} = -\oint_S \boldsymbol{P_k}.\mathbf{a_n}dS$$

(Chu’s power conservation theorem)

$$\mathrm{V_k} = \frac{-\boldsymbol{v_1}.\boldsymbol{v_0}}{|\eta|}$$

(Beam kinetic voltage)

For one-dimensional beam-flow

$$\boldsymbol{v_1}.\boldsymbol{v_0} = v_1\mathbf{a_z}.v_0\mathbf{a_z} = v_1v_0$$

$$\mathrm{V_k} = \frac{-v_1v_0}{|\eta|}$$

$$P_{\mathrm{k}} = \frac{1}{2}\mathrm{Re}\,V_{\mathrm{k}}J_1^{\,*}$$

(Beam kinetic power density)

For one-dimensional beam-flow

$$\boldsymbol{P_k} = P_{\mathrm{k}}\mathbf{a_z} = \frac{1}{2}\mathrm{Re}\,V_{\mathrm{k}}J_1^{\,*}\mathbf{a_z}$$

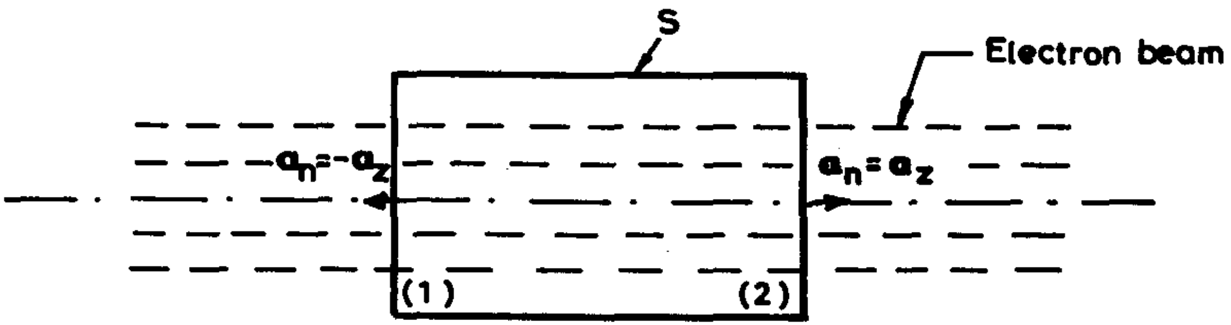
$$\boxed{P_{\mathrm{k}} = \frac{1}{2}\mathrm{Re}\frac{-v_1v_0}{|\eta|}\mathrm{J}_1^{\,*}}$$

(To be recalled later)

$$P_{em} = -\oint_S \boldsymbol{P}_k . \mathbf{a}_n dS$$

Electromagnetic power P_{em} delivered by a portion of the linear electron beam between two cross-sectional plane (1 and 2) perpendicular to the flow of electrons

Flux of the beam kinetic power density $\boldsymbol{P}_K$ through the surface of the volume enclosing the beam must be negative for P_{em} to be positive, that is, for a net electromagnetic power to flow out of the beam surface.



Closed surface (S) of integration enclosing a portion of a linear beam between two reference cross sections 1 and 2.

$$P_{em} = -\left[\oint_\alpha \boldsymbol{P}_k . \mathbf{a}_n d\alpha \Big| + \oint_\alpha \boldsymbol{P}_k . \mathbf{a}_n d\alpha \Big| \right] \longrightarrow P_{em} = -[-(\boldsymbol{P}_k . \mathbf{a}_z)|_{(1)} \alpha + (\boldsymbol{P}_k . \mathbf{a}_z)|_{(2)} \alpha]$$

beam cross section 1

beam cross section 2

$$\boxed{P_{em} = -\alpha(-P_{K(1)} + P_{K(2)}) = \alpha(P_{K(1)} - P_{K(2)})}$$

(To be recalled later)

$$P_k = \frac{1}{2}\mathrm{Re}\frac{-v_1 v_0}{|\eta|}J_1^*$$ (beam kinetic power density recalled)

Contribution to beam kinetic power density by slow and fast space-charge waves

$$P_k = \mathrm{Re}\frac{-v_0(v_{1f}+v_{1s})(J_{1f}^*+J_{1s}^*)}{2|\eta|}$$

$$v_{1f} = \frac{\omega_p}{\omega\rho_0}J_{1f}, \quad v_{1s} = \frac{-\omega_p}{\omega\rho_0}J_{1f}$$ (recalled)

$$P_k = R_e\frac{-v_0\omega_p}{2|\eta|\omega\rho_0}(J_{1f}J_{1f}^* + J_{1f}J_{1s}^* - J_{1s}J_{1f}^* - J_{1s}J_{1f}^*)$$

