Engineering Electromagnetics Essentials

Chapter 4

Basic concepts of static magnetic fields

Objective

Recapitulation of basic concepts of static magnetic fields or magnetostatics

Topics dealt with

Coulomb's and Gauss's laws of static magnetic fields or magnetostatics
Biot–Savart's law
Ampere's circuital law
Lorentz force experienced by a moving charge in a magnetic field
Force experienced by a current-carrying conductor in a magnetic field

Background

Vector calculus expressions developed in Chapter 2
Basic concepts of static fields developed in Chapter 3 as some of these concepts are extended to static magnetic fields in this chapter

Force between a pole of a magnet and a pole of another magnet expressed by a law analogous to Coulomb's law of electrostatics:

$$\vec{F} = \frac{\mu}{4\pi} \frac{q_{m1} q_{m2}}{r^2} \vec{a}_r$$

q_{m1}, q_{m2} : Magnetic charges representing the poles

$\vec{a}_r$: Unit vector directed from the magnetic point charge q_{m1} to q_{m2}

r : Distance between the magnetic point charge q_{m1} to q_{m2}

μ : Permeability of the medium

Relative permeability

$$\mu_r = \frac{\mu}{\mu_0}$$

Permeability of a free-space medium

$$\mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

(That the unit of permeability is H/m will be appreciated from the expression of inductance of a solenoid to be derived later, the unit of inductance being Henry or H)

Magnetic flux density is related to magnetic field as electric flux density or electric displacement is related to electric field

$$\downarrow$$

$$\vec{B} = \mu \vec{H}$$

Force on magnetic point charge q_m in a region of magnetic flux density analogous to force on electric point charge in a region of electric field

$$\downarrow$$

$$\vec{F} = q_m \vec{B}$$

Magnetic flux density due to a magnetic point charge q_m at a distance r is analogous to electric field due to an electric point charge

$$\downarrow$$

$$\vec{B} = \frac{\mu q_m}{4\pi r^2} \vec{a}_r$$

Magnetic field due to a magnetic point charge q_m at a distance r is analogous to electric displacement due to an electric point charge

$$\downarrow$$

$$\vec{H} = \frac{q_m}{4\pi r^2} \vec{a}_r$$

Analogous electrostatic and magnetostatic quantities

Electrostatic quantities/expressions	Magnetostatic quantities/expressions
ε	μ
$\vec{E}$	$\vec{B}$
$\vec{D}$	$\vec{H}$
$\vec{F} = \dfrac{q_1 q_2}{4\pi\varepsilon\, r^2}\vec{a}_r$	$\vec{F} = \dfrac{\mu}{4\pi}\dfrac{q_{m1} q_{m2}}{r^2}\vec{a}_r$
$\vec{F} = q\vec{E}$	$\vec{F} = q_m \vec{B}$
$\vec{E} = \dfrac{q}{4\pi\varepsilon\, r^2}\vec{a}_r$	$\vec{B} = \dfrac{\mu\, q_m}{4\pi r^2}\vec{a}_r$
$\vec{D} = \dfrac{q}{4\pi\, r^2}\vec{a}_r$	$\vec{H} = \dfrac{q_m}{4\pi r^2}\vec{a}_r$
$\vec{D} = \varepsilon\,\vec{E}$	$\vec{B} = \mu\,\vec{H}$

Gauss's law and Poisson's equation of magnetostatics

Free magnetic charges do not exist. A magnetic charge is always accompanied by an equal and opposite magnetic charge as in a permanent magnet. One cannot separate north and south poles of a magnet by breaking the magnet.

As many flux lines leave a volume enclosing a pole (magnetic charge) of a magnet as those enter the volume. Thus, magnetic flux lines always form closed loops.

In other words, magnetic flux lines are continuous

Free magnetic charges do not exist

Electrostatics | **Magnetostatics**

$$\oint_S \vec{D}\cdot d\vec{S} = \oint_S \vec{D}\cdot\vec{a}_n dS = Q \quad \xrightarrow{\text{analogous}} \quad \oint_S \vec{H}\cdot d\vec{S} = \oint_S \vec{H}\cdot\vec{a}_n dS = 0$$

$$\oint_S \vec{E}\cdot d\vec{S} = \oint_S \vec{E}\cdot\vec{a}_n dS = \frac{Q}{\varepsilon} \quad \xrightarrow{\text{analogous}} \quad \oint_S \vec{B}\cdot d\vec{S} = \oint_S \vec{B}\cdot\vec{a}_n dS = 0$$

Gauss's law

Free magnetic charges do not exist

Electrostatics

Magnetostatics

$$\nabla \cdot \vec{D} = \rho \quad \xrightarrow{\text{analogous}} \quad \nabla \cdot \vec{H} = 0$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \quad \xrightarrow{\text{analogous}} \quad \nabla \cdot \vec{B} = 0$$

Poisson's equation

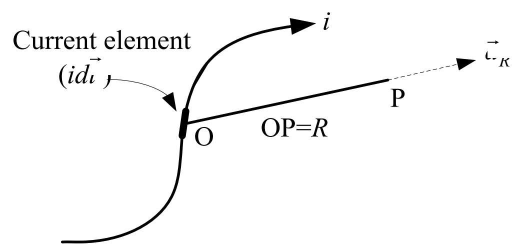
Poisson's equation of magnetostatics is the manifestation of the absence of free magnetic charges (poles); it is another way of stating that magnetic flux lines are continuous.

Electric flux lines (as in an electric dipole) originate from a positive point charge and terminate on a negative point charge whereas magnetic flux lines originating from north pole of a magnet will return to the north pole via south pole.

Biot–Savart's Law

Element of magnetic field and element of magnetic flux density due to a direct current through an element of length of a filamentary conductor (say, a solid-cylindrical conductor of circular cross-section of radius approaching zero) is given by

$$\left.\begin{aligned} d\vec{H} &= \frac{i\,d\vec{l} \times \vec{a}_R}{4\pi R^2} \\ d\vec{B} &= (\mu_0 d\vec{H}) = \mu_0 \frac{i\,d\vec{l} \times \vec{a}_R}{4\pi R^2} \end{aligned}\right\}$$



Length element vector $d\vec{l}$ takes on the direction of current through filamentary conductor

$\vec{a}_R$ is the unit vector directed from the current element to the point at a distance R where the magnetic field and flux density is sought

Magnetic field due to a long thin wire carrying a direct current

Wire is aligned along z axis carrying a direct current along z.

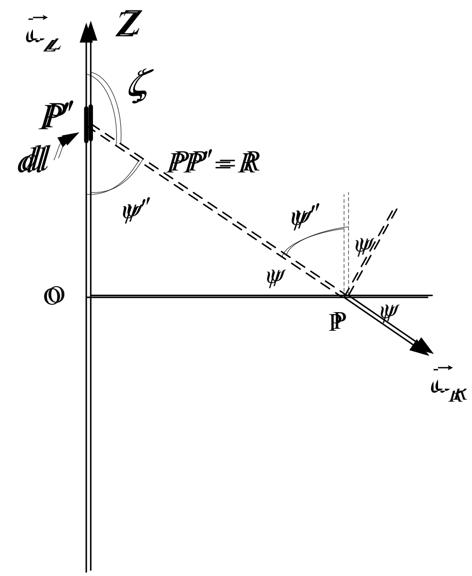
P is point where the magnetic field is sought.

O is the foot of perpendicular form P on the length of the wire.

O is considered as the origin *z* = 0.

OP = *r*

Wire extends from z = -∞ to ∞.



Elements of magnetic field and magnetic flux density are given by:

$$\left.\begin{aligned} d\vec{H} &= \frac{i\,d\vec{l} \times \vec{a}_R}{4\pi R^2} \\ d\vec{B} &= (\mu_0 d\vec{H}) = \frac{i\,d\vec{l} \times \vec{a}_R}{4\pi R^2} \end{aligned}\right\}$$ (Biot-Savart's law)

$i\,d\vec{l} = \text{current element vector considerd at P}'$

$i\,d\vec{l} = idz\,\vec{a}_z$ $\quad d\vec{l}$ takes the direction of current along $\vec{a}_z$

$\vec{a}_z = \text{unit vector considerd along } z$

$$d\vec{H} = \frac{i\,d\vec{l} \times \vec{a}_R}{4\pi R^2}$$

$i\,d\vec{l} =$ current element vector considerd at P′

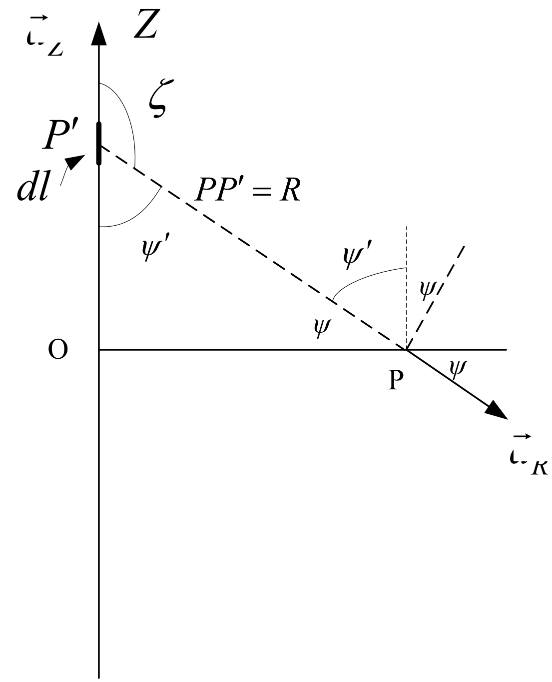
$i\,d\vec{l} = idz\,\vec{a}_z$

$$d\vec{H} = \frac{i\,dz\,\vec{a}_z \times \vec{a}_R}{4\pi R^2}$$

$R =$ distance between P and P′

$\vec{a}_R =$ unit vector directed from P to P′

$\vec{a}_\theta =$ unit vector normal to the plane of the paper in azimuthal direction away from the reader



