Engineering Electromagnetics Essentials

Chapter 11

Summary

Electromagnetics is of relevance and importance to understanding the concepts of electrical, electronics and communication engineering.

Historical timeline of the development in the subject at the outset of the book motivates readers to take part in the further development of the subject.

The topics of the book have been chosen such that by reading them readers would grasp most of the essence of engineering electromagnetics within a reasonable time without being lost in lengthy exercises on each of these topics (Chapter 1).

- The following objectives of writing the book set at the outset in the introductory have been fulfilled:
  - ♣ The subject has been made easy to understand
  - ♣ Attempt has been made to convey the overall essence of the subject to students
  - ♣ The book has uncovered in a reasonable period of time the concepts of the subject as are required to appreciate the relevant engineering application of the subject.

• The vector calculus expressions for the gradient of a scalar and the divergence and curl of a vector have been derived from first principles in generalised curvilinear system of coordinates (Chapter 2).

• The vector calculus expressions in generalised curvilinear system of coordinates are easy to remember in the sense that if we can remember one of the terms of an expression, we can write the remaining terms simply by permutation.

• The vector calculus expressions in generalised curvilinear system of coordinates have been interpreted for the rectangular, cylindrical and spherical polar systems of coordinates to deal with electromagnetic problems that enjoy rectangular, cylindrical and spherical symmetry respectively.

•The expression for the Laplacian of a scalar or a vector quantity is obtained by combining the gradient and the divergence expressions (Chapter 2).

- Coulomb' law can be used to find the force between static point charges and the static electric field due to a point charge or a charge distribution (Chapter 3).

- Gauss's law can be used to find the electric field in electrostatic problems that enjoy geometrical symmetry.

- Elctrostatic field has been found as the negative of the gradient of potential and applied to problems such as finding the electric field in a capacitor (Chapter 3).

- Poisson's and Laplace's equations have been deduced and read in the desired system of coordinates and used to find the electric field and potential due to a line charge, a sheet of charge and a charged conductor (Chapter 3).

- Poisson's equation can be used to deduce Child-Langmuir's law applied to charge-flow problems, for instance, to find a relation between the anode voltage, the anode current, and the distance between the cathode and the anode in a vacuum diode (Chapter 3).

- Magnetostatic quantities analogous to the corresponding electrostatic quantities can be identified and used to develop Coulomb's and Gauss's laws as well as Poisson's and Laplace's equations of magnetostatics on the line of electrostatics (Chapter 4).

- Gauss's law of magnetostatics and Poisson's equation of magnetostatics are the manifestation of the absence of free magnetic charges (poles) or that of the continuity of magnetic flux lines.

- Biot-Savart's law gives the magnetic field due to a steady current element, which may be integrated to find the steady magnetic field due to a current distribution and thus takes the same role as Coulomb's law in electrostatics.
- The concept of magnetic vector potential helps in finding the magnetic field due to a steady current.
- Ampere's circuital law can be used to simplify the problem of finding magnetic field in steady magnetic field problems that enjoy geometrical symmetry, the same as Gauss's law in electrostatic problems that enjoy geometrical symmetry (Chapter 4).
- The continuity equation at a point relating the divergence of the current density with the time rate of the variation of volume charge density at the point can be derived from the conservation of charge in a region of charge flow, the concept marking the beginning of the concepts in time-varying fields (Chapter 5).
- The relaxation time of a medium measures how long a charge injected into a medium will stay in the bulk of the medium.
- The expression for the relaxation time can be derived from the continuity equation.
- The relaxation time depends on the conductivity and the permittivity of the medium.
- The relaxation time of a conductor is very short while that of a dielectric is very long; this explains why a conductor can be charged only at its surface and that a dielectric can be charged throughout its volume.

- Faraday's law for time-varying magnetic fields expressed in both its integral and differential forms helps appreciate how an electric field in electromagnetic induction is associated with a time-varying magnetic field and thus how electric and magnetic fields are coupled in time-varying phenomena.
- The concept of the displacement current can be developed with the help of the continuity equation, Poisson's equation and the differential form of Ampere's circuital law.
- The concept of the displacement current helps understand why a capacitor filled with a non-conducting medium allows time-varying current to pass through it (Chapter 5).
- The loss tangent is the tangent of the angle by which the displacement current density in a lossy dielectric fails to differ in phase from $\pi/2$ from the conduction current density (Chapter 5).
- The loss tangent is also the tangent of the angle by which the time-varying current through a capacitor filled with a lossy dielectric fails to differ in phase from $\pi/2$ from the voltage across the capacitor.
- The loss tangent of a lossy dielectric can be expressed in terms of the frequency of a time-periodic electric field and the conductivity and permittivity of the dielectric.
- Maxwell's equations derived in both integral and differential forms are extensively used in electromagnetics.
- We can modify the electrostatic relation between the electric field and the gradient of potential for time-varying fields with the help of one of Maxwell's equations and the concept of magnetic vector potential (Chapter 5).