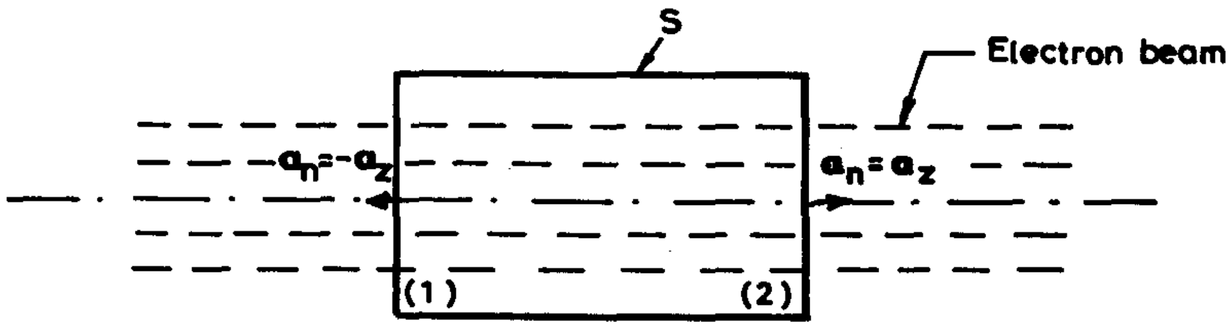
$$J_{1f} = \hat{J}_{1f}(0)\exp j(\omega t - \beta_f z), \quad J_{1s} = \hat{J}_{1s}(0)\exp j(\omega t - \beta_s z)$$

$$\beta_f = \frac{\omega-\omega_p}{v_0}, \quad \beta_s = \frac{\omega+\omega_p}{v_0}$$

$$P_k = R_e\frac{-v_0\omega_p}{2|\eta|\omega\rho_0}(\hat{J}^2_{1f}(0) - \hat{J}^2_{1s}(0) + 2\hat{J}_{1f}(0)\hat{J}_{1s}(0)\,j\sin\beta_p z)$$

$$\rho_0 = -|\rho_0|, \quad |J_0| = |\rho_0|v_0, \quad v_0 = (2\eta V_0)^{1/2}$$

$$\boxed{P_k = \frac{\omega_p V_0}{\omega|J_0|}[\hat{J}^2_{1f}(0) - \hat{J}^2_{1s}(0)]}$$ (To be recalled later)



$$P_{em} = -\alpha(-P_{K(1)} + P_{K(2)}) = \alpha(P_{K(1)} - P_{K(2)})$$

(recalled)

$$P_k = \frac{\omega_p V_0}{\omega |J_0|}[\hat{J}^2_{1f}(0) - \hat{J}^2_{1s}(0)]$$

(recalled)

$$P_{em} = \frac{\omega_p}{\omega}\frac{V_0}{|J_0|}[\hat{J}^2_{1s}(0)_{(2)} - \hat{J}^2_{1s}(0)_{(1)} - (\hat{J}^2_{1f}(0)_{(2)} - \hat{J}^2_{f}(0)_{(1)})]$$

Since

$$\hat{J}_{1s}(0)_{(2)} > \hat{J}_{1s}(0)_{(1)} \text{ and } \hat{J}_{1f}(0)_{(2)} > \hat{J}_{1f}(0)_{(1)}$$

for a growing-wave device like the TWT, P_{em} is positive provided the coupling of the slow space-charge wave to the circuit (interaction structure) predominates over that of the fast space-charge wave.
This corresponds to the transfer of electron beam kinetic energy to electromagnetic waves supported by the interaction structure of the device.

The slow space-charge wave on the electron beam carries a negative kinetic power density the excitation of which is accompanied by the delivery of power from the beam to the circuit (Chu's power conservation theory).

In a growing-wave device like the travelling-wave tube (TWT), the electron beam velocity is made nearly synchronous with the RF phase velocity of the slow-wave structure. In fact, the electron beam velocity is made slightly greater than the RF phase velocity (thereby making the TWT a Cerenkov radiation device). This ensures that the slow space-charge wave on the electron beam predominantly couples to RF waves supported by the slow-wave structure.
According to Chu's theory then the electron beam kinetic energy is transferred to RF waves.
Further, this slight de-synchronization ensures that on the average the electron bunch finds itself in the decelerating phase of RF waves for the transfer of the beam kinetic energy to RF waves.

♣L. J. Chu, "A kinetic power theorem", paper presented at the IRE-PGED Electron Tube Research Conference, Durham, New Hampshire, June, 1951.

♣H. A. Haus and D. Bobroff, "Small-signal power theorem for electron beams", *J. Appl. Phys. 28* (1957), 694-703.

♣ J. W. Kliiver, "Small-signal power conservation theorem for irrotational electron beams", *J. Appl. Phys. 29 (1958), 618-622.*

Thank you!