$\vec{a}_z \times \vec{a}_R = \sin\varsigma\,\vec{a}_\theta$

$\varsigma = \pi - \psi'$

$\psi' = \pi/2 - \psi$

$\vec{a}_z \times \vec{a}_R = \cos\psi\,\vec{a}_\theta$

$\sin\varsigma = \sin(\pi - \psi') = \sin\psi' = \sin(\pi/2 - \psi) = \cos\psi$

$\vec{a}_\theta =$ unit vector in azimuthal direction away from the reader

$$\vec{a}_z \times \vec{a}_R = \cos\psi\, \vec{a}_\theta$$

$$d\vec{H} = \frac{i\, dz\, \vec{a}_z \times \vec{a}_R}{4\pi R^2}$$

$$\downarrow$$

$$d\vec{H} = \frac{i\, dz \cos\psi}{4\pi R^2} \vec{a}_\theta$$

$$z = \mathrm{OP}' = \mathrm{OP}\tan\psi = r\tan\psi$$

$$dz = r\sec^2\psi\, d\psi$$

$$\downarrow$$

$$d\vec{H} = \frac{i\, r\sec^2\psi\, d\psi\, \cos\psi}{4\pi R^2} \vec{a}_\theta$$

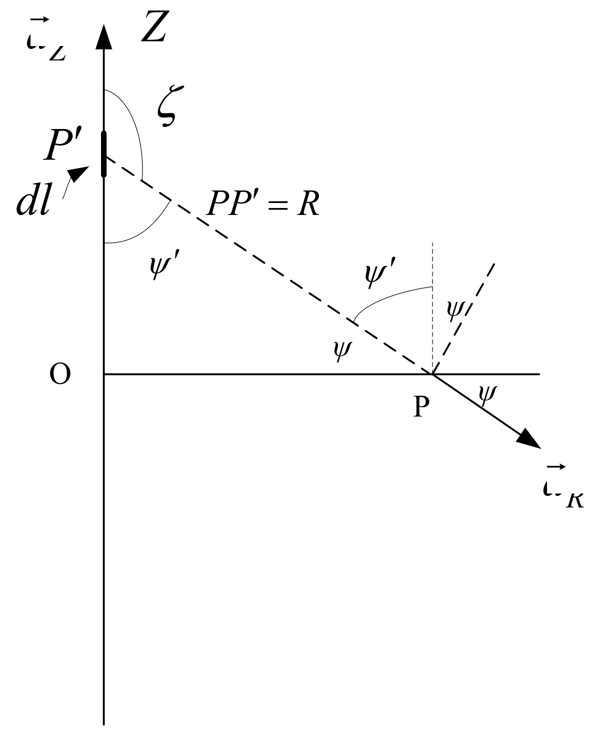
$$d\vec{H} = \frac{i\, r}{4\pi R^2} \sec\psi\, d\psi\, \vec{a}_\theta$$

$$R = r\sec\psi$$

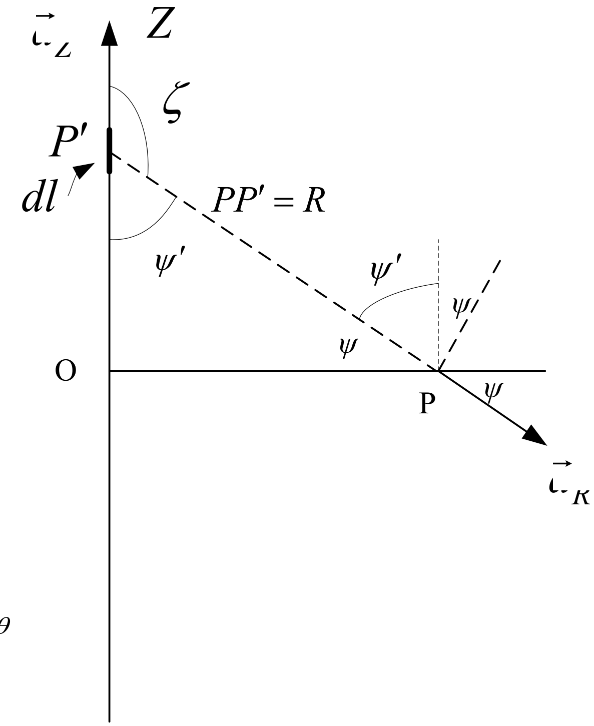
$$\downarrow$$

$$d\vec{H} = \frac{i}{4\pi\, r} \cos\psi\ d\psi\, \vec{a}_\theta$$

$$\vec{H} = \int d\vec{H} = \frac{1}{4\pi}\int \frac{i}{r}\cos\psi\ d\psi\, \vec{a}_\theta = \frac{i}{4\pi r}\int \cos\psi\ d\psi\, \vec{a}_\theta$$



$$d\vec{H} = \frac{i}{4\pi r} \cos\psi \; d\psi \, \vec{a}_\theta$$



Integrating over the length of the wire we get

$$\vec{H} = \int d\vec{H} = \frac{1}{4\pi} \int \frac{i}{r} \cos\psi \; d\psi \, \vec{a}_\theta = \frac{i}{4\pi r} \int \cos\psi \; d\psi \, \vec{a}_\theta$$

For the wire extending from z = -∞ to ∞, the integration limits become $\psi = -\pi/2$ and $\psi = \pi/2$ giving

$$\vec{H} = \frac{i}{4\pi r} \int \cos\psi \; d\psi \, \vec{a}_\theta = \frac{i}{4\pi r} \int_{-\pi/2}^{+\pi/2} \cos\psi \; d\psi \, \vec{a}_\theta$$

$$\vec{H} = \frac{i}{4\pi r}\int \cos\psi \; d\psi \, \vec{a}_\theta = \vec{H} = \frac{i}{4\pi\, r}\int_{-\pi/2}^{+\pi/2}\cos\psi \; d\psi \, \vec{a}_\theta$$

$$\vec{H} = \frac{i}{4\pi\, r}\int_{-\pi/2}^{+\pi/2}\cos\psi \; d\psi \, \vec{a}_\theta = \frac{i}{4\pi r}[\sin\psi]_{-\pi/2}^{+\pi/2}\vec{a}_\theta$$

$$\vec{H} = \frac{i}{4\pi r}[\sin\psi]_{-\pi/2}^{+\pi/2}\vec{a}_\theta = \frac{i}{4\pi r}[\sin(\pi/2) - \sin(-\pi/2)]\vec{a}_\theta = \frac{i}{4\pi r}(1+1)\vec{a}_\theta$$

$$\vec{H} = \frac{i}{4\pi r}(1+1)\vec{a}_\theta = \frac{2i}{4\pi r}\vec{a}_\theta = \frac{i}{2\pi r}\vec{a}_\theta$$

$\vec{a}_\theta$ = unit vector normal to the plane of the paper in azimuthal direction away from the reader

(Magnetic field due to an infinitely long thin wire carrying a direct current)

Magnetic field due to a thin straight wire of finite length carrying a direct current

Point where to find magnetic field is not symmetric with respect to the wire of finite length

Wire length AB = OA + OB = $|l_1| + |l_2|$

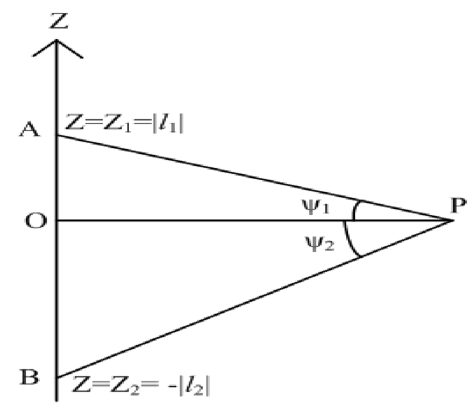
For the wire extending from $z = -|l_2|$ to $|l_1|$, the integration limits become $\psi = \psi_1$ and $\psi = \psi_2$ giving

$\vec{a}_\theta =$ unit vector normal to the plane of the paper in azimuthal direction away from the reader

$$\vec{H} = \frac{i}{4\pi r}\int \cos\psi \; d\psi\, \vec{a}_\theta = \frac{i}{4\pi r}\int_{\psi_2}^{\psi_1} \cos\psi \; d\psi\, \vec{a}_\theta = \frac{i}{4\pi r}[\sin\psi]_{\psi_2}^{\psi_1}\vec{a}_\theta$$

$$\vec{H} = \frac{i}{4\pi r}[\sin\psi]_{\psi_2}^{\psi_1}\vec{a}_\theta = \frac{i}{4\pi r}(\sin\psi_1 - \sin\psi_2)\vec{a}_\theta$$

$$\left.\begin{aligned} \sin\psi_1 &= \frac{|l_1|}{\sqrt{|l_1|^2 + r^2}} \\ \sin\psi_2 &= \frac{-|l_2|}{\sqrt{|l_2|^2 + r^2}} \end{aligned}\right\}$$



$$\vec{H} = \frac{i}{4\pi r}\left(\frac{1}{\sqrt{1 + r^2/|l_1|^2}} + \frac{1}{\sqrt{1 + r^2/|l_2|^2}}\right)\vec{a}_\theta$$

(Magnetic field at a point not symmetric with respect to the wire length)

Point where to find magnetic field is symmetric with respect to the wire length of finite length

Wire length AB = OA + OB = $|l_1| + |l_2|$ = $l/2 + l/2 = l$

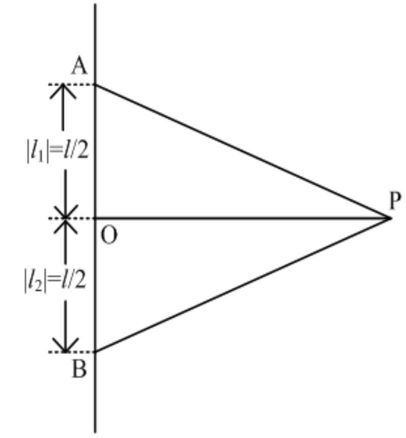
$$\vec{H} = \frac{i}{4\pi r}\left(\frac{1}{\sqrt{1+r^2/|l_1|^2}} + \frac{1}{\sqrt{1+r^2/|l_2|^2}}\right)\vec{a}_\theta$$