- Two of Maxwell's equations have in them electric and magnetic fields coupled.

- Maxwell's equations with electric and magnetic fields coupled can be combined, thereby decoupling these fields to obtain the wave equation in electric field and the wave equation in magnetic field (Chapter 6).

- A uniform plane wavefront is characterised by a plane perpendicular to the direction of propagation over which the amplitude of the field representing the wave remains constant, the wavefront being the locus of points having the same phase of the field of the wave.

- The wave equation in electric field and the wave equation in magnetic field each can be solved for studying the characteristics of propagation of a uniform, plane electromagnetic wave propagating through an unbounded free-space medium.

- The unbounded free-space medium supports transverse electromagnetic (TEM) wave.

- The directions of the electric field, magnetic field and propagation of wave are mutually perpendicular to one another (Chapter 6).

- The intrinsic impedance and the wave phase velocity of the medium are each related to the permeability and the permittivity of free-space (Chapter 6).

- The study of propagation of a uniform, plane electromagnetic wave through a semi-infinite conducting medium making a planar interface with a free-space medium gives the expression for the surface resistance and that of the surface impedance, both depending on the wave frequency and the conductivity of the medium.

- The planar-interface approximation can be made with respect to a round wire if the conductivity of the material of the wire and/or the wave frequency is high, for which the skin depth of the material is much smaller than the radius of the round wire.

- Under planar-interface approximation, the ratio of the ac-to-dc resistance of a round wire is equal to the ratio of the wire radius to twice the skin depth of the material of the wire.

- Lower frequencies, say, ~10 kHz, is preferred to higher frequencies, say, ~10 GHz, in view of comparatively lower attenuation of waves at such lower frequencies for sea-water communication as revealed by studying the propagation through unbounded sea-water of finite conductivity and permittivity.

- Study of wave propagation through a medium of charged particles gives the concepts of sky-wave propagations through ionosphere as well as those of space-charge waves and cyclotron waves on an electron beam (Chapter 6).

- General electromagnetic boundary conditions can be deduced at the interface between two media in terms of a unit vector directed from one medium to another (Chapter 7).

While deducing general electromagnetic boundary conditions at the interface between media, two definitions have emerged as follows (Chapter 7):

- The surface charge density is the product of the volume charge density and the infinitesimal thickness over which the charge is spread at the interface in the limit of the infinitesimal thickness tending to zero.
- The current density is the product of the current density and the infinitesimal thickness at the interface over which the current density is significant in the limit of the infinitesimal thickness tending to zero.

General electromagnetic boundary conditions have been interpreted at the dielectric-dielectric interface and at the conductor-dielectric interface based on the following findings:

- ♣ The surface charge density is nil at the dielectric surface for both time-independent and time-dependent situations.
- ♣ A finite surface charge density develops at a conductor surface for both time-independent and time-dependent situations.
- ♣ Electric field or electric displacement is absent in a good conductor for both time-independent and time-dependent situations.

- ♣ Finite magnetic field or magnetic flux density can be established inside a dielectric independently of electric field for both time-independent and time-dependent situations.

- ♣ Magnetic field and the magnetic flux density each become zero inside a conductor for time- dependent situations.

- ♣ Finite magnetic field or magnetic flux density can be established independently of an electric field in a conductor for time-independent situations.

- ♣ Surface current density at a dielectric surface is nil for both time-independent and time-dependent situations.

- ♣ Surface current density at a conductor surface is nil for time-independent situations.

- ♣ Finite surface current density can be established at the surface of a good conductor for time-dependent situations (Chapter 7).

• Using a relevant electromagnetic boundary condition, we can understand the phenomenon of the formation of a standing wave when a uniform plane electromagnetic wave is incident from a free-space region on a conducting surface. in which the minima of the amplitudes of electric field is found to coincide with the maxima of the amplitudes of magnetic field, and vice versa. Also, there is no power flow in such a standing wave (Chapter 7).