$\vec{a}_\theta =$ unit vector normal to the plane of the paper in azimuthal direction away from the reader

$$\vec{H} = \frac{i}{4\pi r}\left(\frac{1}{\sqrt{1+r^2/(l/2)^2}} + \frac{1}{\sqrt{1+r^2/(l/2)^2}}\right)\vec{a}_\theta$$

$$\vec{H} = \frac{i}{4\pi r}\left(\frac{2}{\sqrt{1+r^2/(l/2)^2}}\right)\vec{a}_\theta$$

$$= \frac{i}{2\pi r}\left(\frac{1}{\sqrt{1+(2r/l)^2}}\right)\vec{a}_\theta$$

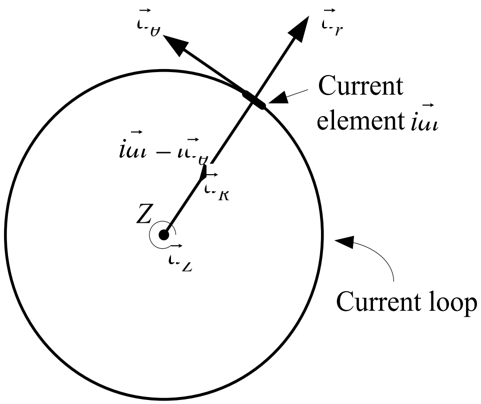


(Magnetic field at a point symmetric with respect to the wire length)

Magnetic field at the centre of a a circular loop of wire carrying a direct current

$$d\vec{H} = \frac{i\,d\vec{l} \times \vec{a}_R}{4\pi R^2}$$ (Biot-Savart's law)

$$\begin{cases} d\vec{l} = dl\vec{a}_\theta \\ \vec{a}_R = -\vec{a}_r \\ R = a \end{cases}$$



$$d\vec{H} = \frac{i\,d\vec{l} \times \vec{a}_R}{4\pi R^2} = \frac{i(dl\,\vec{a}_\theta)\times(-\vec{a}_r)}{4\pi a^2} = \frac{-idl\,\vec{a}_\theta \times \vec{a}_r}{4\pi a^2} = \frac{idl\,\vec{a}_z}{4\pi a^2}$$

$$\int dl = 2\pi a$$

$$\vec{H} = \int d\vec{H} = \frac{i\vec{a}_z}{4\pi a^2}\int dl$$

$$\vec{H} = \int d\vec{H} = \frac{i\vec{a}_z}{4\pi a^2}\int dl = \frac{i\vec{a}_z}{4\pi a^2}2\pi a$$

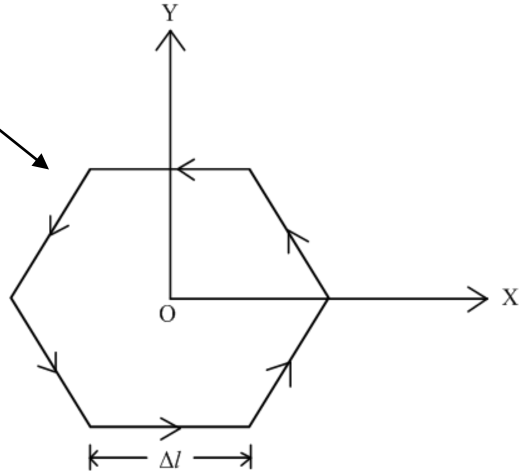
$$\vec{H} = \frac{i}{2a}\vec{a}_z$$

(magnetic field at the centre of a a circular loop of wire carrying a direct current)

Magnetic field at the centre of a regular polygonal loop of wire carrying a direct current

a = perpendicular distance of each side of the polygon from its centre

Δl = length of each side of the polygon



Recall the expression for magnetic field due to a straight wire of finite length

$$\vec{H} = \frac{i}{2\pi r}\left(\frac{1}{\sqrt{1+(2r/l)^2}}\right)\vec{a}_\theta$$

(Magnetic field at a point symmetric with respect to the wire length)

$$\vec{H} = \frac{i}{2\pi r}\left(\frac{1}{\sqrt{1+(2a/\Delta l)^2}}\right)\vec{a}_\theta \qquad \left.\begin{array}{l} l = \Delta l \\ r = a \end{array}\right\}$$

(Expression for the magnetic field at the centre of the polygon loop of wire carrying a direct current due to a single side of the polygon)

$$\vec{H} = \frac{i}{2\pi r}\left(\frac{1}{\sqrt{1+(2a/\Delta l)^2}}\right)\vec{a}_\theta$$

(Expression for the magnetic field at the centre of the polygon loop of wire carrying a direct current due to a single side of the polygon)

$$\downarrow$$

$$\vec{H} = \sum_n \frac{i}{2\pi a}\left(\frac{1}{\sqrt{1+(2a/\Delta l)^2}}\right)\vec{a}_z$$

(Expression for the magnetic field at the centre of the polygon loop of wire carrying a direct current due to *n* sides of the polygon obtained by summing up the contributions of sides)

$$\downarrow \leftarrow \begin{cases} n >> 1 \\ a >> \Delta l \end{cases}$$ for large number of sides of the polygon

$$\vec{H} = \sum_n \frac{i}{2\pi a}\left(\frac{1}{\sqrt{(2a/\Delta l)^2}}\right)\vec{a}_z = \frac{i}{4\pi a^2}\sum_n \Delta l\,\vec{a}_z = \frac{i}{4\pi a^2}2\pi a\vec{a}_z = \frac{i}{2a}\vec{a}_z$$

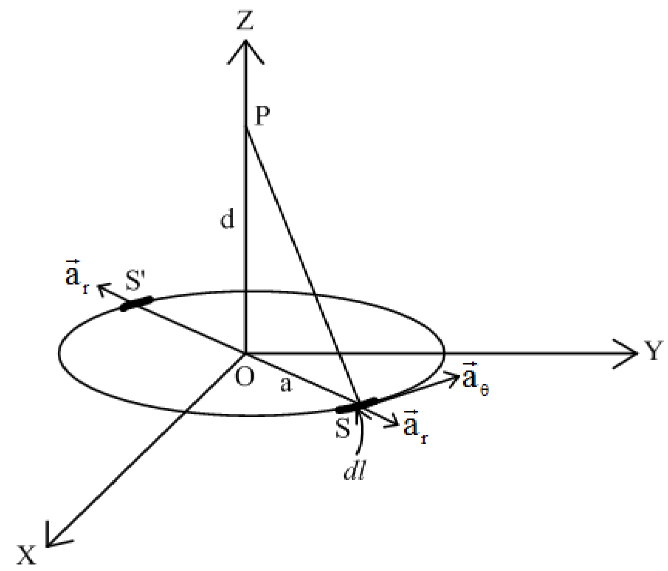
Expression becomes identical with the expression for magnetic field at the centre of a circular loop of wire carrying a direct current, for large number of sides of the polygon when the latter tends to become a circle!

Magnetic field due to a circular loop of wire carrying a direct current at a point lying on the axis of the loop off from its centre

P is the point on the z-axis, at a distance *d* from the centre O of the circular current loop, where the magnetic field is to be found

$$\mathrm{OP} = d$$

Two current elements are considered at two opposite points S and S′ on diameter.



Element of magnetic field at P due to current element at S:

$$d\vec{H} = \frac{i\,d\vec{l} \times \vec{a}_R}{4\pi R^2}$$ (Biot-Savart's law)

$$\begin{cases} i\,d\vec{l} = idl\vec{a}_\theta \\ \vec{R} = R\vec{a}_R \\ \vec{R} = \overrightarrow{\mathrm{SP}} \end{cases}$$

$$d\vec{H} = \frac{i\,d\vec{l} \times \vec{a}_R}{4\pi\, R^2} = \frac{idl\,\vec{a}_\theta \times \vec{R}}{4\pi\, R^3}$$

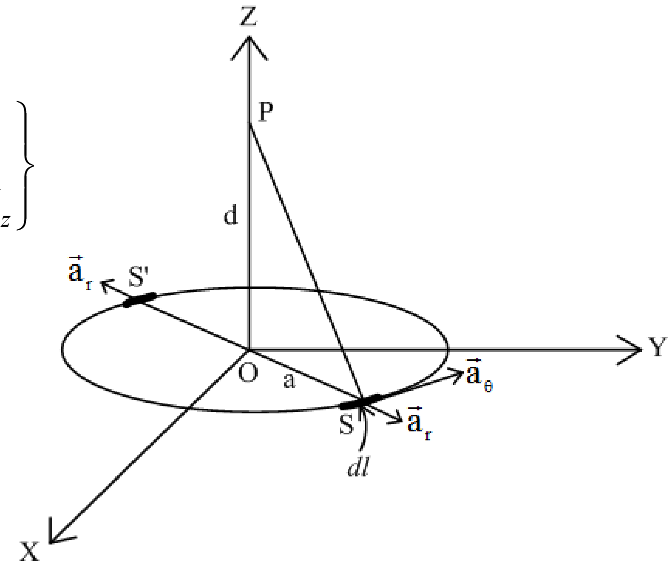
$$d\vec{H} = \frac{idl\,\vec{a}_\theta \times \overrightarrow{\mathrm{SP}}}{4\pi(\mathrm{SP})^3}$$

Element of magnetic field at P due to current element at S:

$$d\vec{H} = \frac{idl\,\vec{a}_\theta \times \overrightarrow{\mathrm{SP}}}{4\pi(\mathrm{SP})^3}$$

$a =$ radius of the circular current loop

$$\left.\begin{aligned} \mathrm{SP} &= R = (a^2 + d^2)^{1/2} \\ \overrightarrow{\mathrm{SP}} &= \overrightarrow{\mathrm{SO}} + \overrightarrow{\mathrm{OP}} = -\mathrm{SO}\vec{a}_r + \mathrm{OP}\vec{a}_z = -a\vec{a}_r + d\vec{a}_z \end{aligned}\right\}$$



$$d\vec{H} = \frac{i\,dl\vec{a}_\theta \times (-a\vec{a}_r + d\vec{a}_z)}{4\pi(a^2 + d^2)^{3/2}}$$

$$\begin{cases} \vec{a}_\theta \times \vec{a}_r = -\vec{a}_z \\ \vec{a}_\theta \times \vec{a}_z = \vec{a}_r \end{cases}$$

$$d\vec{H} = \frac{i\,dl(a\vec{a}_z + d\vec{a}_r)}{4\pi(a^2 + d^2)^{3/2}}$$

(element of magnetic field at P due to current element at S)

Element of magnetic field at P due to current element at S

Element of magnetic field at P due to current element at S'

$$d\vec{H} = \frac{i\,dl(a\vec{a}_z + d\vec{a}_r)}{4\pi(a^2+d^2)^{3/2}}$$

(with $\vec{a}_r$ in opposite directions at S and S′ on diameter)

While summing up the contribution to the magnetic field at P due to all diametrically opposite current elements, the second term containing the unit vector will cancel out since unit vectors at diametrically opposite points though equal in magnitude are in opposite directions.

$$\vec{H} = \int d\vec{H} = \int \frac{i\,dl a\vec{a}_z}{4\pi(a^2+d^2)^{3/2}} = \frac{i\,a\vec{a}_z}{4\pi(a^2+d^2)^{3/2}}\int dl$$

$$\int dl = 2\pi a$$

$$\vec{H} = \frac{i\,a^2\vec{a}_z}{4\pi(a^2+d^2)^{3/2}}2\pi a$$

$$\vec{H} = \frac{i\,a^2}{2(a^2+d^2)^{3/2}}\vec{a}_z$$

(magnetic field due to a circular loop of wire carrying a direct current at a point lying on the axis of the loop off from its centre)

$$\vec{H} = \frac{i\,a^2}{2(a^2+d^2)^{3/2}}\,\vec{a}_z$$ (magnetic field due to a circular loop of wire carrying a direct current at a point lying on the axis of the loop off from its centre)

$$\alpha = \pi a^2 = \text{area of circular current loop}$$

$$\vec{H} = \frac{i\,a}{4\pi(a^2+d^2)^{3/2}}\,2\pi a\vec{a}_z = \frac{i\,\pi a^2}{2\pi(a^2+d^2)^{3/2}}\,\vec{a}_z = \frac{i\,\alpha}{2\pi(a^2+d^2)^{3/2}}\,\vec{a}_z$$

Special case: *d* = 0

$$\vec{H} = \frac{i}{2a}\,\vec{a}_z$$

$$p_m = i\,\alpha = \text{magnetic dipole moment}$$

$$\vec{H} = \frac{i\,\alpha}{2\pi a^3}\,\vec{a}_z$$

$$\vec{H} = \frac{p_m}{2\pi a^3}\,\vec{a}_z$$

(magnetic field at the centre of a a circular loop of wire carrying a direct current as a special case *d* = 0)

Magnetic dipole moment

$$\vec{E} = \frac{p}{2\pi\varepsilon_0 r^3}\vec{a}_r$$

$$p = ql \text{ (electric dipole moment)}$$

(expression for electric field due to a short dipole at a distance *r* on the axis of the dipole in terms of the dipole moment *p*)

$$\vec{H} = \frac{p_m}{2\pi a^3}\vec{a}_z$$

$$p_m = i\alpha \text{ (magnetic dipole moment)}$$

(expression for magnetic field at the centre of a a circular current loop of wire)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3} p(2\cos\theta\vec{a}_r + \sin\theta\vec{a}_\theta)$$

(expression for electric field due to a short dipole at a distance *r* at an angle θ from the axis of the electric dipole in terms of the electric dipole moment *p* (already derived)

$$\vec{D} = \varepsilon_0\vec{E}$$

$$\vec{D} = \frac{1}{4\pi r^3} p(2\cos\theta\vec{a}_r + \sin\theta\vec{a}_\theta)$$

$\vec{D}$ is analogous to $\vec{H}$

$$\vec{H} = \frac{1}{4\pi r^3} p_m(2\cos\theta\vec{a}_r + \sin\theta\vec{a}_\theta)$$

(expression for magnetic field due to a short dipole at a distance *r* at an angle θ from the axis of the magnetic dipole in terms of the electric dipole moment p_m)

$$\vec{H} = \frac{1}{4\pi r^3} p_m (2\cos\theta \vec{a}_r + \sin\theta \vec{a}_\theta)$$

(expression for magnetic field due to a short dipole at a distance *r* at an angle θ from the axis of the magnetic dipole in terms of the electric dipole moment p_m)

$$B = \mu_0 \vec{H}$$

$$\vec{B} = \frac{\mu_0}{4\pi} p_m \frac{1}{r^3} (2\cos\theta \vec{a}_r + \sin\theta \vec{a}_\theta)$$

Analogous electric and magnetic dipole quantities

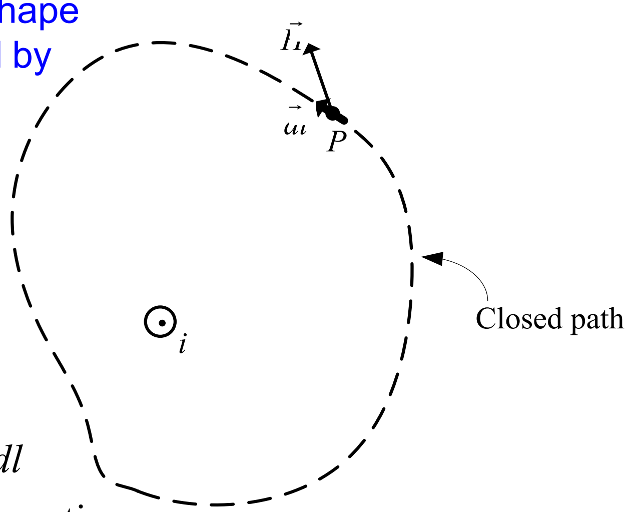
Electric dipole	Magnetic dipole
$\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3} p (2\cos\theta \vec{a}_r + \sin\theta \vec{a}_\theta)$	$\vec{B} = \frac{\mu_0}{4\pi} p_m \frac{1}{r^3} (2\cos\theta \vec{a}_r + \sin\theta \vec{a}_\theta)$
$1/4\pi\varepsilon_0$	$\mu_0/4\pi$
ε_0	$1/\mu_0$
$p = ql$	$p_m (= i\alpha = i\pi a^2)$
$\vec{D} = \frac{1}{4\pi r^3} p (2\cos\theta \vec{a}_r + \sin\theta \vec{a}_\theta)$	$\vec{H} = \frac{1}{4\pi r^3} p_m (2\cos\theta \vec{a}_r + \sin\theta \vec{a}_\theta)$

Ampere's circuital law in integral form

Line integral of magnetic field around a closed path of any arbitrary shape on a plane perpendicular to a current is equal to the current enclosed by the path.

$$\oint_l \vec{H} \cdot d\vec{l} = i$$

Ampere's circuital law



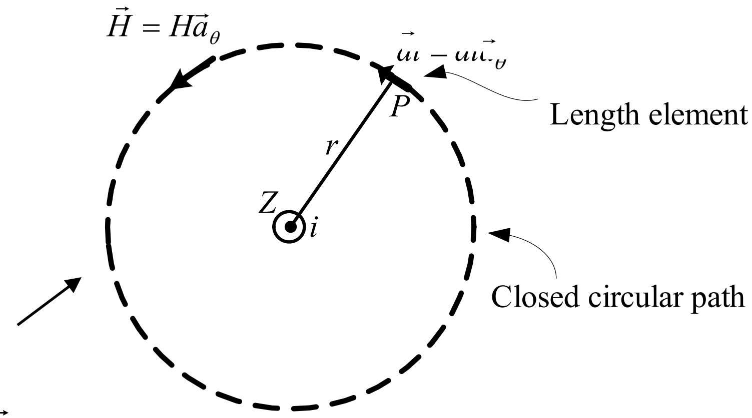
$\vec{H}$ = magnetic field at a point P on the closed path considered

$d\vec{l}$ = length element vector at the point such that the magnitude of $d\vec{l}$ is dl and its direction is the sense of rotation of a screw that will cause an axial motion of the screw in the direction of the current i

Ampere's circuital law simplifies the problems that enjoy geometrical symmetry in magnetostatics as does Gauss's law of electrostatics.

Let us take up the problem of an infinitely long current-carrying wire

A long straight wire along z carrying current *i* and a circular closed path of radius *r* considered on a plane perpendicular to Z axis passing through the point P where the magnetic field is to be found



$$\oint_l \vec{H}\cdot d\vec{l} = i \quad \text{(Ampere's circuital law)} \longleftarrow \begin{cases} \vec{H} = H\vec{a}_\theta \\ d\vec{l} = dl\vec{a}_\theta \end{cases}$$

$$\oint_l H\vec{a}_\theta \cdot dl\vec{a}_\theta = i$$

$$\downarrow$$

$$\oint_l H dl = i$$

The problem enjoys cylindrical symmetry and the magnetic field magnitude remains constant at all points on the closed path of radius *r*.