- Reflection and refraction of electromagnetic waves at a dielectric-dielectric interface can be studied with the help of the relevant electromagnetic boundary conditions for parallel and perpendicular polarisations.

- Brewster's phenomenon takes place for parallel polarisation in which there is no reflection at a dielectric-dielectric interface for parallel polarisation.

- Total internal reflection at a dielectric-dielectric interface for the angle of incidence greater than the critical angle can be understood for both parallel and perpendicular polarisations.

- Circuit law of parallel resistances can be appreciated from the electromagnetic boundary condition that the tangential component of electric field is continuous at the interface between two media.

- Boundary conditions at the interface between two conducting media yield the law of refraction of current for time-independent situations relating the angles of incidence and refraction of current with the conductivities of the media (Chapter 7).

- Expressions for energy and energy density in electrostatic field can be derived in terms of the electric field and electric displacement (or electric flux density) and the analogous expressions for energy and energy density in magnetostatic field appreciated (Chapter 8).
- Poynting theorem encapsulates the phenomenon of the storage, loss and flow of electromagnetic energy.
- Poynting theorem can be used to appreciate Joule's circuit law for the power loss in a wire of circular cross section and of finite resistance carrying a direct current (Chapter 8).
- Poynting theorem has been

  (i) used to derive the expression for energy density in electric field with reference to the problem of a parallel-plate capacitor of circular cross section and

  (ii) applied to the problem of an inductor in the form of a solenoid of circular cross section to derive an expression for energy density in magnetic field (Chapter 8).
- Complex Poynting theorem gives the concept of time averaged electromagnetic power flow and can be used to study time-averaged power flow through a bounded or an unbounded medium and associated power loss due to the presence of a lossy conducting medium.

- Complex Poynting vector is half the cross product of the electric field vector and the complex conjugate of magnetic field vector, and the ‘time averaged’ complex Poynting vector is the real part of the complex Poynting vector.

- The outward flux of the time averaged complex Poynting vector through a volume enclosure gives the average power going out of the volume enclosure.

- The reactive power flowing into a volume enclosure is of relevance to average energies stored in electric and magnetic fields in the volume has been appreciated.

- Concepts of power flow developed can be applied to

  ♣ develop the concepts of the gain and effective aperture area of an antenna

  ♣ establish Friis transmission equation relating the radiated power from a transmitting antenna to the power delivered to the load connected to a receiving antenna and

  ♣ study of conduction current antennas exemplified by predicting the characteristics of Hertzian infinitesimal dipole, finite-length dipole, antenna arrays, including broadside array, end-fire array and Yagi-Uda array, and loop-current antennas.

  ♣ derive an expression for the power loss per unit area in a conductor in terms of the surface resistance and surface current density of the conductor (Chapter 8).

- The wave equation in a bounded medium can be solved to study the characteristics of a waveguide— a hollow pipe made of a conducting material, which is extensively used for the transmission of power in the microwave frequency range (Chapter 9).

- Waveguides

  ♣ can support transverse electric (TE) mode, which is characterized by non-zero axial magnetic field and zero axial electric field

  ♣ can support transverse magnetic TM mode, which is characterized by non-zero axial electric field and zero axial magnetic field

  ♣ cannot support transverse electromagnetic (TEM).

- We can obtain the characteristic equation or the dispersion relation of a waveguide with the help of field solutions and electromagnetic boundary condition that the tangential component of the electric field is nil at the conducting surface of the waveguide wall.

- One and the same ω-β dispersion relation is obtained for both TE and the TM modes, for both rectangular and cylindrical waveguide, with appropriate interpretation of cutoff frequency ω_c in terms of waveguide dimensions (ω_c being the value of angular frequency ω corresponding to zero value of the phase propagation constant β).

- ω-β dispersion plots of identical nature are generated for rectangular and cylindrical waveguides, irrespective of TE or TM mode of excitation.

- A waveguide behaves as a high-pass filter supporting propagating waves above cutoff frequency ω_c that is related to waveguide dimensions.

- A waveguide supports evanescent mode cutoff frequency ω_c associated with nil component of the average Poynting vector in the direction of wave propagation corresponding to nil power flow in the waveguide (Chapter 9).

- The characteristic parameters of a waveguide are guide wavelength, phase propagation constant, phase velocity, group velocity, and wave impedance, each of them depending on the operating frequency relative to the cutoff frequency of the waveguide. (Chapter 9)

- Dominant mode of a waveguide, characterized by the lowest value of the cutoff frequencies of all the TE_{mn} and the TM_{mn} modes of the waveguide, is mode TE_{10} for a rectangular and mode TE_{11} for a cylindrical waveguide.