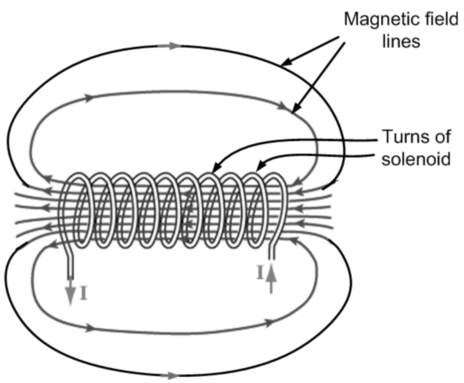
$$H\oint_l dl = i \longleftarrow \oint_l dl = 2\pi r$$

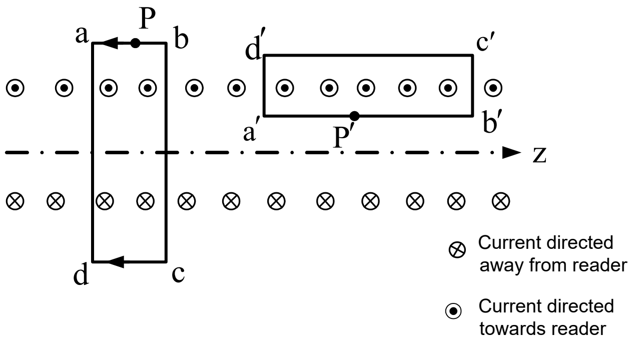
$$H 2\pi r = i \longrightarrow H = \frac{i}{2\pi r} \longrightarrow \vec{H} = H\vec{a}_\theta = \frac{i}{2\pi r}\vec{a}_\theta$$

$$\vec{H} = \frac{i}{2\pi r}\vec{a}_{\theta}$$

Expression for magnetic field due to a long straight current carrying wire has been obtained so simply by Ampere's circuital law doing away with the evaluation of integrals required in the application Biot-Savart's law to find the expression.

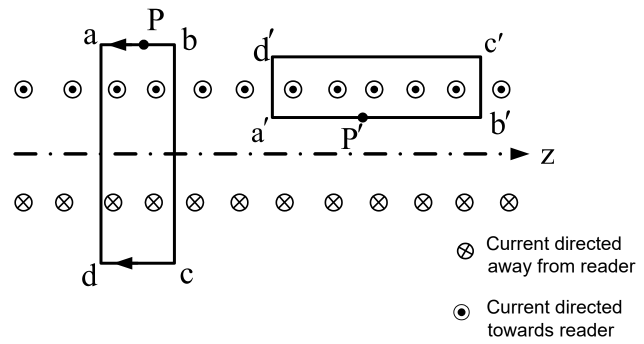
Let us find the magnetic field both inside and outside a long solenoid carrying a direct current and also its inductance





Length of the solenoid is considered to be very large compared to the radius of the and distance between two consecutive turns of the solenoid. Therefore, we can take the magnetic field due to the solenoid to be independent of *z*.

Biot-Savart's law can be used to appreciate that the direction of the magnetic field due to currents, both out of and into the plane of the paper, will be azimuthal, as in the problem of a long straight current carrying wire.



Magnetic field outside the solenoid

At a point outside the solenoid, at a radial distance much larger than the solenoid radius, the magnetic field contributed by the anticlockwise flux lines due to the currents out of the plane of the paper will balance the magnetic field contributed by the clockwise flux lines due to currents into the plane of the paper, thereby making the magnetic field outside the solenoid nil.

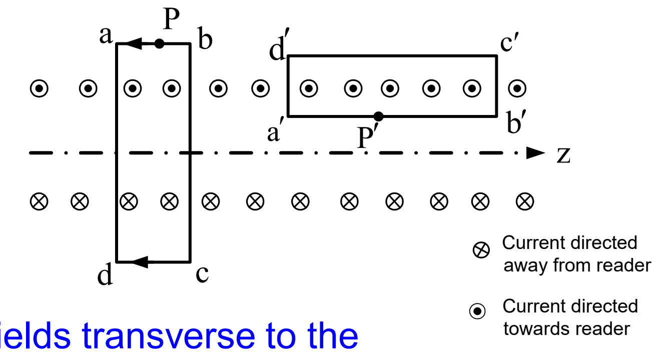
We can also appreciate this with the help of Ampere's circuital law.
For the application of Ampere's circuital law, take a rectangular path abcd in the cross-sectional plane passing through the point P outside the solenoid where the magnetic field is sought. We purposely take the side cd of the rectangle to be at a different distance from its side ab

$$\int_{abcd} \vec{H}\cdot d\vec{l} = \int_{ab} \vec{H}\cdot d\vec{l} + \int_{bc} \vec{H}\cdot d\vec{l} + \int_{cd} \vec{H}\cdot d\vec{l} + \int_{da} \vec{H}\cdot d\vec{l} = 0 \leftarrow \oint_{l} \vec{H}\cdot d\vec{l} = i$$

(Ampere's circuital law applied to path abcd)

Current *i* enclosed by the path abcd is nil since the number of turns corresponding to the current going out of the plane of the paper is equal to the number of turns corresponding to the current going into the plane of the paper.

$$\int_{abcd} \vec{H}\cdot d\vec{l} = \int_{ab} \vec{H}\cdot d\vec{l} + \int_{bc} \vec{H}\cdot d\vec{l} + \int_{cd} \vec{H}\cdot d\vec{l} + \int_{da} \vec{H}\cdot d\vec{l} = 0$$



Magnetic fields transverse to the solenoid axis due to the current turns cancel out and therefore the integrands of the second and fourth integrals each becomes equal to zero.

$$\int_{ab} \vec{H}\cdot d\vec{l} + \int_{cd} \vec{H}\cdot d\vec{l} = 0$$

$$\int_{ab} H_{\text{outside}}\vec{a}_z\cdot(dl\vec{a}_z) + \int_{cd} H'_{\text{outside}}\vec{a}_z\cdot(-dl\vec{a}_z) = 0$$

H_{outside} and H'_{outside} are the magnitudes of magnetic fields at the sides ab and cd respectively which are each along z but oppositely directed

$$H_{\text{outside}}\int_{ab} dl - H'_{\text{outside}}\int_{cd} dl = 0$$

$\text{ab} = \text{cd} = l$, say

$$H_{\text{outside}}l - H'_{\text{outside}}l = 0$$

$$H_{\text{outside}} = H'_{\text{outside}}$$

Contradicts the fact that if present H_{outside} will be different from H'_{outside} since ab and cd are at different distances from the solenoid axis.

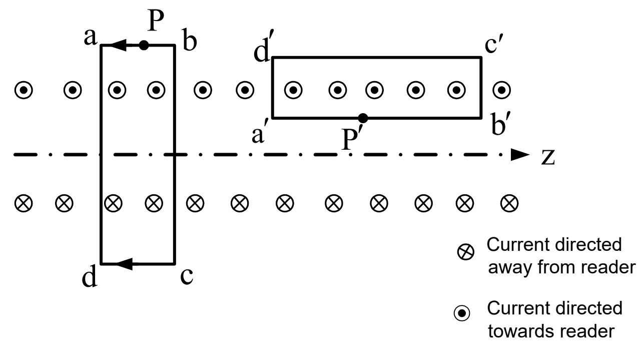
Therefore, we have to put H_{outside} and H'_{outside} each equal to nil.

$$H_{\text{outside}} = H'_{\text{outside}} = 0$$

(magnetic field outside the solenoid is nil)

Magnetic field inside the solenoid

Rectangular path $a'b'c'd'$ passes through the point P' and encloses the currents of the turns out of paper directed towards reader.



$$\oint_l \vec{H}\cdot d\vec{l} = \text{current enclosed (Ampere's circuital law)}$$

$$\int_{a'b'c'd'} H\cdot d\vec{l} = \int_{a'b'} \vec{H}\cdot d\vec{l} + \int_{b'c'} \vec{H}\cdot d\vec{l} + \int_{c'd'} \vec{H}\cdot d\vec{l} + \int_{d'a'} \vec{H}\cdot d\vec{l} = nli$$

$a'd' = b'c' = l$

$n =$ number of turns per unit length of the solenoid

$i =$ current through the solenoid

$$\int_{d'a'} \vec{H}\cdot d\vec{l} = \int_{a'b'} \vec{H}\cdot d\vec{l} = \int_{c'd'} \vec{H}\cdot d\vec{l} = 0$$

Magnetic field outside the solenoid being zero, the integrand of the third term becomes zero. Magnetic fields transverse to the solenoid axis due to the current turns canceling out, the integrands of the second and fourth terms each become nil.

$$\int_{a'b'} \vec{H}\cdot d\vec{l} = nli \leftarrow \begin{cases} \vec{H} = H\vec{a}_z \\ d\vec{l} = dl\vec{a}_z \end{cases}$$

$$\int_{a'b'} H\vec{a}_z \cdot dl\vec{a}_z = nli$$

$$\int_{a'b'} dl = l$$

$$H\int_{a'b'} dl = nli \longrightarrow Hl = nli \longrightarrow H = ni$$

$$\vec{H} = H\vec{a}_z$$

$$\vec{H} = ni\vec{a}_z$$

Expression for magnetic field inside the solenoid

Inductance of a solenoid

$\vec{H} = ni\vec{a}_z$ (magnetic field inside the solenoid)

$\vec{B} = \mu_0 \vec{H} = \mu_0 ni\vec{a}_z$ (magnetic flux density inside the solenoid found thus uniform over the cross section of the solenoid)

$\phi_{B,\text{ singlet turn}} = \int_S \vec{B}\cdot\vec{a}_n dS$ (magnetic flux linked with a single turn of the solenoid)

$\int_S dS = \alpha$ (cross-sectional area of the solenoid)

$\phi_{B,\text{ singlet turn}} = \int_S \mu_0 ni\vec{a}_z \cdot \vec{a}_z dS = \mu_0 ni \int_S dS = \mu_0 ni\alpha$ (magnetic flux linked with a single turn of the solenoid)

$\phi_B = (\phi_{B,\text{ singlet turn}})(nl)$ (magnetic flux linked with *nl* turns of length *l* of the solenoid, *n* being the number of turns per unit length)

$\phi_B = (\phi_{B,\text{ singlet turn}})(nl) = (\mu_0 ni\alpha)(nl)$ (Weber or Wb)

$L = \frac{\phi_B}{i} = \frac{(\mu_0 ni\alpha)(nl)}{i} = \mu_0 n^2 \alpha l$ (Wb/A or henry or H)

(expression for inductance of a solenoid)

That the practical unit of permeability is H/m is clear from this expression for inductance.