- For a rectangular waveguide, the mode number *m* indicates the number of maxima of the field component along the broad dimension of the waveguide, while the mode number *n* indicates the number of maxima of the field component along the narrow dimension of the waveguide. Alternatively, *m* and *n* can be interpreted as the numbers of half-wave field patterns across the broad and the narrow dimensions of the waveguide respectively.

- For a cylindrical waveguide, the mode number *m* indicates the number of half-wave field pattern around the half circumference and *n* indicates the number of positive or negative maxima of the field component across the waveguide radius.

- The expression for the power propagating through a rectangular waveguide above cutoff frequency can be found and interpreted to predict the power handling capability of a waveguide, which shows its dependence on the breakdown voltage of the medium filling the hollow region of the waveguide, the operating frequency and the waveguide dimensions (studied with respect to a rectangular waveguide excited in the dominant mode).

- The expression for the power loss per unit length of the walls of a rectangular waveguide due to the finite resistivity of the material making up the walls has been developed and used to find the expression for the attenuation constant of the waveguide (studied with respect to a rectangular waveguide excited in the dominant mode).

- Attenuation constant of a waveguide depends on the operating frequency and the waveguide dimensions, which should be taken into consideration while choosing the waveguide mode and frequency from the standpoint of lower values of attenuation in the waveguide (Chapter 9).

- A hollow-pipe waveguide of an appropriately chosen length with both its ends either closed by a conductor or kept open makes a waveguide resonator (Chapter 10).

- Transmission line theory, which is an easy approach of treating a waveguide resonator, is worth developing. This has been done encompassing
  - the basic concepts such as distributed line parameters;
  - telegrapher's equation,
  - condition for distortionless transmission,
  - input impedance of the line terminated in a load impedance,
  - characteristic impedance of the line,
  - voltage standing-wave ratio (VSWR) of the line,
  - impedance matching such as in radome for the protection of an antenna and branch-type radar duplexer of a radar system, and
  - theory of Smith chart and its application to transmission line problems to make them simpler.
- Resonator length of the waveguide resonator has been found by transmission line theory as an integral multiple of half the guide wavelength for both closed-ended and open-ended resonators. Its resonant frequency can be found with the help of transmission line theory using the dispersion relation of the waveguide.
- Field solutions and electromagnetic boundary conditions typically for a rectangular closed-ended waveguide resonator has yielded
  - ♣ resonator length that is same as that predicted by the transmission line theory and
  - ♣ additional mode number *p* of the waveguide resonator, to be read with reference to TE_{10}- mode excitation of the waveguide as TE_{10p} mode of the resonator (which may be generalised as TE_{mnp} mode of the resonator with reference to TE_{mn}-mode excitation of the waveguide).

- Field solution and relevant electromagnetic boundary conditions can be used to obtain

  ♣ expression for the time-averaged energy stored in electric and magnetic fields and

  ♣ expression for the power loss in resonator walls (Chapter 10).

- Expression for the quality factor of a resonator in the TE_{101} mode at the resonant frequency, in terms of the resonator dimensions and the surface resistance of the conducting material making up the resonator, can be derived with the help of the expressions for the time-averaged energy stored and power loss in the resonator (Chapter 10).

- Relation between the unloaded quality factor, external quality factor and loaded quality factor of a cavity has been obtained keeping in view some part of energy stored in a cavity being coupled out from it to an external load in practice.

- An alternative expression for quality factor can be arrived at by considering the waveguide resonator as a resonant circuit comprising an inductor, a reactance, a capacitor and a resistor, all connected in parallel. Then the quality factor can be expressed in terms of the resonant frequency of the circuit and the bandwidth of the frequency response of $(1/2)(|Zeq|/R)^2$, i.e., half the value of the square of ratio of the magnitude of the equivalent impedance to resistance of the resonant circuit (Chapter 10).

- ▲ The book covers all the elementary concepts with detailed derivation of the steps of analysis making the learning of the subject interesting and enjoyable.
- ▲ The book has been substantiated by a large number of worked-out examples and appendixes in the text of the chapters.
- ▲ The book provides thought-provoking chapter-end summarising notes and review questions with answers/solutions of narrative, numerical and multiple-choice, objective types.
- ▲ The book inspires students to take up more challenging problems of practical relevance.