Inductance per unit length of a coaxial cable

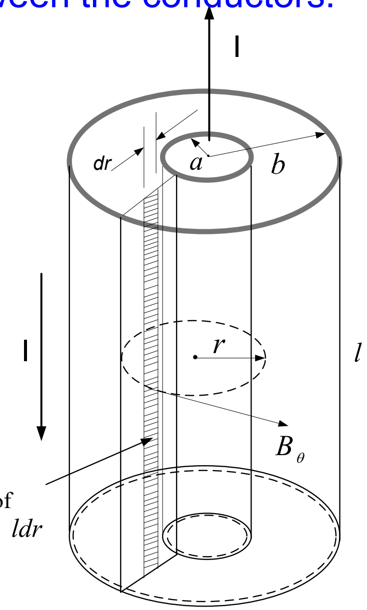
Apply Ampere's circuital law to a closed circular path on a cross-sectional plane perpendicular to the axis of the cable of radius r between *r* = *a* (radius of the inner conductor) and *r* = *b* (radius of the outer conductor) carrying current *I*. You can get the following expression for magnetic flux density in the region between the conductors:

$$B_\theta (= \mu_0 H_\theta) = \frac{\mu_0 I}{2\pi r}$$

$$d\phi_B = B_\theta l dr$$

(element of magnetic flux density linked with a rectangular element of strip of length *l*, width *dr* and area *ldr* in the region between the conductors on a radial cross-sectional plane)

$$d\phi_B = \frac{\mu_0 I l}{2\pi}\frac{dr}{r}$$



Magnetic flux linked with the rectangular area on a radial plane between the conductors can be obtained by integration as follows:

$$\phi_B = \int d\phi_B = \frac{\mu_0 I}{2\pi} l \int_a^b \frac{dr}{r} = \frac{\mu_0 I}{2\pi} l [\ln r]_a^b$$

$$= \frac{\mu_0 I}{2\pi} l (\ln b - \ln a) = \frac{\mu_0 I}{2\pi} l \ln \frac{b}{a}$$

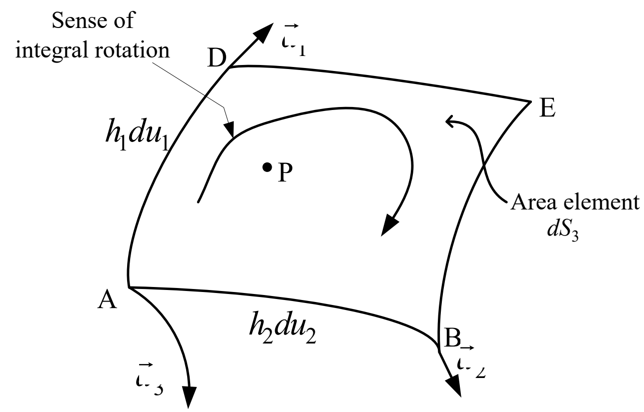
$$\text{Inductance} = \frac{\phi_B}{I} = \frac{\frac{\mu_0 I}{2\pi} l \ln \frac{b}{a}}{I} = \frac{\mu_0}{2\pi} l \ln \frac{b}{a}$$

$$\text{Inductance per unit length} = \frac{\mu_0}{2\pi} \ln \frac{b}{a}$$

(expression for inductance and inductance per unit length of a coaxial cable)

Ampere's circuital law in differential form

In electrostatics, we obtained Poisson's equation in point form or differential form from Gauss's law in integral form. Similarly, in magnetostatics, we can obtain Ampere's circuital law in differential or point form from its integral form.



$\oint_l \vec{H}\cdot d\vec{l} = i$ (Ampere's circuital law in integral form)

$i \cong \vec{J}.\vec{a}_3 dS_3$ current density regarded approximately as constant over element of area dS_3

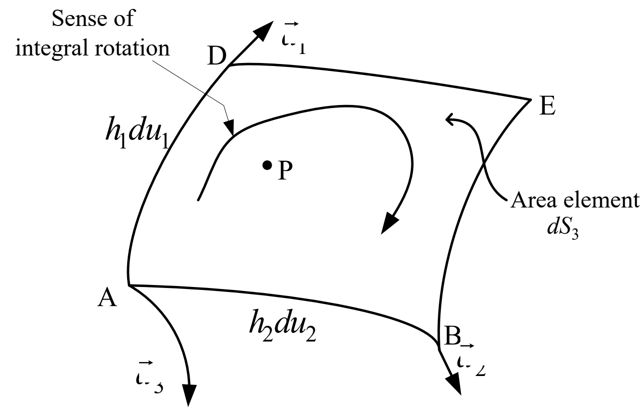
Apply Ampere's circuital law to a closed element of area $dS_3 (= h_1 du_1 h_2 du_2)$ which is taken normal to the unit vector $\vec{a}_3$ in generalised curvilinear system of coordinates, and which encloses the point P where to obtain Ampere's circuital law in point or differential form.

$$\oint_l \vec{H}\cdot d\vec{l} \cong \vec{J}.\vec{a}_3 dS_3$$

$$\oint_l \vec{H} \cdot d\vec{l} \cong \vec{J}.\vec{a}_3 dS_3$$

$$\frac{\oint_l \vec{H} \cdot d\vec{l}}{dS_3} \cong \vec{J}.\vec{a}_3$$

$$\vec{J} = J_1\vec{a}_1 + J_2\vec{a}_2 + J_3\vec{a}_3$$



$$\frac{\oint_l \vec{H} \cdot d\vec{l}}{dS_3} \cong (J_1\vec{a}_1 + J_2\vec{a}_2 + J_3\vec{a}_3).\vec{a}_3 = J_3$$

In the limit $\Delta S_3 \to 0$, corresponding to the area element shrinking to the point P, the current density may be taken as exactly constant and the approximation can be removed from the sign of equality.

$$\underset{\Delta S_3 \to 0}{Lt} \frac{\oint_l \vec{H} \cdot d\vec{l}}{dS_3} = J_3$$

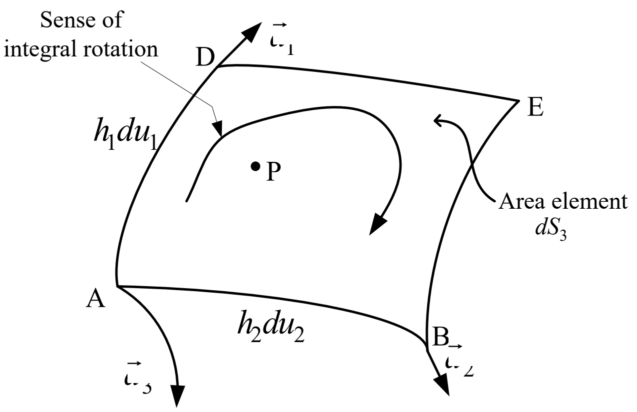
(by the definition of the component of the curl of a vector)

$$(\nabla \times \vec{H})_3 = J_3$$

We have obtained

$$(\nabla \times \vec{H})_3 = J_3$$

(taking the area element perpendicular to $\vec{a}_3$)



Similarly, we can obtain

$$(\nabla \times \vec{H})_1 = J_1$$

(taking the area element perpendicular to $\vec{a}_1$)

Similarly, we can also obtain

$$(\nabla \times \vec{H})_2 = J_2$$

(taking the area element perpendicular to $\vec{a}_2$)

Combining the components we then obtain the differential form of Ampere's circuital law as follows

$$(\nabla \times \vec{H})_1 \vec{a}_1 + (\nabla \times \vec{H})_2 \vec{a}_2 + (\nabla \times \vec{H})_3 \vec{a}_3 = J_1 \vec{a}_1 + J_2 \vec{a}_2 + J_3 \vec{a}_3$$

$$\nabla \times \vec{H} = \vec{J}$$

(Ampere's circuital law in differential form)

Lorentz force on a moving point charge in a magnetic field

A point charge moving with a velocity in a region of magnetic field experiences a force called Lorentz force given by

$\vec{F} = q\vec{v} \times \vec{B}$ (Lorentz force)

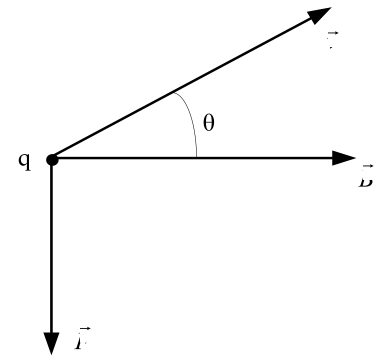
$q =$ amount of point charge

$\vec{v} =$ velocity of point charge

$\vec{B} =$ magnetic flux density in which point charge is placed

$F = qvB\sin\theta$ is the magnitude of $\vec{F}$

Direction of $\vec{F}$ is the direction of the cross product $\vec{v} \times \vec{B}$



Cyclotron frequency

Show that an electron when it is shot with a dc velocity perpendicular to a uniform dc magnetic field of flux density of magnitude *B* executes a circular motion with an angular frequency called the electron cyclotron frequency.

You can balance Lorentz force with the centrifugal force to obtain

$$|e|vB = \frac{mv^2}{r}$$

$$\downarrow$$

$$v = \frac{|e|Br}{m} = |\eta|Br$$

$|e| =$ magnitude of electronic charge

$m =$ electronic mass

$|\eta| =$ magnitude of charge-to-mass ratio of electron

$v =$ velocity of electron in circular motion

$r =$ radius of circular motion of electron

Time period *T* of electronic circular motion is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{|\eta|Br} = \frac{2\pi}{|\eta|B} \longrightarrow \omega_c = \frac{2\pi}{T} = \frac{2\pi}{\frac{2\pi}{|\eta|B}} = |\eta|B$$

(angular electron cyclotron frequency)

Relation between the current density, volume charge density and beam velocity in a beam of charge flow

Consider a current due to the flow of charge particles, for instance electrons, taken as point charges all with a constant velocity *v*. This current is called the conduction current in a conducting medium or convection current in a fee-space medium.

Consider two identical cross-sectional planes each of cross-sectional area α separated by a distance numerically equal to *v* being the distance a charged particle covers during a second. Consequently, all the electrons within the volume $v\alpha$ will flow through a cross-sectional plane of area α in a second. The number of such electrons is equal to the number of electrons per unit volume *n* multiplied by the volume $v\alpha$, that is $n\,v\alpha$. Multiplying this number $n\,v\alpha$ by the charge *e* carried by each electron we get the current $i = n\,v\alpha\,e$ through the cross-sectional area α and dividing *i* by α we get current density *J:*

$$i = nv\alpha e \qquad\qquad ne = \rho \text{ (volume charge density)}$$

$$J = (nv\alpha e)/\alpha = nev = \rho v \text{ (current density)}$$

(relation between current density, volume charge density and beam velocity in a beam of charge flow)

Force on a current carrying conductor placed in a magnetic field

Consider a current element placed in a region of a steady magnetic field (a wire of infinitesimal length *dl* and cross-sectional area α through which a direct current *i* passes. Let us find the element of Lorentz force on the moving charge particles constituting the current element. In the length element *dl*, the number of charge particles is $(n)(dl\alpha) = n\,dl\alpha$, where *n* is the number of charge particles per unit volume of the current element and $dl\alpha$ is its volume. Magnetic field is assumed over the length element

$\vec{F} = q\vec{v}\times\vec{B}$ (Lorentz force)

$q = -|e|$

$|e|$ is the magnitude of electronic charge carried by each of $ndl\alpha$ number of electrons

Element of Lorentz force on $ndl\alpha$ number of electrons:

$$d\vec{F} = -ndl\alpha|e|\vec{v}\times\vec{B}$$

$\vec{v} = v\vec{a}_v$

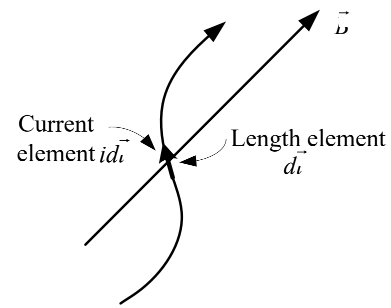
$\vec{a}_v$ is the unit vector in the direction of $\vec{v}$

$$d\vec{F} = -ndl\alpha|e|v\vec{a}_v\times\vec{B}$$

$d\vec{l} = dl\vec{a}_v$

$i = nv\alpha e = -nv\alpha|e|$

$$d\vec{F} = (-nv\alpha|e|)(dl\vec{a}_v)\times\vec{B}$$



$d\vec{F} = i\,d\vec{l}\times\vec{B}$ (element of force on a current element carrying a direct current in a magnetic field)

Magnetic field even if it is non-uniform may be regarded as constant over the element of length *dl*. However, we have to integrate the element of force over the entire length of the conductor to find the force on it due to the magnetic field:

$$\vec{F} = \int d\vec{F} = \int i\,d\vec{l}\times\vec{B}$$

Appreciate that like currents attract and unlike currents repel.

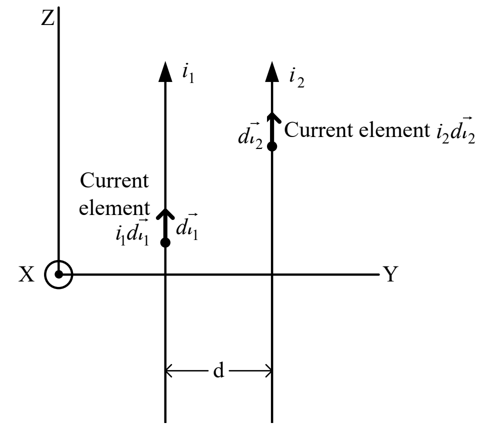
Consider two long straight parallel conducting wires carrying direct currents i_1 and i_2 each parallel to z axis. You can express the force on a current element on the wire carrying current i_2 due to the magnetic field of the current i_1 as

$$d\vec{F}_{21} = i_2\, d\vec{l}_2 \times \vec{B}_{21}$$

$d\vec{F} = i\, d\vec{l} \times \vec{B}$ (element of force on a current element carrying a direct current in a magnetic field)

$$\vec{B}_{21} = -\mu_0 \frac{i_1}{2\pi d}\vec{a}_x \text{ (obtainable using Ampere's circuital law)}$$

$$d\vec{l}_2 = dl_2\vec{a}_z$$



$$d\vec{F}_{21} = i_2\,(dl_2\vec{a}_z)\times(\mu_0 \frac{i_1}{2\pi d}\vec{a}_x)$$

$$d\vec{F}_{21} = i_2\,(dl_2)(-\mu_0 \frac{i_1}{2\pi d})(\vec{a}_z \times \vec{a}_x)$$

$$d\vec{F}_{21} = -i_2\,(dl_2)(\mu_0 \frac{i_1}{2\pi d})(\vec{a}_y)$$

$$d\vec{F}_{21} = -\frac{\mu_0 i_1 i_2\, dl_2}{2\pi d}\vec{a}_y$$

Negative sign in the above force expression indicates that the force experienced by the wire carrying current i_2 is in the negative y direction, that is, towards the wire carrying current i_1. In other words, the force between the wires is that of attraction which means that 'like currents attract'. If the direction of current i_2 is reversed to make the two currents unlike, the direction of the element of length vector on the wire carrying i_2 will also reverse giving $d\vec{l}_2 = dl_2\vec{a}_z$

which in turn gives

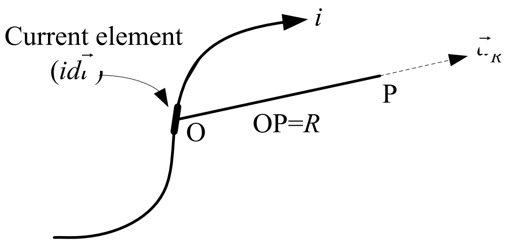
$$d\vec{F}_{21} = \frac{\mu_0 i_1 i_2\, dl_2}{2\pi d}\vec{a}_y$$

which indicates that the force is in the positive y direction or the force is that of attraction. This, in other words means that 'unlike currents attract'.

Magnetic Vector Potential due to a Steady Current

Finding magnetic field from magnetic vector potential is an alternative approach to Biot-Savarts' law. We will see that magnetic field can be found from magnetic vector potential as electric field can be found from electric potential.

Recall Biot-Savarts' law:



$$
\left.
\begin{aligned}
d\vec{H} &= \frac{i\,d\vec{l}\times\vec{a}_R}{4\pi R^2} \\
d\vec{B} &= (\mu_0 d\vec{H}) = \mu_0 \frac{i\,d\vec{l}\times\vec{a}_R}{4\pi R^2}
\end{aligned}
\right\} \leftarrow \quad \vec{a}_R = \vec{R}/R
$$

$$\downarrow$$

$$d\vec{B} = \frac{\mu_0}{4\pi} i\,d\vec{l}\times\frac{\vec{R}}{R^3} \quad \leftarrow \quad \nabla\left(\frac{1}{R}\right) = -\frac{1}{R^3}\vec{R}$$

$$\downarrow$$

$$d\vec{B} = -\frac{\mu_0}{4\pi} i\,d\vec{l}\times\nabla\left(\frac{1}{R}\right) \quad \leftarrow \quad \left\{
\begin{aligned}
&\vec{G}\times\nabla\psi = \psi\nabla\times\vec{G} - \nabla\times(\psi\vec{G}) \ \ \text{(vectoridentity)} \\
&\vec{G} = i\,d\vec{l} \\
&\psi = 1/R
\end{aligned}
\right.$$

$$\downarrow$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\left[\nabla\times\left(\frac{1}{R} i\,d\vec{l} - \frac{1}{R}\nabla\times i\,d\vec{l}\right)\right]$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\left[\nabla\times\left(\frac{1}{R}\,i\,d\vec{l} - \frac{1}{R}\nabla\times i\,d\vec{l}\right)\right]$$

However, you may note that the second term of the right-hand side of the above expression becomes zero since in that term the curl operation is taken with respect to the field-point variables (*x*,*y*,*z*) while the current element term on which this operation is taken is represented by the source point variables ($x^{/}$,$y^{/}$,$z^{/}$, say): $\nabla\times d\vec{l} = 0$

$$d\vec{B} = \nabla\times\left(\frac{\mu_0}{4\pi}\frac{i\,d\vec{l}}{R}\right) = \nabla\times d\vec{A}$$

where the element of vector potential die to a current element turns out to be

$$d\vec{A} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{l}}{R}$$

We can find the magnetic vector potential by integration:

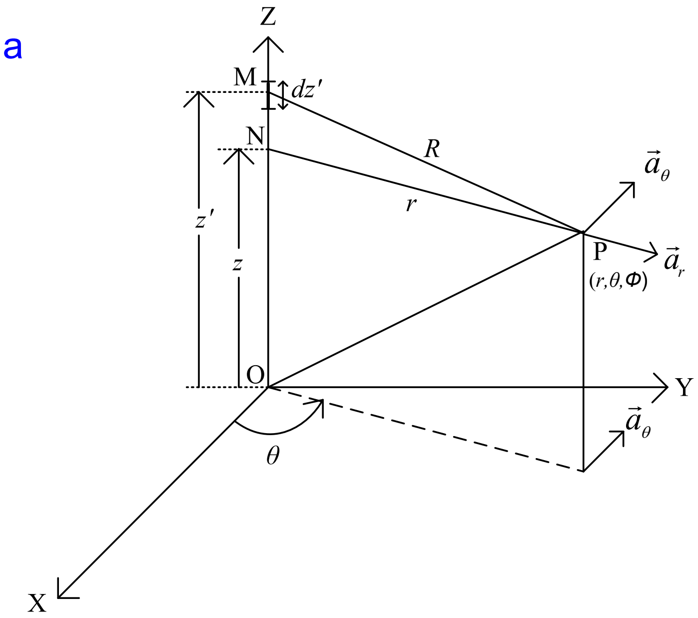
$$\vec{A} = \int d\vec{A} = \int \frac{\mu_0}{4\pi}\frac{i\,d\vec{l}}{R}$$

$$\downarrow$$

$$\vec{B} = \int d\vec{B} = \int \nabla\times d\vec{A} = \nabla\times\vec{A}$$

Find magnetic field due to a long current carrying straight wire using vector potential approach

Consider current element at the point M on z axis at a distance z' from the origin O. The problem enjoys cylindrical symmetry. Element of magnetic vector potential due to the current element at P (r,θ,z) is



$$d\vec{A} = \frac{\mu_0}{4\pi}\frac{i\,dz'\,\vec{a}_z}{R}$$

$$R = \mathrm{PM} = [(\mathrm{NM})^2 + (\mathrm{NP})^2]^{1/2} = [(z'-z)^2 + r^2]^{1/2}$$

$$d\vec{A} = \frac{\mu_0}{4\pi}\frac{i\,dz'\,\vec{a}_z}{[(z'-z)^2 + r^2]^{1/2}}$$

$$\vec{B} = \int \nabla\times d\vec{A} = \frac{\mu_0 i}{4\pi}\int \nabla\times\frac{dz'\vec{a}_z}{[(z'-z)^2 + r^2]^{1/2}}$$

Expansion of curl in the right hand side in cylindrical system of coordinates and remembering azimuthal symmetry of the problem $\partial/\partial\theta = 0$

$$\vec{B} = \nabla\times\frac{dz'\vec{a}_z}{[(z'-z)^2 + r^2]^{1/2}} = -\frac{\partial}{\partial r}\frac{dz'}{[(z'-z)^2 + r^2]^{1/2}}\vec{a}_\theta$$

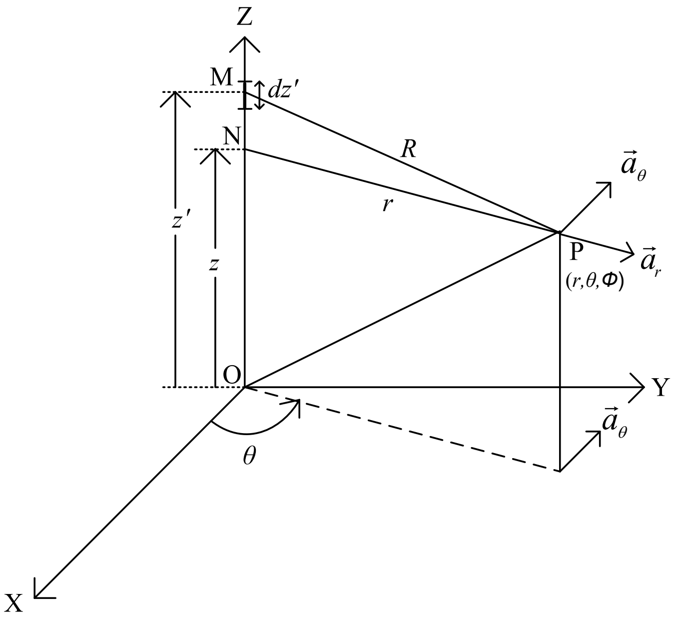
$$R = \mathrm{PM} = [(z'-z)^2 + r^2]^{1/2}$$

$$\vec{B} = \frac{\mu_0 i}{4\pi}\int\frac{r}{R^3}dz'\vec{a}_\theta$$

$$\vec{B} = \frac{\mu_0 i}{4\pi} \int \frac{r}{R^3} dz' \vec{a}_\theta$$ ← Integration limits taken as z/ = -∞ and z/ = ∞ for a long wire

$$\vec{B} = \frac{\mu_0 i}{4\pi} \int_{z'=-\infty}^{z'=\infty} \frac{r}{R^3} dz' \vec{a}_\theta$$

$$\begin{cases} z' - z = r\tan\psi \\ dz' = r\sec^2\psi d\psi \\ R = r\sec\psi \end{cases}$$



$$\vec{B} = \frac{\mu_0 i}{4\pi} \int_{\psi=-\pi/2}^{\psi=\pi/2} \frac{r}{r^3\sec^3\psi} r\sec^2\psi\, d\psi\, \vec{a}_\theta = \frac{\mu_0 i}{4\pi r} \int_{\psi=-\pi/2}^{\psi=\pi/2} \cos\psi\, d\psi\, \vec{a}_\theta$$

$$\vec{B} = \frac{\mu_0 i}{4\pi r} [\sin\psi]_{-\pi/2}^{\pi/2} \vec{a}_\theta$$

$$= \frac{\mu_0 i}{4\pi r} [\sin(\pi/2) - \sin(-\pi/2)] \vec{a}_\theta$$

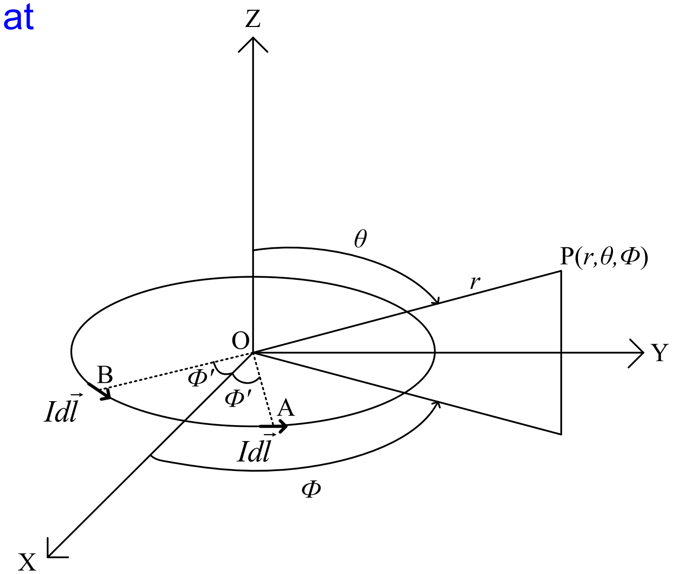
$$= \frac{\mu_0 i}{4\pi r} [1 - (-1)] \vec{a}_\theta = \frac{\mu_0 i}{4\pi r} 2\vec{a}_\theta$$

$$\vec{B} = \frac{\mu_0 i}{2\pi r} \vec{a}_\theta$$ ← $\vec{H} = \vec{B} / \mu_0$

$$\vec{H} = \frac{i}{2\pi r} \vec{a}_\theta$$

(expression for ,agnetic field due to a long current carrying straight wire which is one and the same as that obtained by Biot-Savarts law)

You can find magnetic field due to a circular current loop at a point off the axis using the magnetic vector potential approach however now treating the problem in spherical polar system of coordinates unlike cylindrical system of coordinates used to treat the problem of a long straight current carryong wire.



Hope you will be motivated to deduce the following expression using the vector potential approach in terms of the magnetic dipole moment $p_m = i\pi a^2$, the other symbols having their usual significance (see the text of the book for the details of deduction):

$$\vec{B} = \frac{\mu_0 i\pi a^2}{4\pi r^3}(2\cos\theta\,\vec{a}_r + \sin\theta\,\vec{a}_\theta)$$

$$\vec{H} = \frac{1}{4\pi r^3}p_m(2\cos\theta\vec{a}_r + \sin\theta\vec{a}_\theta)$$

(expressions which become identical with the expressions predicted earlier from the corresponding expressions derived for electric field quantities—obtained by drawing an analogy between the electric and magnetic dipoles)

Summarising Notes

√ Basic concepts of static magnetic fields or magnetostatics have been developed.

√ Magnetostatic quantities analogous to the corresponding electrostatic quantities have identified.

√ Coulomb's and Gauss's laws as well as Poisson's and Laplace's equations of magnetostatics have been developed on the same line of electrostatics.

√ Absence of free magnetic charges (poles) makes it possible to write Gauss' law and Poisson's equation of magnetostatics analogous to those of electrostatics.

√ Gauss's law of magnetostatics and Poisson's equation of magnetostatics have been appreciated as the manifestation of the absence of free magnetic charges (poles) or that of the continuity of magnetic flux lines.

√ Biot-Savart's law predicts magnetic field due to a current element as does Coulomb's law predict electric field due to a point charge.

√ Use of Biot-Savart's law has been demonstrated in illustrative examples, for instance, in the problem of finding the magnetic fields due to a long filamentary steady current, a filamentary current of finite length, a circular loop of current and a polygonal loop of current.

√ Ampere's law has made the problem of finding the magnetic field due to a steady current simpler in problems that enjoy geometrical symmetry, as has done Gauss's law in electrostatic problems.

√ Ampere's circuital law in integral form has been extended to derive the law in differential form.

√ Force experienced by a moving point charge placed in a magnetic field is given by Lorentz force equation.

√Concept of Lorentz force equation has been extended to find the force on a current carrying conductor placed in a magnetic field.

√ Concept of magnetic vector potential due to a steady current has been developed and its application to finding magnetic field as an alterative to Biot-Savarts's law discussed.

Readers are encouraged to go through Chapter 4 of the book for more topics and more worked-out examples and review questions.